Vectors

Business Mathematics

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VECTORS

A vector is a column of numbers or variables

Notation

- we often use roman bold non-italic letters to refer to them
- e.g., $\mathbf{q} = \begin{pmatrix} 3 \\ -2 \\ 12.5 \end{pmatrix}$, $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ \dots \\ a_n \end{pmatrix}$

Terminology

- these vectors consist of 3 respectively n elements
- the order (or size is) 3 respectively n
- by default, we consider column vectors. A row vector looks like $\mathbf{q} = (3 \quad -2 \quad 12.5)$ or $\mathbf{q} = (3, -2, 12.5)$

OPERATIONS WITH VECTORS

We can define some basic operation with vectors:

Addition: $\begin{pmatrix} a_1 \\ a_2 \\ \dots \\ a_n \end{pmatrix} + \begin{pmatrix} b_1 \\ b_2 \\ \dots \\ b_n \end{pmatrix} = \begin{pmatrix} a_1 + b_1 \\ a_2 + b_2 \\ \dots \\ a_n + b_n \end{pmatrix}$

- for any two vectors $\mathbf{a}$ and $\mathbf{b}$ of equal order

written as $\mathbf{a} + \mathbf{b}$

Multiplication: $c \times \begin{pmatrix} a_1 \\ a_2 \\ \dots \\ a_n \end{pmatrix} = \begin{pmatrix} c \times a_1 \\ c \times a_2 \\ \dots \\ c \times a_n \end{pmatrix}$

- for any constant c

written as $c\mathbf{a}$

OPERATIONS WITH VECTORS

Negative vector: $-\mathbf{a} = -1 \times \mathbf{a}$

Subtraction: $\mathbf{a} - \mathbf{b} = \mathbf{a} + (-\mathbf{b})$

- for any two vectors **a** and **b** of equal order

if and only if ...
we sometimes say: iff

Equality: $\mathbf{a} = \mathbf{b}$ if and only if $a_i = b_i, i = 1, \ldots, n$

- for any two vectors **a** and **b** of equal order n

Most other operations not defined

- e.g., ~~$\frac{1}{\mathbf{a}}$ or $\ln \mathbf{a}$ or $\sqrt{\mathbf{a}}$~~
- but a mathematician can define almost anything

SPECIAL VECTORS

Zero vector: $\mathbf{0} = \begin{pmatrix} 0 \\ 0 \\ \dots \\ 0 \end{pmatrix}$

All-ones vector: $\mathbf{1} = \begin{pmatrix} 1 \\ 1 \\ \dots \\ 1 \end{pmatrix}$

Standard basis vectors: $\mathbf{e}_1 = \begin{pmatrix} 1 \\ 0 \\ \dots \\ 0 \end{pmatrix}$, $\mathbf{e}_2 = \begin{pmatrix} 0 \\ 1 \\ \dots \\ 0 \end{pmatrix}$, etc.

Sometimes, their order is added as a subscript

- e.g., $\mathbf{0}_3$, $(\mathbf{e}_2)_5$

INNER PRODUCT

Let $\mathbf{a}$ and $\mathbf{b}$ be two vectors of order n

We define the <u>inner product</u> of $\mathbf{a}$ and $\mathbf{b}$ as

$$\mathbf{a} \cdot \mathbf{b} = \sum_{i=1}^{n} a_i b_i$$

- alternatively written as $(\mathbf{a}, \mathbf{b})$
- but do not write ~~$\mathbf{a} \times \mathbf{b}$ or $\mathbf{ab}$~~

Notice well:

- the result of adding two vectors or multiplying a vector with a constant is another vector
- but the result of the inner product is not a vector but a number

INNER PRODUCT

It follows that (with suitable $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c}$)

- $\mathbf{a} \cdot \mathbf{b} = \mathbf{b} \cdot \mathbf{a}$ (commutative property)
- $\mathbf{a} \cdot (\mathbf{b} + \mathbf{c}) = \mathbf{a} \cdot \mathbf{b} + \mathbf{a} \cdot \mathbf{c}$ (distributive property)

But not that

- $\mathbf{a} \cdot (\mathbf{b} \cdot \mathbf{c}) = (\mathbf{a} \cdot \mathbf{b}) \cdot \mathbf{c}$ (associative property)
- why not?

Also:

- $\mathbf{a} \cdot \mathbf{a} \geq 0$
- when $= 0$ and when > 0?

GEOMETRIC INTERPRETATION

A vector can be seen as an abstract order set of numbers

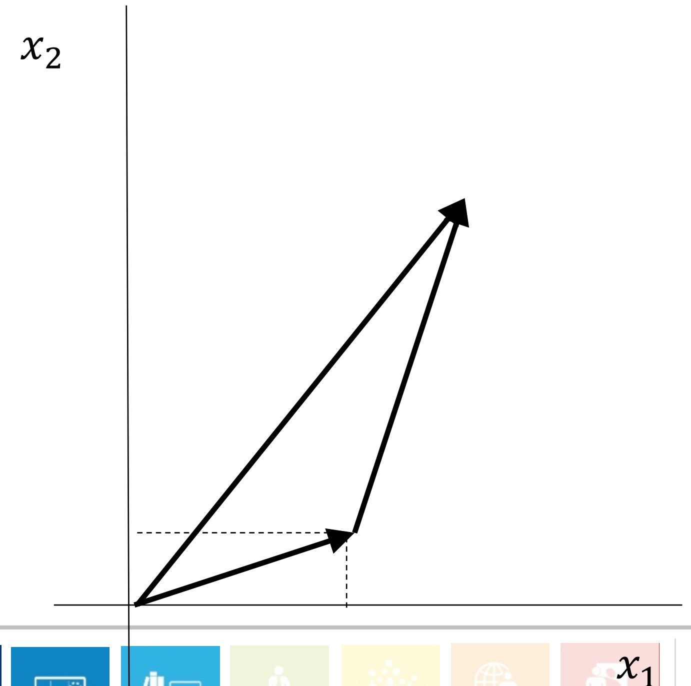
But a vector can also be interpreted as a line from one point to another point

- example: $(x_1, x_2) = (2{,}1)$
- from the origin to $(2{,}1)$

Adding means "stacking" vectors

- example: $(2{,}1) + (1{,}3) = (3{,}4)$

Multiplying is just extending

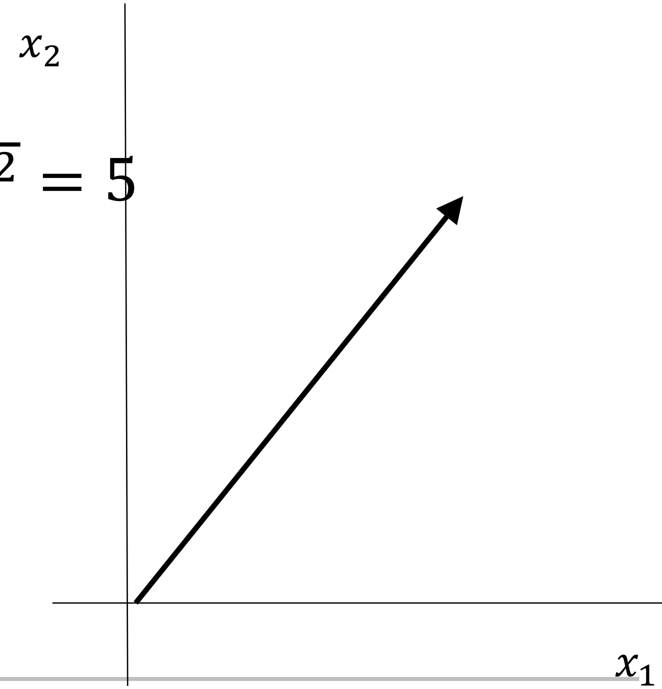


GEOMETRIC INTERPRETATION

We define the <u>norm</u> of a vector $\mathbf{x}$ as

$$\|\mathbf{x}\| = \sqrt{\sum_{i=1}^{n} (x_i)^2}$$

- example: vector $\mathbf{x} = (3,4\ )$ is $\|\mathbf{x}\| = \sqrt{3^2 + 4^2} = 5$
- it is the <u>Euclidean length</u> of the arrow



FEWEB

GEOMETRIC INTERPRETATION

Recall inner product of two vectors: $\mathbf{x} \cdot \mathbf{y} = \sum_{i=1}^{n} x_i y_i$

So for a vector with itself: $\mathbf{x} \cdot \mathbf{x} = \sum_{i=1}^{n} (x_i)^2$

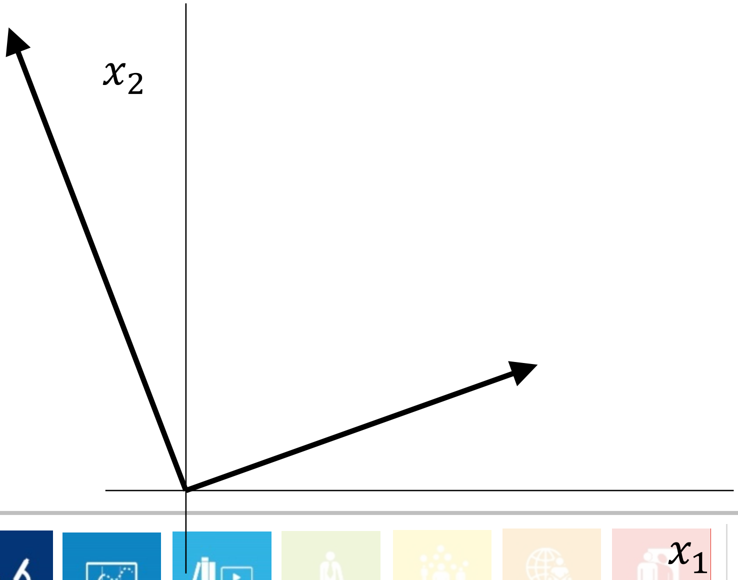
Conclusion: $||\mathbf{x}|| = \sqrt{\mathbf{x} \cdot \mathbf{x}}$

Also, consider the vectors $\mathbf{x} = (3,1)$ and $\mathbf{y} = (-2,6)$

Their inner product $\mathbf{x} \cdot \mathbf{y}$ is 0

Geometric interpretation:

- they are <u>orthogonal</u>



OLD EXAM QUESTION

22 October 2014, Q2d

The stock of products is indicated by a vector $\mathbf{s} = \binom{s_1}{s_2}$, where s_1 is the stock of product 1, and s_2 is the stock of product 2. The prices are as before: p_1 and p_2. Define a suitable price vector, and give a vector/matrix-formula for the total value of the stock. (4 points)