Elasticities and approximations

Business Mathematics

VU UNIVERSITY AMSTERDAM
Faculty of Economics and Business Administration

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ELASTICITIES

Let $D(p)$ denote the demand for a commodity at price p. Changing the price by Δp yields a relative (or proportional) change

$$\frac{D(p+\Delta p)-D(p)}{D(p)}$$

The relative change of price is $\frac{\Delta p}{p}$

Hence the proportional change of demand on relative change in price is

$$\frac{p}{\Delta p}\frac{D(p+\Delta p)-D(p)}{D(p)}=\frac{p}{D(p)}\frac{D(p+\Delta p)-D(p)}{\Delta p}$$

ELASTICITIES

Letting Δp tend to zero yields $\frac{p}{D(p)}\frac{d}{dp}D(p)$, which is called the elasticity of $D(p)$ with respect to p.

Definition

Let $f(x)$ be differentiable at x and $f(x) \neq 0$, then

$$\boxed{\mathrm{El}_x f(x) = \frac{x}{f(x)} f'(x)}$$

is the elasticity of f with respect to x

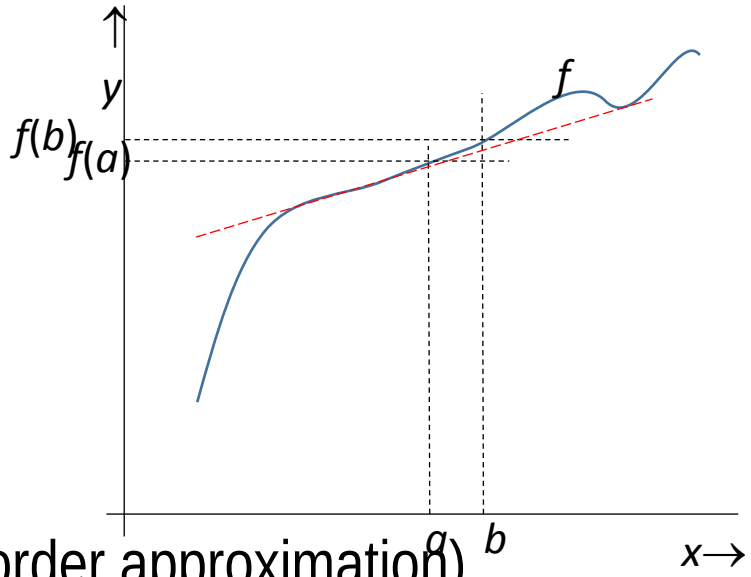
El_x is not a variable, but an operator that works on a function, like $\frac{d}{dx}$

ELASTICITIES

Example:

- let $f(x) = Ax^b$ for some constant $A \neq 0$ and b.
- then $\mathrm{El}_x f(x) = b$.
- interpretation: if x increases by 1%, $f(x)$ increases approximately by $b\%$.

APPROXIMATIONS



Let $f(x)$ be a "smooth" function

Suppose we know $f(a)$

What is the best guess of $f(b)$

- where b is "close to" a?

Easiest guess

- $f(b) \approx f(a)$

Better guess (linear approximation, or first-order approximation)

- $\boxed{f(b) \approx f(a) + f'(a)(b-a)}$

Even better guess (quadratic approximation, or second-order approximation)

- $f(b) \approx f(a) + f'(a)(b-a) + \frac{1}{2}f''(a)(b-a)^2$

APPROXIMATIONS

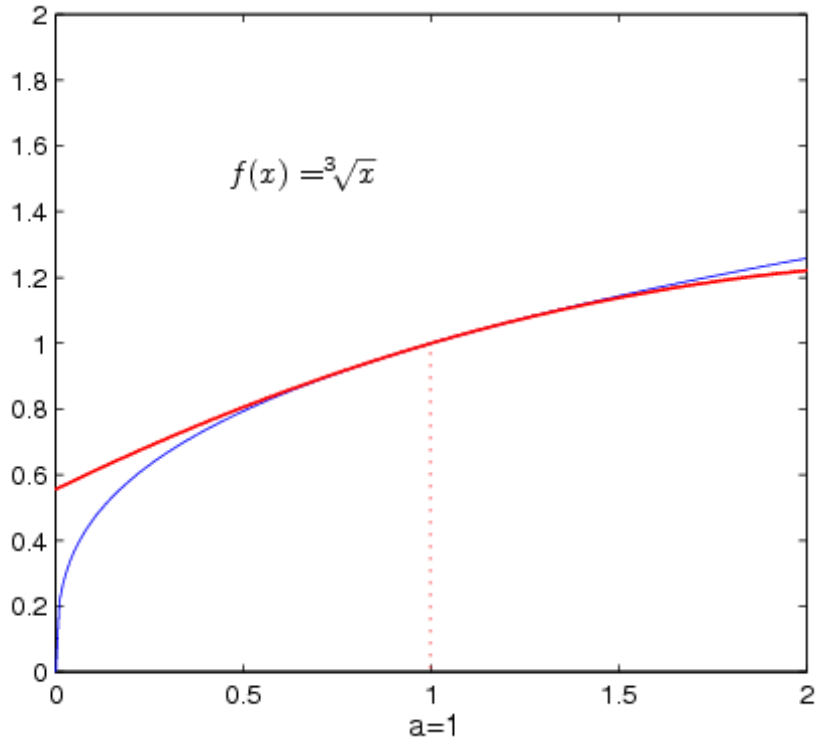
Example:

$f(x) = \sqrt[3]{x}$ about $x = 1$

- linear: $f(x) \approx f(1) + f'(1)(x-1) = 1 + \frac{1}{3}1^{-\frac{2}{3}}(x-1)$
- quadratic: $f(x) \approx f(1) + f'(1)(x-1) + \frac{1}{2}f''(1)(x-1)^2 = \cdots$

Check:

- exact: $f(1.1) = 1.03228$
- linear: $f(1.1) \approx 1.03333$
- quadratic: $f(1.1) \approx 1.03222$



OLD EXAM QUESTION

22 October 2014, Q3d

For a certain country, the debt/capita D in a certain year t is described by a function $f(t)$. The form of $f(t)$ is not known, but it is known that $f'(2014) = 3$ and $f''(2014) = -2$. Use a second-order approximation to find the debt/capita in 2016, if it is known than the debt/capita in 2014 is 350 USD. (5 points)

OLD EXAM QUESTION

22 October 2014, Q2b

The company operates in a market where the demand function for product 1 is given by $x_1 = 40 - 6p_1 + 4\sqrt{p_2}.$, where p_1 is the price of product 1 and p_2 the price of product 2. Determine the elasticity of the demand of product 1 with respect to the price of product 2. Simplify the expression as far as possible. (5 points)