Implicit functions

Business Mathematics

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CONTENTS

IMPLICIT FUNCTIONS

A simple function maps an input x onto an output y: $y = f(x)$

- to each x belongs not more than one y
- each y can originate from more than one x
- e.g., $f(x) = x^2$: $f(3) = f(-3) = 9$

But there are also cases where we have no explicit form for $y = f(x)$

- e.g., $y^3 + 3x^2y = 13$
- this is an example of an implicit function for y (or for x)
- y is not defined explicitly as $y = f(x)$, but yet follows from x (e.g., from $x = 2$ follows $y = 1$)
- to one x may belong several y (at given x only several (or one, or no) y possible)

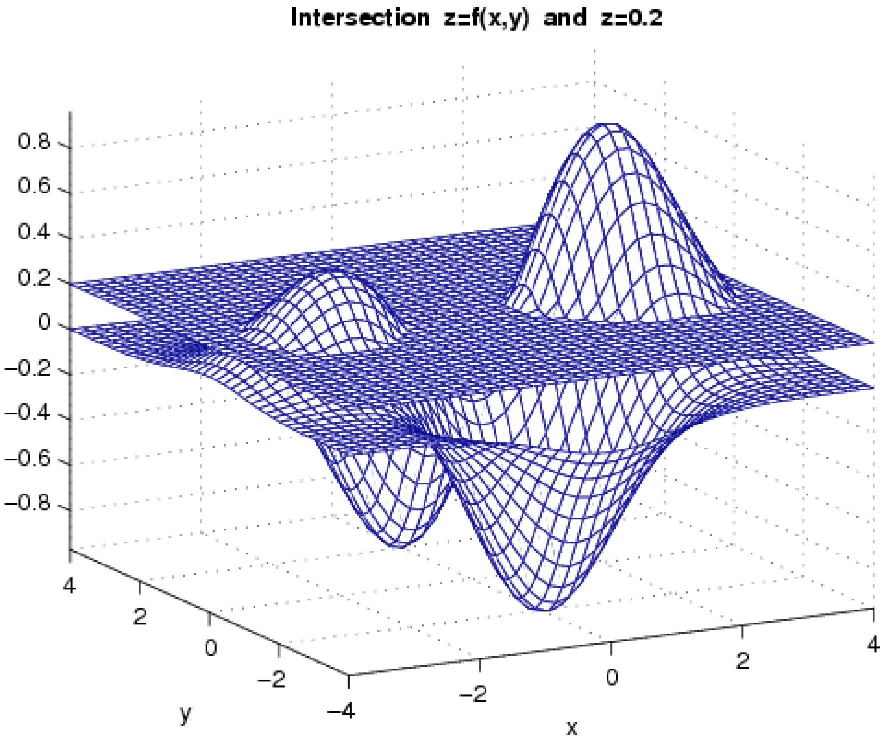
IMPLICIT FUNCTIONS

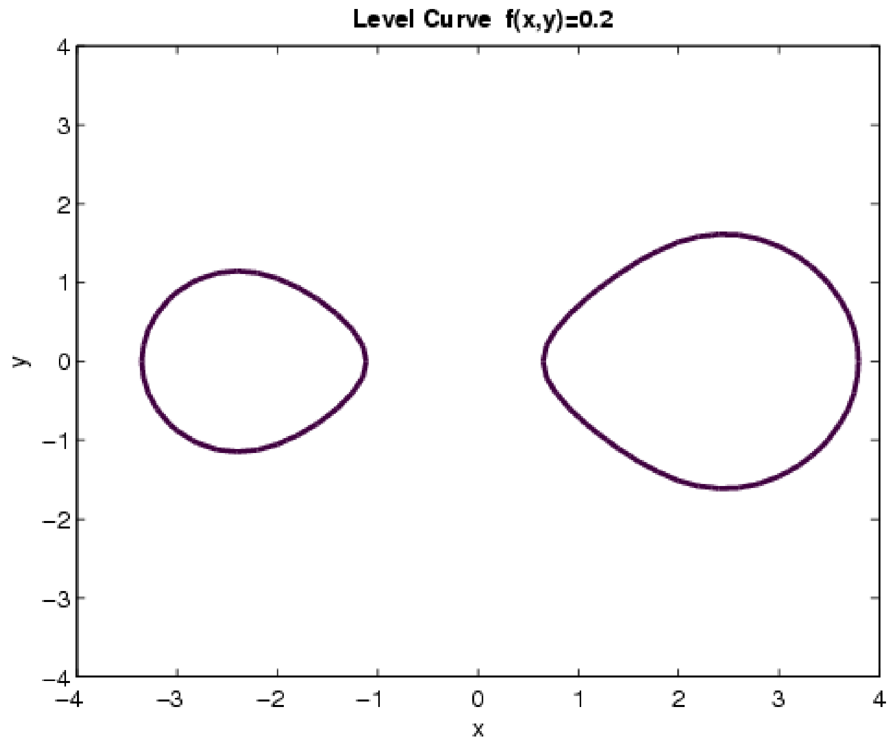
Some functions of two variables may be treated as implicit functions

- take $z = f(x, y)$
- we can analyse different outcomes $z = c$
- so, we analyse $f(x, y) = c$ for a given value of c
- $f(x, y)$ then yields an implicit function for y

LEVEL CURVES

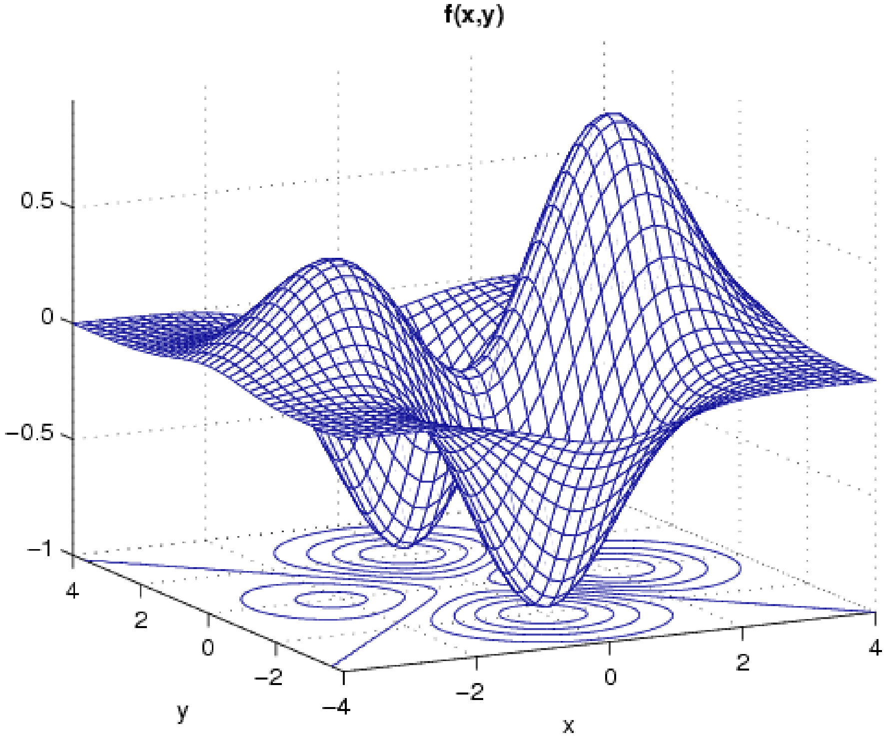
One way of representing is by level curves

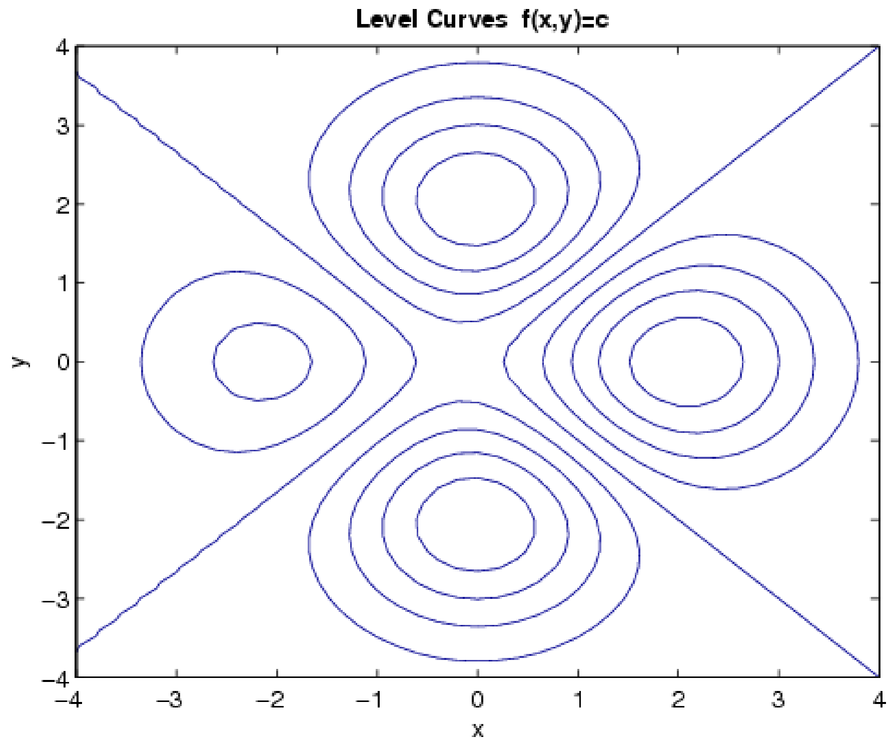




LEVEL CURVES

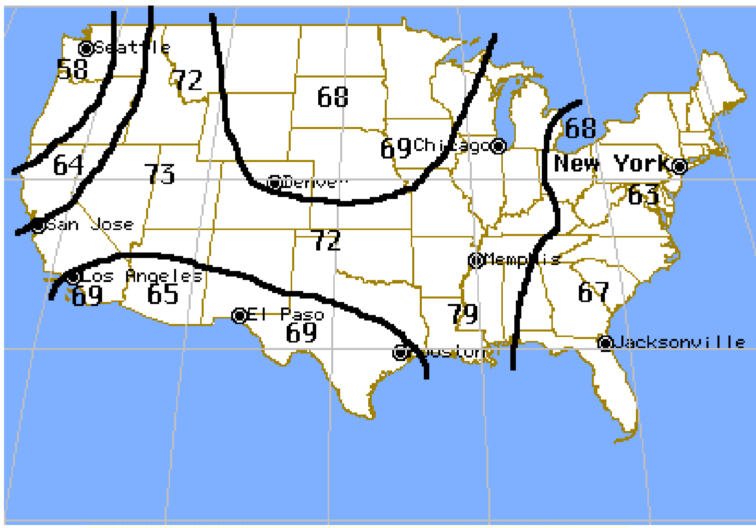
For a different value of c the function $y(x)$ can be completely different

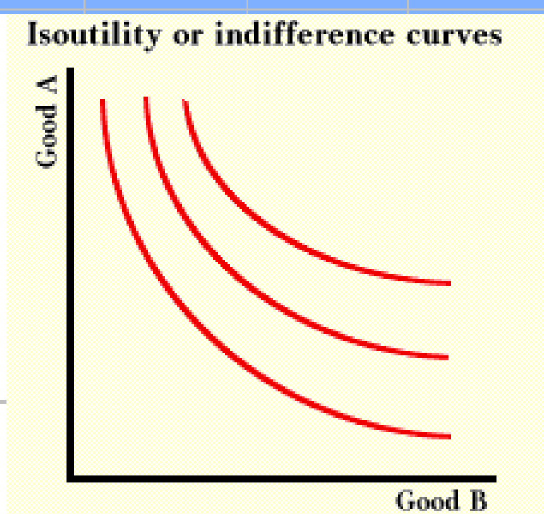


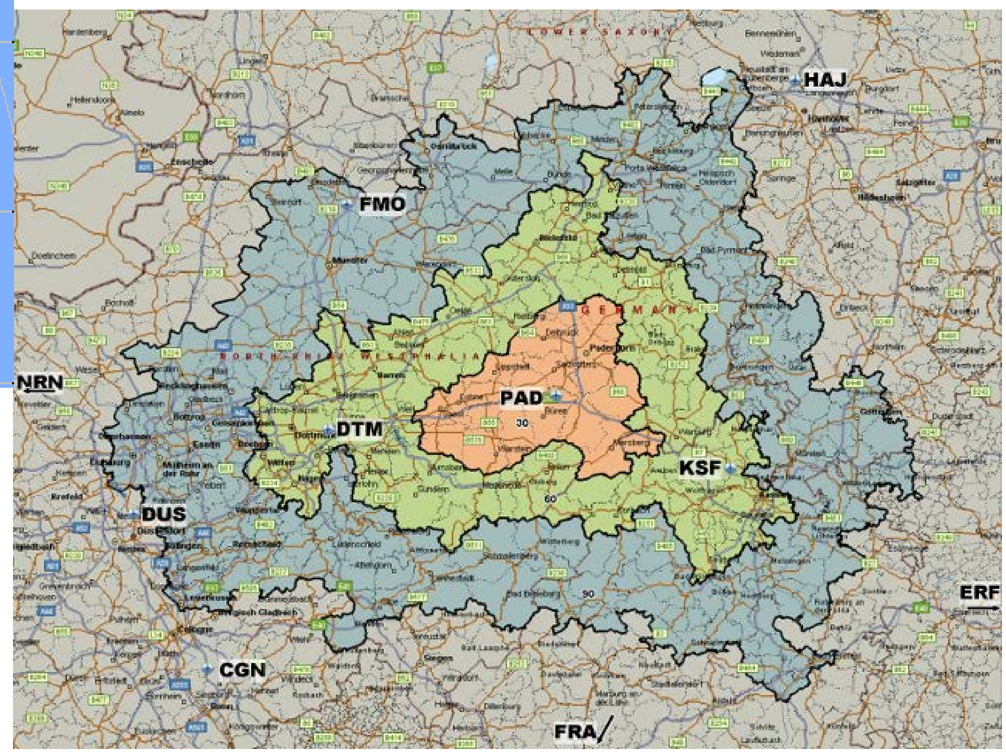


LEVEL CURVES

Used a lot in meteorology, logistics, welfare economics, ...







IMPLICIT DERIVATIVES

Can we find a derivative even if we have no explicit function?

- take again $y^3 + 3x^2y = 13$
- and differentiate with respect to x (so $\frac{d}{dx}$) on both sides
- so $\frac{d}{dx}(y^3 + 3x^2y) = \frac{d}{dx}(13)$
- thus $3y^2\frac{dy}{dx} + 6xy + 3x^2\frac{dy}{dx} = 0$
- solve for $\frac{dy}{dx}$: this gives $\frac{dy}{dx} = \frac{-2xy}{x^2+y^2}$
- so, the derivative at the point $(2,1)$ is $\left.\frac{dy}{dx}\right]_{(2,1)} = -\frac{4}{5}$

Implicit differentiation

IMPLICIT DERIVATIVES

Only if the point is on a level curve

- so, the derivative at the point $(1,1)$ makes no sense!
- check $f(x_a, y_a) = c$, and only then find $\left.\frac{dy}{dx}\right]_{(x_a, y_a)}$

IMPLICIT DERIVATIVES

Alternative method for implicit differentating $f(x,y) = c$ (book, p.420)

$$\frac{dy}{dx} = -\frac{\frac{\partial f(x,y)}{\partial x}}{\frac{\partial f(x,y)}{\partial y}}$$

Example: $y^3 + 3x^2y = 13$

- $f(x,y) = y^3 + 3x^2y$
- $\frac{dy}{dx} = -\frac{6xy}{3y^2+3x^2} = -\frac{2xy}{y^2+x^2}$
- at the point $(2{,}1)$: $\left.\frac{dy}{dx}\right]_{(2,1)} = -\frac{4}{5}$

OLD EXAM QUESTION

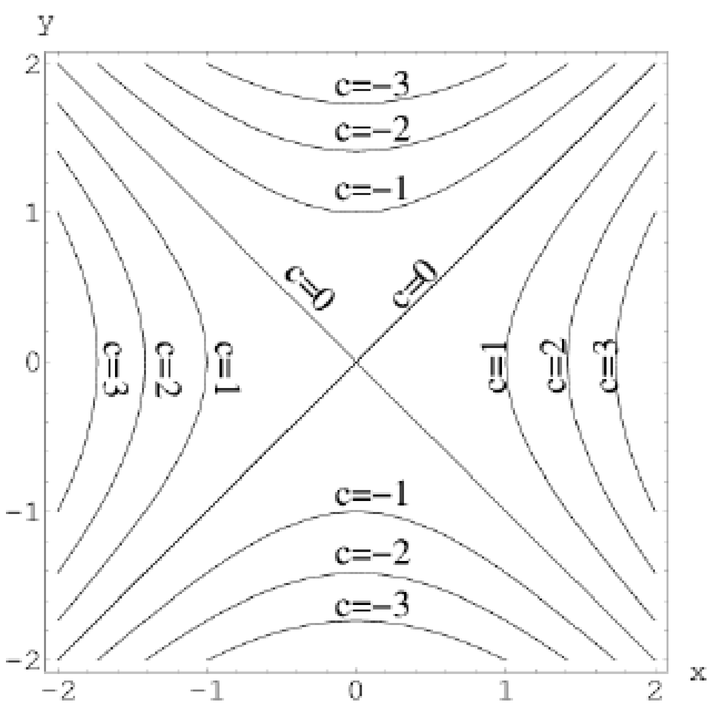
10 December 2014, Q1g

Given is the equation of a curve in the (x, y)-plane $xy^3 + x^2y^2 = 2$. Calculate $\frac{dy}{dx}$ of this curve in point $(x, y) = (1,1)$. (exact)

OLD EXAM QUESTION

10 December 2014, Q1g

Given is a function $f(x, y)$ on $\mathbb{R} \times \mathbb{R}$. For a small part of its domain, it is visualized in a with the level curves below:



(A) $-\infty$.

(B) -1.

(C) 0.

(D) 1.

(E) ∞.

The derivative $\frac{dy}{dx}$ of $f(x, y) = -1$ in the point $(0, -1)$ is approximately: (choose one)