Summation

Business Mathematics

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CONTENTS

DEFINITION OF SUMMATION

We introduce the summation operator $\sum$ as follows:

$$\sum_{i=m}^{n} x_i = x_m + x_{m+1} + \cdots + x_{n-1} + x_n$$

- example 1: take $x_i = i$, $m = 1$, and $n = 10$:

$$\sum_{i=m}^{n} x_i = \sum_{i=1}^{10} i = 1 + 2 + \cdots + 10 = 55$$

- example 2: take $x_i = 2$, $m = 0$, and $n = 10$:

$$\sum_{i=m}^{n} x_i = \sum_{i=0}^{10} 2 = 2 + 2 + \cdots + 2 = 22$$

DEFINITION OF SUMMATION

Notice the alternative use of display form and inline form:

$$\sum_{i=1}^{n} x_i = \textstyle\sum_{i=1}^{n} x_i$$

When unambiguous you may simplify:

$$\sum_{i=1}^{n} x_i = \sum_{i} x_i = \sum x_i = \sum x$$

DEFINITION OF SUMMATION

Index symbol "disappears":

$$\sum_{i=1}^{n} x_i = q$$

$$\sum_{i=1}^{n} x_{ij} = q_j$$

so not q_i!

so not q or q_i!

Index symbol is arbitrary:

$$\sum_{i=1}^{n} x_i = \sum_{j=1}^{n} x_j = \sum_{\alpha=1}^{n} x_\alpha$$

MOTIVATION

Why use $\sum$?

- in business analyses, we often need to add over years, over departments, etc.
- it helps us to prepare for later subjects (statistics, matrices)
- it is important in applied courses (finance, accounting)

PROPERTIES

Additivity: $\sum(x_i + y_i) = \sum x_i + \sum y_i$

Homogeneity: $\sum c x_i = c \sum x_i$

Extending terms: $\sum_{i=1}^{n} x_i + \sum_{i=n+1}^{n+m} x_i = \sum_{i=1}^{n+m} x_i$

Constant term: $\sum_{i=1}^{n} c = nc$

Telescopic series: $\sum_{i=1}^{n}(x_i - x_{i-1}) = x_n - x_0$

PROPERTIES

Most are easy to see

- e.g.,

$$\sum_{i=1}^{4} cx_i = cx_1 + cx_2 + cx_3 + cx_4$$
$$= c(x_1 + x_2 + x_3 + x_4)$$
$$= c\sum_{i=1}^{4} x_i$$

- but we will skip most proofs

DOUBLE SUMMATION

Straightforward to generalize:

$$\sum_{i=1}^{n}\left(\sum_{j=1}^{m} x_{ij}\right) = \sum_{j=1}^{m}\left(\sum_{i=1}^{n} x_{ij}\right)$$

therefore no ambiguity in writing $\sum_{i=1}^{n}\sum_{j=1}^{m} x_{ij}$

$$\boxed{\sum_{i=1}^{n}\sum_{j=1}^{m} x_{ij} = x_{11} + \cdots + x_{1m} + x_{21} + \cdots + x_{2m} + \cdots + x_{nm}}$$

- Example: take $x_{ij} = i + 2j$ and $n = 4$ and $m = 3$:

$$\sum_{i=1}^{n}\sum_{j=1}^{m} x_{ij} = (1+2) + (1+4) + \cdots + (4+6) = 78$$

PRODUCT OPERATOR

We also define the product operator Π as follows:

$$\prod_{i=m}^{n} x_i = x_m \times x_{m+1} \times \cdots \times x_{n-1} \times x_n$$

- example 1: take $x_i = i$, $m = 1$, and $n = 10$:

$$\prod_{i=m}^{n} x_i = \prod_{i=1}^{10} i = 1 \times 2 \times \cdots \times 10 = 10! = 3628800$$

- example 2: take $x_i = 2$, $m = 0$, and $n = 10$:

$$\prod_{i=m}^{n} x_i = \prod_{i=0}^{10} 2 = 2 \times 2 \times \cdots \times 2 = 2^{11} = 2048$$

OLD EXAM QUESTION

27 March 2015, Q1h

Given is the function $S(k) = \sum_{i=k}^{2k} (k + i)$. Compute $S(3)$. (formula or exact)

OLD EXAM QUESTION

10 December 2014, Q2d

A customer card is introduced to increase turnover. After a customer's n^{th} visit, the discount at the next visit is $\sqrt[3]{10n}$ percent. Give a formula for the cumulative discount for a person who has had dinner m times, and who always orders meals and drinks which have a full (undiscounted) price of $60€$. (6 points)