

Business Mathematics (BK/IBA) – Quantitative Research Methods I (EBE)
Formula sheet (August 2015)

Summation

$$\sum_{i=1}^n x_i = x_1 + x_2 + \dots + x_n$$

$$\sum_{i=1}^n \sum_{j=1}^m x_{i,j} = \sum_{i=1}^n \left(\sum_{j=1}^m x_{i,j} \right) = \sum_{j=1}^m \left(\sum_{i=1}^n x_{i,j} \right)$$

Derivatives

$$\frac{df(x)}{dx} = f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$\frac{d^2f(x)}{dx^2} = f''(x) = \frac{d}{dx} \left(\frac{df(x)}{dx} \right)$$

$$\frac{\partial f(x,y)}{\partial x} = \lim_{h \rightarrow 0} \frac{f(x+h,y) - f(x,y)}{h}$$

Function	Derivative	Remark
A	0	constant function
$Af(x)$	$Af'(x)$	$A \in \mathbb{R}$
x^a	ax^{a-1}	$a \neq 0$
$f(x) + g(x)$	$f'(x) + g'(x)$	sum rule
$f(x) \times g(x)$	$f'(x)g(x) + f(x)g'(x)$	product rule
$\frac{f(x)}{g(x)}$	$\frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$	quotient rule
$f(g(x))$	$f'(g(x)) \times g'(x)$	chain rule
e^x	e^x	exponential function
$\ln x $	$\frac{1}{x}$	$x \neq 0$, logarithmic function

Descriptive statistics

$$\bar{x} = \frac{1}{n} \sum_{i=1}^n x_i$$

$$s_x^2 = \frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})^2$$

$$s_{x,y} = \frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})$$

$$CV_x = \frac{s_x}{\bar{x}} \quad r_{x,y} = \frac{s_{x,y}}{s_x s_y}$$

Functions and equations

$$a^x = e^{x \ln a}$$
$$ax^2 + bx + c = 0 \Rightarrow x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Integrals

$$F'(x) = f(x) \Leftrightarrow \int f(x) dx = F(x) + C$$
$$\int_a^b f(x) dx = F(b) - F(a)$$

Matrices

$$(\mathbf{AB})' = \mathbf{B}'\mathbf{A}' \quad (\mathbf{A}')^{-1} = (\mathbf{A}^{-1})' \quad (\mathbf{AB})^{-1} = \mathbf{B}^{-1}\mathbf{A}^{-1}$$

Approximations and elasticities

$$f(x) \approx f(a) + f'(a)(x - a) + \frac{1}{2}f''(a)(x - a)^2$$

$$\text{El}_x f(x) = \frac{x}{f(x)} f'(x)$$

Extreme values

$$\left(\frac{\partial^2 f}{\partial x^2}\right)\left(\frac{\partial^2 f}{\partial y^2}\right) - \left(\frac{\partial^2 f}{\partial x \partial y}\right)^2 > 0$$

Constrained optimization

$$\begin{cases} \max f(\mathbf{x}) \\ \text{subject to } \mathbf{g}(\mathbf{x}) = \mathbf{c} \end{cases}$$
$$\mathcal{L}(\mathbf{x}, \boldsymbol{\lambda}) = f(\mathbf{x}) - \boldsymbol{\lambda} \cdot (\mathbf{g}(\mathbf{x}) - \mathbf{c}) \quad \frac{df^*}{dc} = \boldsymbol{\lambda}$$

Curve fitting

$$y = ax + b$$
$$a = \frac{n\sum xy - \sum x \sum y}{n\sum x^2 - (\sum x)^2} \quad b = \frac{\sum y - a\sum x}{n}$$

Linear programming

$$\begin{cases} \max f(\mathbf{x}) = \mathbf{c} \cdot \mathbf{x} \\ \text{subject to } \mathbf{Ax} \leq \mathbf{b} \\ \text{and } \mathbf{x} \geq \mathbf{0} \end{cases}$$