

Definite integrals

Business Mathematics

CONTENTS

The indefinite integral

The definite integral

Properties of definite integrals

Definite integrals as a function

Old exam question



THE INDEFINITE INTEGRAL

Recall that

$$\int f(x)dx = F(x) + C$$

where $F'(x) = f(x)$



THE DEFINITE INTEGRAL

The definite integral of f over the interval $[a, b]$ is defined as

$$\int_a^b f(x)dx = F(b) - F(a)$$

Alternative notations for $F(b) - F(a)$:

- $F(x)]_a^b$
- $[F(x)]_a^b$
- $\left|_a^b F(x)\right.$

Motivation (with applications) will come later



THE DEFINITE INTEGRAL

The definite integral is a number

- in contrast to the indefinite integral, which is a function
- on the interpretation of this number, see Applications of integrals

Once you know the indefinite integral, it is easy to calculate the definite integral

Example:

- let $f(x) = x^2$
- then $\int f(x)dx = \frac{1}{3}x^3 + C$ (because $(\frac{1}{3}x^3 + C)' = x^2$)
- and $\int_1^3 f(x)dx = \frac{1}{3}3^3 - \frac{1}{3}1^3 (\approx 8.667)$

Observe that the unknown constant C is in the indefinite integral, but not in the definite integral



PROPERTIES OF DEFINITE INTEGRALS

Same as those of indefinite integrals

In addition:

- $\int_a^b f(x)dx = -\int_b^a f(x)dx$ (switching boundaries)
- $\int_a^a f(x)dx = 0$ (integrating over "zero" interval)
- $\int_a^b f(x)dx + \int_b^c f(x)dx = \int_a^c f(x)dx$ (additivity)



DEFINITE INTEGRALS AS A FUNCTION

An indefinite integral is a function, while a definite integral is a number

But a definite integral can sometimes be a function

Two important cases:

Integrating a function of two variables

- example: $\int_1^3 e^{x+y} dx = e^y (e^3 - e) = G(y)$

Integrating over a variable integration interval

- example: $\int_1^y e^x dx = e^y - e = H(y)$

Although y is unknown, this is a definite integral, and we should not write the constant C



OLD EXAM QUESTION

10 December 2014, Q1a

If $f(q) = q + q^3$, calculate $\int_{-1}^0 f(q) dq$. (exact)



OLD EXAM QUESTION

22 October 2014, Q1c

Given is the expression $\int_s^t e^{\sqrt{x+z}} dz$. Which statement(s) is (are) correct? (choose one or more)

- (A) The result is a function of s .
- (B) The result is a function of t .
- (C) The result is a function of x .
- (D) The result is a function of z .
- (E) None of the answers above is correct.

