Indefinite integrals

Business Mathematics

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CONTENTS

DERIVATIVES AND ANTIDERIVATIVES

Let functions $F(x)$ and $f(x)$ be given such that $f(x) = F'(x)$

- for example: $f(x) = 3x^2$ and $F(x) = x^3 + 12$

Can we find $F(x)$ given $f(x)$?

In other words, can we find a function $F(x)$ such that $f(x)$ will be a derivative $F'(x)$ of $F(x)$?

- Note that we can determine $F(x)$ only up to a constant

DERIVATIVES AND ANTIDERIVATIVES

Definition

We write $F(x)$ for the antiderivative of $f(x)$

In formula

$$\int f(x)dx = F(x) + C$$

Where

- $f(x)$ is the integrand
- x is the variable of integration
- C is the constant of integration

DERIVATIVES AND ANTIDERIVATIVES

The antiderivative is also known as the indefinite integral or the primitive or primitive function

It is a function, not a number

Note that we can also write:

the derivative of the primitive is the original function

$$\frac{d}{dx}\left(\int f(x)dx\right) = f(x)$$

the primitive of the derivative is the original function, up to a constant

and

$$\int \left(\frac{dF(x)}{dx}\right) dx = F(x) + C$$

THE INDEFINITE INTEGRAL

Given two functions $f(x)$ and $g(x)$ and their primitive functions $F(x)$ and $G(x)$

- then $F(x) = G(x) \Rightarrow f(x) = g(x)$
- but $f(x) = g(x) \nRightarrow F(x) = G(x)$

because the constants may be different

Example: the function

$$f(x) = g(x) = \sqrt{x}$$

may have different primitive functions

$$F(x) = \frac{2}{3}x^{\frac{3}{2}} + 13 \text{ and } G(x) = \frac{2}{3}x^{\frac{3}{2}} - 7$$

THE INDEFINITE INTEGRAL

Notation:

- $\int f(x)dx = F(x) + C$ when $F'(x) = f(x)$

or

- $\int q(\beta)d\beta = Q(\beta) + C$ when $Q'(\beta) = q(\beta)$

Mind well the role of the variables

Also mind the role and meaning of "dx" (or similar)

- $\frac{dF(x)}{dx} = f(x) \Leftrightarrow F(x) + C = \int f(x)dx$

BASIC INDEFINITE INTEGRALS

$\int a dx = ax + C$

$\int x^a dx = \frac{1}{a+1} x^{a+1} + C$ (with $a \neq -1$)

- example $\int x^4 dx = \frac{1}{5} x^5 + C$, because $\left(\frac{1}{5} x^5 + C\right)' = x^4$

$\int \frac{1}{x} dx = \ln(|x|) + C$

$\int e^x dx = e^x + C$

$\int e^{ax} dx = \frac{1}{a} e^{ax} + C$ (with $a \neq 0$)

$\int a^x dx = \frac{1}{\ln(a)} a^x + C$ (with $a > 0$ and $a \neq 1$)

- example $\int 10^x dx = \frac{1}{\ln 10} 10^x + C$, because $\left(\frac{1}{\ln 10} 10^x + C\right)' = 10^x$

PROPERTIES OF INDEFINITE INTEGRALS

$$\int (af(x))dx = a\int f(x)dx$$

$$\int (f(x) + a)dx = \int f(x)dx + ax$$

$$\int (f(x) + g(x))dx = \int f(x)dx + \int g(x)dx$$

PLAYING WITH CONSTANTS

Suppose that $\int f(x)dx = F(x) + C$

and that $\int g(x)dx = G(x) + D$

then:

$$\int (f(x) + g(x))dx =$$

$$\int f(x)dx + \int g(x)dx =$$

$$F(x) + C + G(x) + D =$$
$$F(x) + G(x) + E$$

but we often write here "$+C$" acknowledging that this C has nothing to do with the previous C

OLD EXAM QUESTION

22 October 2014, Q1f

For a function $f(x)$, it is known that $\int f(x)dx = 2(x-1)^5 + \ln x + C$. Determine $f(x)$.
(formula)

OLD EXAM QUESTION

27 March 2015, Q1d

Find $\int_0^x (3-2y)dy$. (formula or exact)