

Mathematical modelling

Business Mathematics

CONTENTS

How to solve it

Example 1

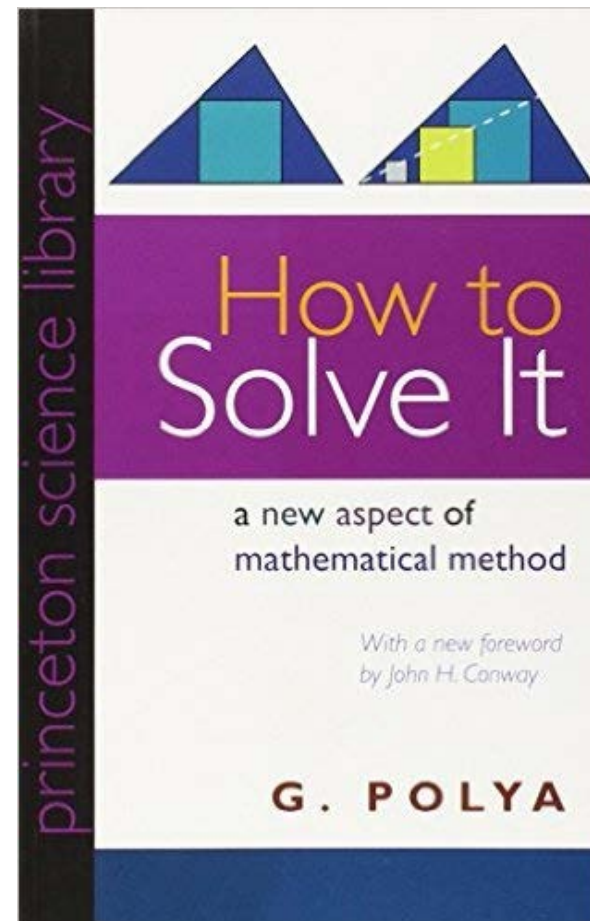
Example 2

Points of attention



HOW TO SOLVE IT?

George Pólya (1887-1985)
How to solve it (1945)



HOW TO SOLVE IT?

Understand the problem

- what is the unknown? what are the data? what is the condition?

Devise a plan

- find the connection between the data and the unknown

Carry out the plan

- check each step

Look back

- check the result



EXAMPLE 1

27 March 2015, Q3a

A bookshop owner sells newspapers for 2€. He buys them from the publisher for €1.20, and he can return unsold copies for €0.10. Indicating total profit in € by π , the number of sold copies by s and the number of returned copies by r , write an expression for $\pi(s, r)$. (4 points)



EXAMPLE 1

Understand the problem

what is unknown?

- profit (π)

what are the data?

- per copy selling price (2.00), buying price (1.20), return price (0.10)
- total sold copies (s) and unsold copies (r)

what is the condition?

- $\pi = \text{proceeds} - \text{costs}$
- $\text{total} = \text{number} \times \text{per copy}$



EXAMPLE 1

Devise a plan

- find the connection between the data and the unknown
- find an expression for total proceeds
- find an expression for total costs
- find an expression for total profit



EXAMPLE 1

Carry out the plan

- total proceeds: $2.00 \times s + 0.10 \times r$
 - total costs: $1.20 \times (s + r)$
 - profit: $2.00 \times s + 0.10 \times r - 1.20 \times (s + r)$
 - simplify: $\pi = 0.80 \times s - 1.10 \times r$
- check each step



EXAMPLE 1

Look back

- $\pi = 0.80 \times s - 1.10 \times r$

check the result

- is the formula plausible? (more sold copies increase the profit, more returned copies decrease the profit)
- are the units right? (0.80 and 1.20 in dollar/copy, r and s in number of copies, so result in dollars)



EXAMPLE 2

10 December 2014, Q3d

The popularity of Christmas depends on climate: in Sweden it is much more popular than in Dubai. Spending (S , in dollar/capita) appears to be described by a function $f(T_F) = a + bT_F$, where T_F is the average December temperature (in Fahrenheit), and a and b are coefficients obtained by OLS regression.

To move from Fahrenheit to Celsius, subtract 32 and multiply the result by $\frac{5}{9}$. Restate the function $f(T_F)$ as a function $g(T_C)$, so as $g(T_C) = c + dT_C$, where T_C is the average December temperature at the Celsius scale. Also write down the units of the coefficients c and d . (5 points)



EXAMPLE 2

Understand the problem

what is unknown?

- c and d

what are the data?

- $f(T_F) = a + bT_F$
- $g(T_C) = c + dT_C$
- $T_C = \frac{5}{9}(T_F - 32)$

what is the condition?

- $S = f(T_f) = g(T_C)$



EXAMPLE 2

Devise a plan

find the connection between the data and the unknown

- use $S = f(T_F) = g(T_C)$
- fill in T_C in terms of T_F
- elaborate to find the formula for $g(T_C)$



EXAMPLE 2

Carry out the plan

- take $S = f(T_F) = a + bT_F$
- fill in $T_F = \frac{9}{5}T_C + 32$
- to get $S = a + b\left(\frac{9}{5}T_C + 32\right)$
- simplify to $S = a + \frac{9}{5}bT_C + 32b$
- rearrange into $S = \underbrace{a + 32b}_c + \underbrace{\frac{9}{5}b}_d T_C$

check each step



EXAMPLE 2

Look back

- $S = g(T_C) = a + 32b + \frac{9}{5}bT_C$

check the result

- try, e.g., $T_F = 42$: $T_C = \frac{50}{9}$, $S = f(42) = a + 42b$, $S = a + 32b + \frac{9}{5}b \times \frac{50}{9} = a + 32b + 10b = a + 42b$
- did you answer the question? No, you forgot the units of c (dollar/capita) and d (dollar/(capita \times degree Celsius))



POINTS OF ATTENTION

What is the question?

What is given?

Watch for keywords

- prove
- solve
- calculate
- simplify
- maximize
- approximate
- ...



POINTS OF ATTENTION

Ignore “too much” data

- a common but realistic pitfall to use all data

Do not automatically equate a derivative (or other expression) to zero

- this happens more often than you think

Never use a “normal” derivative ($\frac{dy}{dx}$ or f') for a function of two (or more) variables

Use the formula sheet, but always with care

- do you need to replace x by q ?
- or a derivative by a partial derivative?

