Descriptive statistics

Business Mathematics

VU UNIVERSITY AMSTERDAM
Faculty of Economics and Business Administration

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DATA REDUCTION

If we have n measurements on the same variable x, we can organize this in a data set $x_i, i = 1, \ldots, n$

- or in a data vector $\mathbf{x}$ of size n

This data set can be summarized by means of descriptive statistics (data reduction):

- measures of location (mean, median, etc.)
- measures of disperion (variance, standard deviation, etc.)

MEASURES OF LOCATION (CENTRAL TENDENCY)

Mean:

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

- alternative names: average, arithmetic mean




MEASURES OF DISPERSION (VARIABILITY)

Measure of dispersion (variability)

Variance:

$$s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$$

- also denoted by ${s_x}^2$ to emphasize that it is s^2 of x

Why $\frac{1}{n-1}$ and not $\frac{1}{n}$?
Not clear for this moment, but we will return to this in the statistics course

MEASURES OF DISPERSION (VARIABILITY)

Two more measures of dispersion

In original units:
Standard deviation:

$$\boxed{s = \sqrt{s^2}}$$

- or s_x if there are more data vectors

Relative (dimensionless) measure of dispersion:
Coefficient of variation:

$$\boxed{CV = \frac{s}{\bar{x}}}$$

- or CV_x if there are more data vectors

MEASURES OF ASSOCIATION

Measures of association between two paired data vectors

- so, considering $(x_1, y_1), (x_2, y_2), \cdots, (x_n, y_n)$, condensed in two data vectors $\mathbf{x}$ and $\mathbf{y}$ of equal size

Covariance:

$$s_{x,y} = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})$$

What happens if you calculate the covariance of a data vector and the same data vector?

- $s_{x,x} = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})(x_i - \bar{x}) = {s_x}^2$

so the covariance between x and x is the variance of x

MEASURES OF ASSOCIATION

Alternative (relative, dimensionless) measure of association

Correlation coefficient:

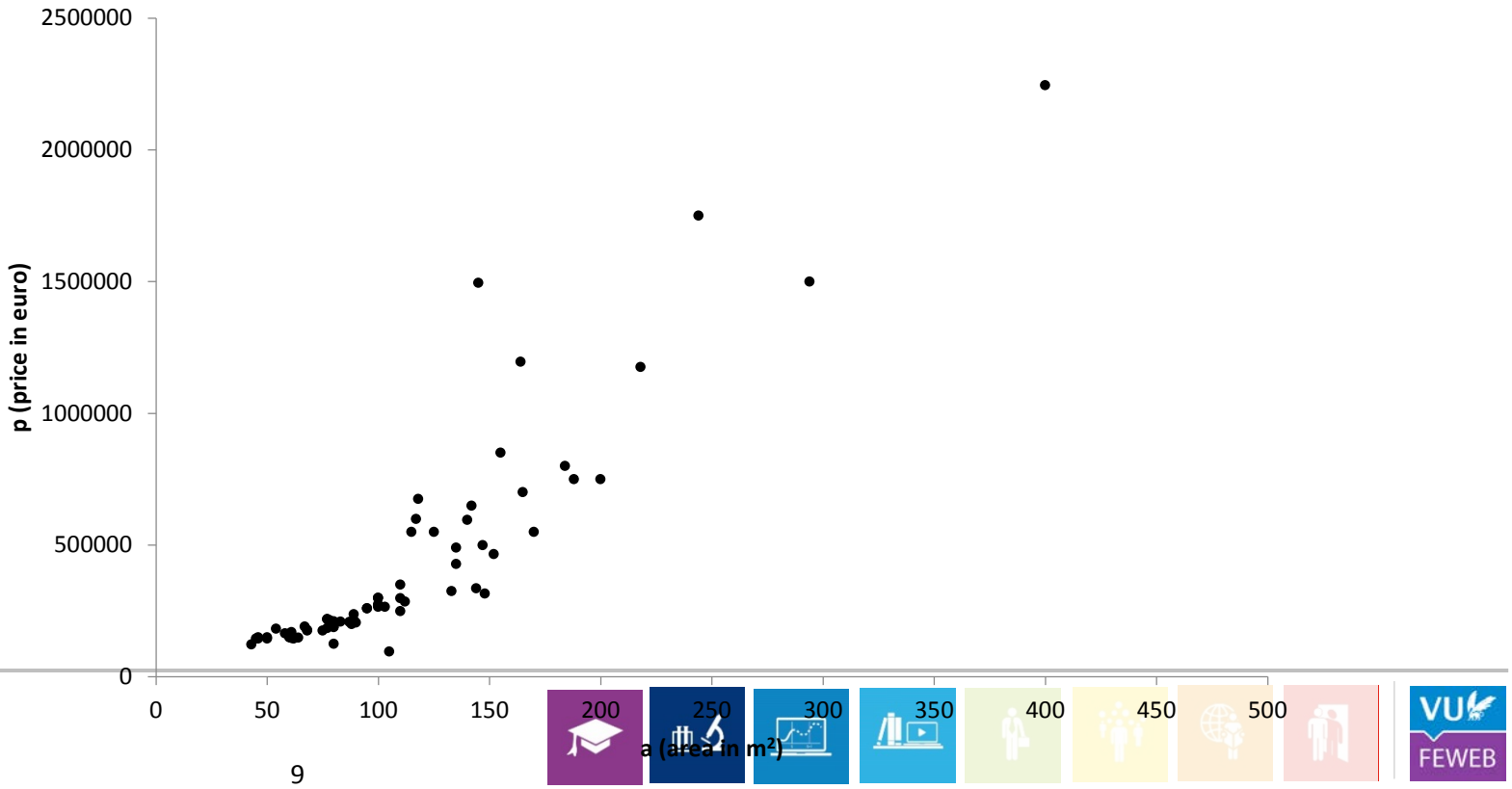
$$r_{x,y} = \frac{s_{x,y}}{s_x s_y}$$

- alternative name: Pearson product-moment correlation coefficient
- always: $-1 \leq r \leq 1$

EXAMPLE ON HOUSING MARKET

Data on Amstelveen 2014 ($n = 71$):

- house prices in €: p: $(649000,125000,599000,145000, \cdots)$
- floor areas in m²: a: $(142,80,117,46, \cdots)$



EXAMPLE ON HOUSING MARKET

Mean: $\bar{p} = 414014$ €; $\bar{a} = 110.4$ m^2

Variance: ${s_p}^2 = 1.69 \times 10^{11}$ €^{2}; ${s_a}^2 = 3673$ m^4

Standard deviation: $s_p = 411864$ €; $s_a = 60.6$ m^2

Coefficient of variation: $CV_p = 0.99$; $CV_a = 0.55$

Covariance: $s_{p,a} = 2.26 \times 10^7$ €×m^2

Correlation coefficient: $r_{p,a} = 0.90$

OLD EXAM QUESTION

22 October 2014, Q2c

Price data for products 1 and 2 are available, by observations on 251 days. The average prices are $\overline{p_1} = 23.6$ euro and $\overline{p_2} = 71.2$ euro, with variances $s_{p_1}^2 = 5.4$ euro^2 and $s_{p_2}^2 = 8.3$ euro^2. The correlation coefficient between the price vectors is $r_{p_1,p_2} = 0.71$. Compute the coefficient of variation for the price data of product 1 (CV_{p_1}) as well as the covariance between the two price vectors s_{p_1,p_2}. (6 points)

OLD EXAM QUESTION

27 March 2015, Q1j

Someone claims that a data vector $\mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \\ \dots \\ x_n \end{pmatrix}$ has $\bar{x} = -23.6$ and $\sum_{i=1}^{n} x_i = 498.3$. Which statements can you deduce? (choose one or more)

(A) $n > 10$

(B) all data elements x_i are ≥ 0

(C) there must be a mistake in $\bar{x}$ and/or $\sum_{i=1}^{n} x_i$

(D) no data element x_i is > 500

(E) None of the above is correct.