Numbers and units

Business Mathematics

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Faculty of Economics and Business Administration

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NOTATION

Natural numbers: 0, 1, 2, etc.

Fractions: $\frac{2}{3}$, $3\frac{1}{4}$, etc.

- clearly distinguish $3\frac{1}{4}$ and $\frac{31}{4}$, don't write $3\,{}^{1}/_{4}$ or ${}^{31}/_{4}$

Decimal numbers: 1.82, 0.034, etc.

- mind the confusion between decimal point and decimal comma
- mind the optional use of a thousand separator (as in $1{,}234.567{,}8$)
- so what is $1{,}234.567$?

Non-computed numbers: $\sqrt{2}$, π, etc.

NOTATION

Negative numbers: -3, $-4\frac{1}{2}$, -0.012, etc.

- mind that $-4\frac{1}{2} < -4$
- mind that the minus sign is long (not -4 but -4)

Scientific notation: $3 \cdot 10^{6}$, $-2 \cdot 10^{-5}$, etc.

- also written as 3×10^{6}, but not as $3 * 10^{6}$ or as 3.10^{6}
- in software as $3\mathrm{E}6$, $-2\mathrm{E}-5$, etc.
- mind that 10^{9} is not $10\mathrm{E}9$ but rather $1\mathrm{E}9$
- and what is $3\mathrm{E}0$?

Words

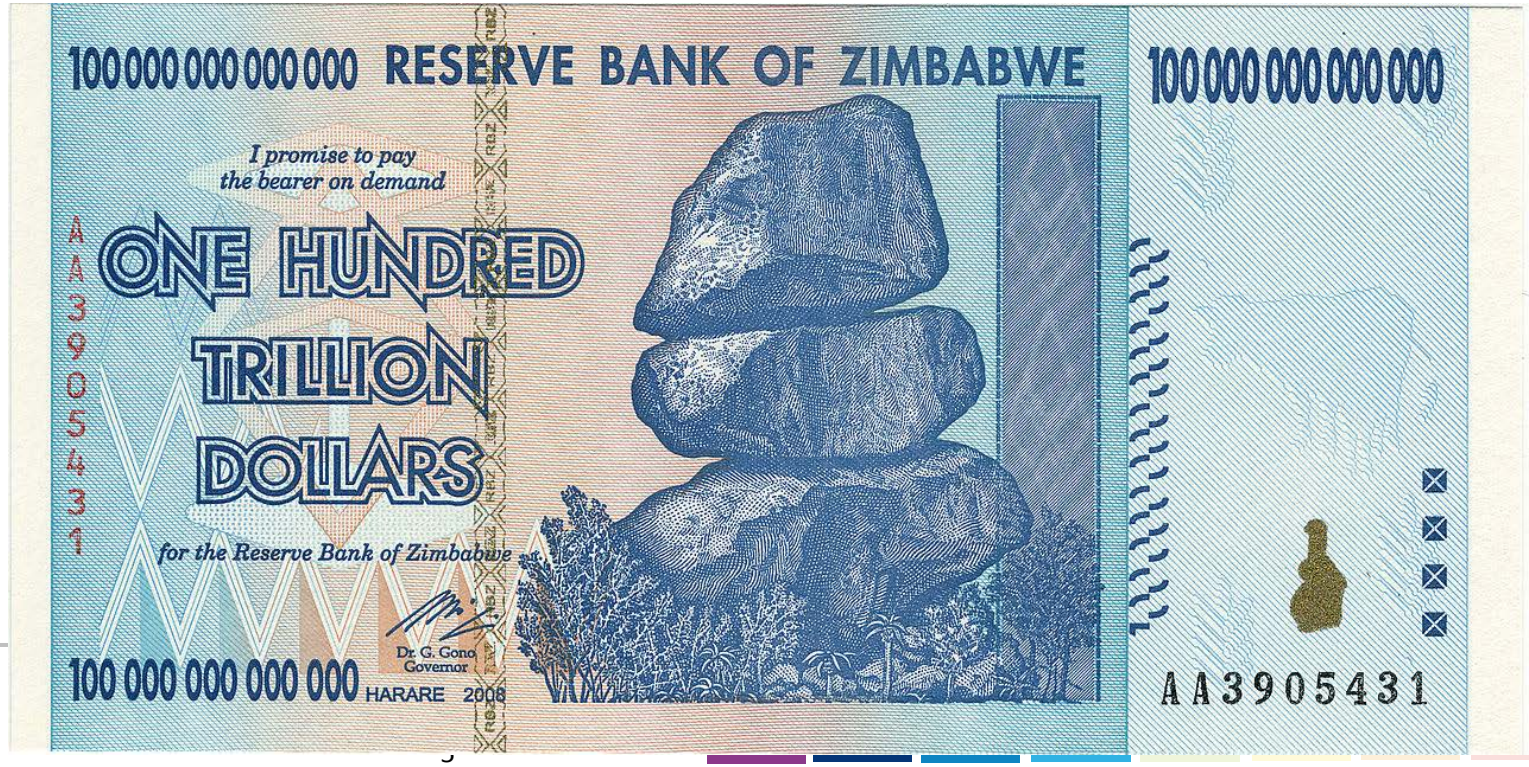
- billion (US, modern UK): 10^{9}
- billion (continental, older UK): 10^{12}
- trillion, etc.

NOTATION

Trillion?

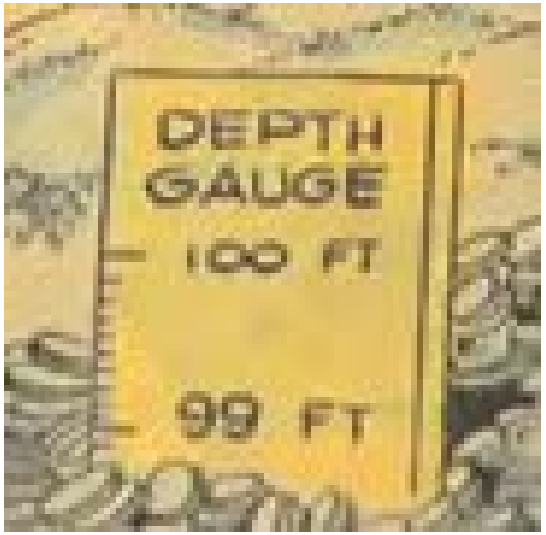
How many zeros?

Prefer “scientific notation”: $1 \cdot 10^{14}$ ZWD



SIGNIFICANT DIGITS

From Uncle Scrooge



- how much does Uncle Scrooge have? 98.8?
- every measurement has a certain precision
- specifying too many digits suggests too much precision

SIGNIFICANT DIGITS

From Apple's press release (first quarter 2009)

Apple sold 2,524,000 Macintosh® computers during the quarter, representing nine percent unit growth over the year-ago quarter. The Company sold a record 22,727,000 iPods during the quarter, representing three percent unit growth over the year-ago quarter. Quarterly iPhone units sold were 4,363,000, representing 88 percent unit growth over the year-ago quarter.

- did they really sell exactly $4{,}363{,}000$ iPhones?
- or perhaps $4{,}363{,}037$? or $4{,}362{,}813$?
- I guess they sold $4{,}363{,}000 \pm 500$ iPhones
- there are 4 significant digits (4363) reported
- better specify this as $4{,}363 \cdot 10^3$

SIGNIFICANT DIGITS

From Funtowicz & Ravetz, Ecological Economics (1994)

> It is about a museum attendant, who was heard telling visitors that some fossil dinosaur bones were 56 000 012 years old. When asked how he knew so precisely, he explained that when he came on the job 12 years previously, he was told that they were 56 000 000 years old then. No one

Adding numbers

- so, $56{,}000{,}000 + 12 = 56{,}000{,}012$?
- no, $56 \cdot 10^6 + 12 = 56 \cdot 10^6$

SIGNIFICANT DIGITS

Facts

- IKEA's value: 11.5 billion $
- 1 $ = 0.747213 €

So, IKEA's value in €:

- 8 billion 592 million 944 thousand 567 euro and 58 cents
- wrong! least precise number has 3 significant digits, so the result has 3 significant digits as well
- 8 billion 590 million euro
- or $8.59 \cdot 10^9$ euro

SIGNIFICANT DIGITS

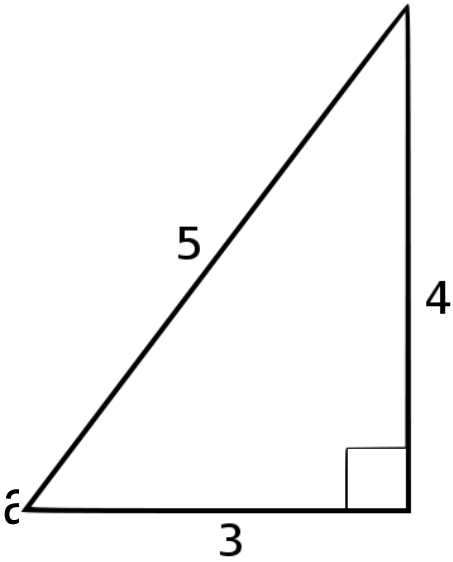
Examples:

$$8 + 8 = 1.6 \cdot 10^1$$
$$8.0 + 8 = 1.6 \cdot 10^1$$
$$8.0 + 8.0 = 16$$
$$8.00 + 8.00 = 16.0$$

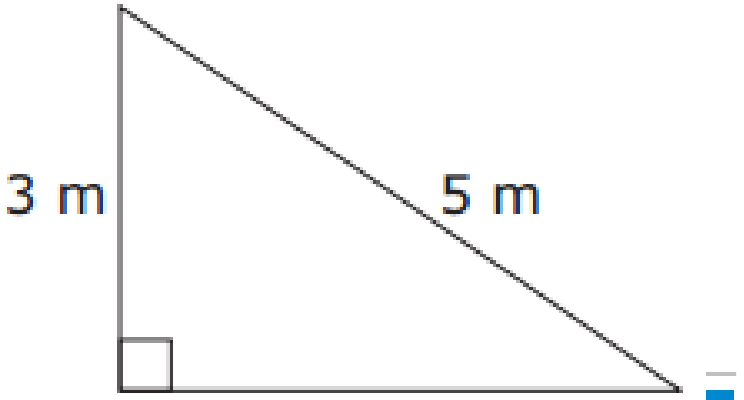
$$8 \times 8 = 6.4 \cdot 10^1$$
$$8.0 \times 8 = 6.4 \cdot 10^1$$
$$8.0 \times 8.0 = 64$$
$$8.00 \times 8.00 = 64.0$$

UNITS

In mathematics variables represent numbers



But in the real world most variables represent numbers plus a



UNITS

A unit is an arbitrarily chosen yardstick, and the number represents how many of such yardsticks are needed

- example: $c = 5\ \mathrm{m}$, so 5 yardsticks of unit length $(1\ \mathrm{m})$

The unit is just processed with the variable

- example: $c = \sqrt{(3\ \mathrm{m})^2 + (4\ \mathrm{m})^2} = \sqrt{9\ \mathrm{m}^2 + 16\ \mathrm{m}^2} = \sqrt{25\ \mathrm{m}^2} = 5\ \mathrm{m}$

UNITS

There is often a choice of yardsticks

kilometre vs. mile

- 1 mile = 1.6 km

litre vs. gallon

- 1 gallon = 3.8 l

25 mile per gallon = how many km per l?

- $25\ \frac{\text{mile}}{\text{gallon}} = 25\ \frac{1.6\ \text{km}}{3.8\ \text{l}} = \frac{25 \times 1.6}{3.8}\frac{\text{km}}{\text{l}} = 10.5\ \frac{\text{km}}{\text{l}}$

UNITS

Some conversions are not by a fixed factor

- 1 € = ??? $ = ??? £ = ??? ¥
- 1 yr = ??? s
- 1 tonne/ton = ???

Full name(s)	Common name	Quantity
long ton,[6] weight ton, gross ton	"ton" (UK)[a]	2,240 lb (1,016.047 kg)
short ton,[7] net ton	"ton" (US)	2,000 lb (907.1847 kg)
tonne[8]	"tonne";[a] "metric ton" (mainly UK)	1,000 kg (2,204.623 lb)
ton shortweight[b]		2240 lb
ton longweight[b]		2400 lb[c]

UNITS

Algebraic operations:

Multiplication: always

- $5\ \mathrm{m} \times 3\ \mathrm{m} = 15\ \mathrm{m} \times \mathrm{m} = 15\ \mathrm{m}^2$
- $5\ \mathrm{m} \times 3\ \mathrm{g} = 15\ \mathrm{m} \times \mathrm{g}$

Addition: only for equal units

- $5\ \mathrm{m} + 3\ \mathrm{m} = 8\ \mathrm{m}$
- $5\ \mathrm{m} + 3\ \mathrm{g} = \mathrm{nonsense}$

Logarithm: doubtful, but often done

UNITS

Units can have a prefix (k=kilo, m=milli, etc.)

It is treated as a number

- $5\ \mathrm{km} = 5 \times (1000\ \mathrm{m}) = (5 \times 1000)\ \mathrm{m} = 5000\ \mathrm{m}$
- $5\ \mathrm{km}^2 = 5 \times (1000\ \mathrm{m})^2 = 5 \times 1000^2\ \mathrm{m}^2 = 5{,}000{,}000\ \mathrm{m}^2$

UNITS

Most crucial for economics and business:

- stocks (deposit, asset)
- flows (income, interest)

Flows have a “per time” in the unit

- so if you change the period, the number changes
- examples: income, sales, GDP

Stocks do not

- examples: savings, debt, balance

UNITS

From The World Bank

GDP (current US$)

DATAB

GDP at purchaser's prices is the sum of gross value added by all resident producers in the economy plus any product taxes and minus any subsidies not included in the value of the products. It is calculated without making deductions for depreciation of fabricated assets or for depletion and degradation of natural resources. Data are in current U.S. dollars.

- wrong! GDP is in $/yr

From tradingeconomics.com

United States Government Debt to GDP 1940-2014 | Data | Chart | Calendar

The United States recorded a Government Debt to GDP of 101.53 percent of the country's Gross Domestic Product in 2013. Government Debt To GDP in the United States averaged 60.81 Percent from 1940 until 2013, reaching an all time high of 121.70 Percent in 1946 and a record low of 31.70 Percent in 1974. Government Debt To GDP in the United States is reported by the U.S. Bureau of Public Debt.

- wrong! Debt to GDP is in $/($/yr), so in yr, not in %

UNITS

JET'S FUEL RAN OUT AFTER METRIC CONVERSION ERRORS

By RICHARD WITKIN (The New York Times); National Desk
July 30, 1983, Saturday
Late City Final Edition, Section 1, Page 7, Column 4, 819 words

[DISPLAYING ABSTRACT]

Air Canada said yesterday that its Boeing 767 jet ran out of fuel in midflight last week because of two mistakes in figuring the fuel supply of the airline's first aircraft to use metric measurements. After both engines lost their power, the pilots made what is now thought to be the first successful emergency "dead stick" landing of a commercial jetliner. The pilots of the Ottawa-to-Edmonton flight came in over the end of the runway at Gimli, Manitoba, at an abnormally high speed of about 180 knots because the engine failure made it impossible to use the flaps to make a slower approach. But the only serious damage was a collapsed nose gear, and the only casualties among the 69 people on board were two passengers who suffered minor injuries.

UNITS

Mars Climate Orbiter

Mishap Investigation Board

Phase I Report

November 10, 1999

The MCO MIB has determined that the root cause for the loss of the MCO spacecraft was the failure to use metric units in the coding of a ground software file, "Small Forces," used in trajectory models. Specifically, thruster performance data in English units instead of metric units was used in the software application code titled SM_FORCES (small

OLD EXAM QUESTION

27 March 2015, Q1m

Calculate: $1500\ \text{million} + 2.5E8 + 3.5 \times 10^{10} + \frac{1}{10^{-9}}$. (3 significant digits)

OLD EXAM QUESTION

23 April 2015, Q1i

The cost of flying a certain airplane is known to be $C = \phi d + \gamma d^2$, where C is costs in euros, d is distance in km, and ϕ and γ are constants. What is the unit of γ? (text)