

Business Mathematics (BK/IBA) – Quantitative Research Methods I (EBE)

Computer tutorial 2

1 Introduction

In this tutorial session, we will continue to work on two topics:

- typesetting mathematical expressions (formulas) using Microsoft Word;
- using Microsoft Excel for quantitative modelling.

The way of working is the same as last week. Please review it in case you forgot.

2 Equations in Microsoft Word – advanced features

Today, we will be addressing a number of new features. We start by adding “ornaments” to symbols. The mostly used ornament is the apostroph, to which mathematicians refer as a prime, as in y' . This is simply done by typing a subscript apostroph to the symbol y .

Slightly more involved are ornaments on top of letters, as in \bar{x} or \hat{y} . Here, we use the Accent option from the equation toolbar. It contains many types: the dot (\dot{x}), the arrow (\vec{x}), the bar (\bar{x}), the tilde (\tilde{x}), the hat (\hat{x}), etc. There is also the option to stack symbols, as in

$$a^n = \underbrace{a \times a \times \dots \times a}_{n \text{ terms}}$$

Finally, it also contains the box that can contain an equation:

$$\boxed{E = mc^2}$$

Of course you must take care whether or not to include subscripts or superscripts in such ornaments. Compare \bar{a}_i and \bar{a}_i . Also compare $\overline{x + y}$ and $\bar{x} + \bar{y}$.

For parentheses the differences are more important. For instance, consider $\left(\frac{1}{2}x^2 - \frac{1}{2}\right)$ versus $\left(\frac{1}{2}x^2 - \frac{1}{2}\right)$. Especially when parentheses at different levels co-exist, the size can help to make the distinction. Consider, for instance,

$$\left((x - y)^2 + \frac{(x + y)}{x} \right)^2$$

The Bracket menu offers also other types of parenthesis (like [these] and {these}), as well as some other symbols.

The equation editor can also be used for typing vectors, matrices, and equations that include several lines.

Examples are $\begin{pmatrix} 3 \\ -4 \end{pmatrix}$, $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$, and $|x| = \begin{cases} x & \text{if } x > 0 \\ 0 & \text{if } x = 0 \\ -x & \text{if } x < 0 \end{cases}$. Mind that it may be needed to arrange the

contents with respect to alignment: compare $\begin{pmatrix} 12345 \\ 6 \end{pmatrix}$ and $\begin{pmatrix} 12345 \\ 6 \end{pmatrix}$. The right button of the mouse helps

you to arrange this. Finally, in some cases we need matrices with lines separating the columns $\begin{pmatrix} a & b \\ c & d \end{pmatrix} \left| \begin{matrix} e \\ f \end{matrix} \right.$.

You may achieve this by selecting brackets with separators.

Long equations may span several lines. For instance, the equation $1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 = 55$ spans two lines. Line breaks go automatically, but you may wish to control them. In a long derivation in display style, for instance, you may write

$$(a + b)^2 =$$

$$\begin{aligned}
 (a + b)(a + b) &= \\
 a(a + b) + b(a + b) &= \\
 a^2 + ab + ba + b^2 &= \\
 a^2 + 2ab + b^2 &=
 \end{aligned}$$

Do some experiments with writing such long formulas and how to outline them in different ways.

Now, practice typing the following formulas:

Exercise 1: $y' \equiv \frac{dy}{dx} = \frac{d}{dx} \left(x + \frac{1}{x} \right)$

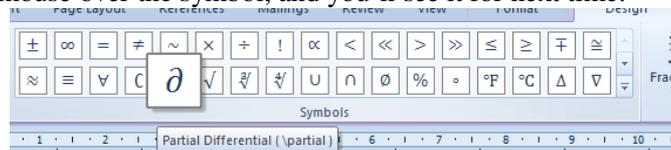
Exercise 2: $\hat{\mu} = \bar{x} = \frac{1}{n} \sum_{i=1}^n x_i$

Exercise 3: $\det \mathbf{A} = \det \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} = a_{11}a_{22} - a_{21}a_{12}$

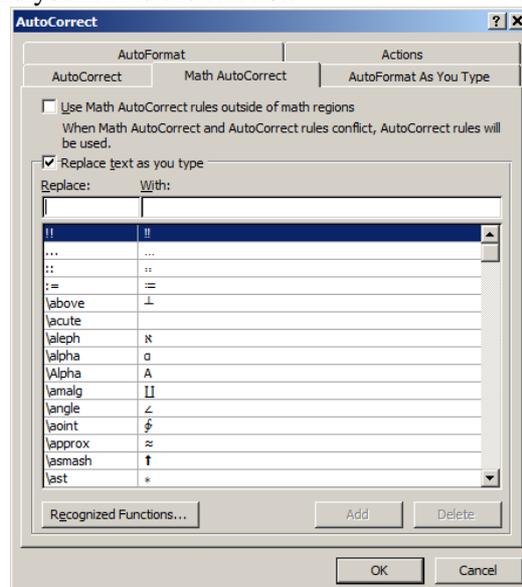
Exercise 4: $\frac{1}{\left(\frac{1}{x}\right)} = \begin{cases} \text{undefined} & \text{if } x = 0 \\ x & \text{otherwise} \end{cases}$

Exercise 5: $\left(\frac{x \ln x}{fg} \right)' = \frac{x' \ln x}{f'g} + \frac{x(\ln x)'}{fg'} = \ln x + 1$

Finally, you may speed up typing equations and omit using the mouse by using programming code. For instance, when you type “x^2” the equation may convert this automatically into x^2 . Similar for “\alpha” which gives α . And “1/2” gives $\frac{1}{2}$. In some cases, you first need to add a space (or = or so) before this automatic conversion works. And you may have to turn some options (autocorrect?) on. Which shortcuts to use? Move with the mouse over the symbol, and you’ll see it for next time:



Or go to Math Autocorrect, and you’ll find the full list:



3 Mathematical modelling in Excel – graphs

Today, we'll explore how to make graphs. We restrict the discussion to graphs in a numerical context, which includes two types of graphs:

- function graphs;
- data graphs.

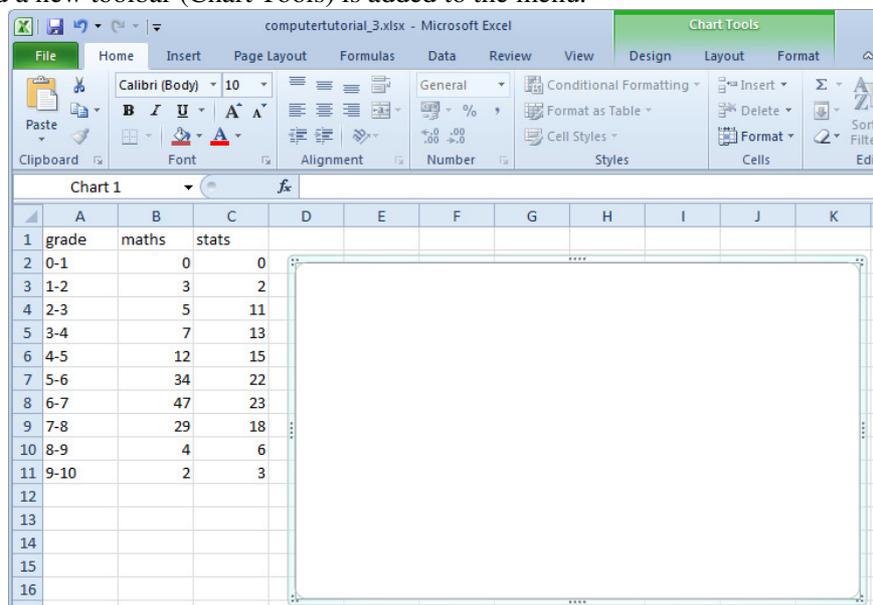
Function graphs are the well-known graphs in a book on mathematics: straight lines, parabolas, the exponential function, etc. Data graphs present the information from empirical data, for instance a histogram of the results of an exam, or a piechart of which share of the population uses a car for commuting and which share travels by train.

4 Mathematical modelling in Excel – data graphs

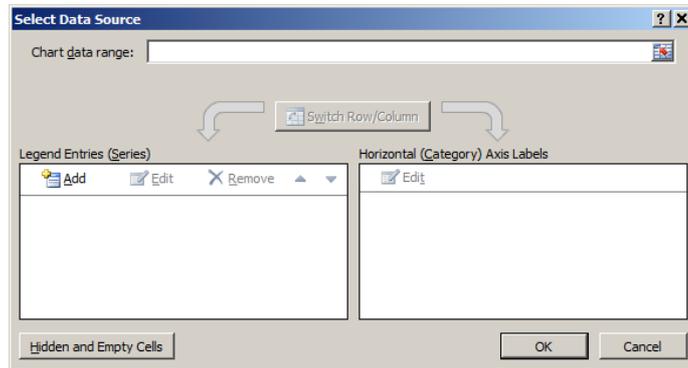
To make a data graph, we must have a worksheet filled with data. We fill a worksheet with our data:

	A	B	C
1	grade	maths	stats
2	0-1	0	0
3	1-2	3	2
4	2-3	5	11
5	3-4	7	13
6	4-5	12	15
7	5-6	34	22
8	6-7	47	23
9	7-8	29	18
10	8-9	4	6
11	9-10	2	3

Now click Insert - Charts - Column and two things happen: an empty graph area is added to the worksheet, and a new toolbar (Chart Tools) is added to the menu.



We must now tell Excel which data we would like to display. Click Select Data.

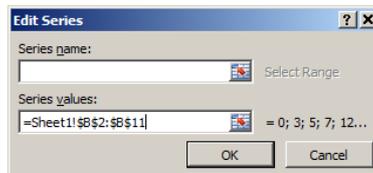


Now, there is an easy way and a difficult way. We'll take the difficult way, because the easy way doesn't always work well.

Click Add in the left pane (Legend Entries (Series)).

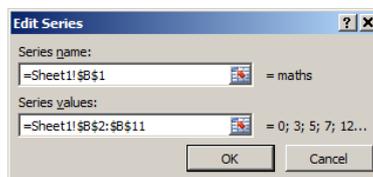


A series is a column of a data matrix that we wish to display. Here the series is in the range from B2 until B11, so we insert that range as Series Values.

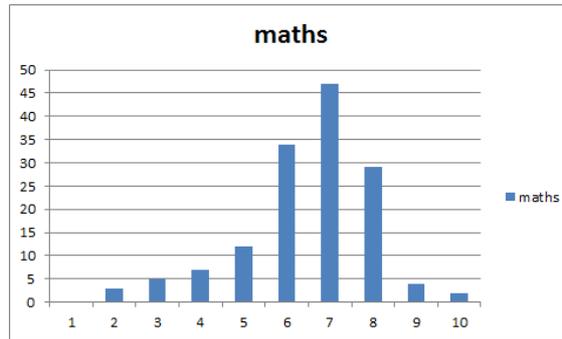


Note that we must add "Sheet1!" in front of the range, to indicate that we take the range from that specific worksheet. In our case its name is Sheet1, but it might also be called Sheet2, Students2015, or any other name.

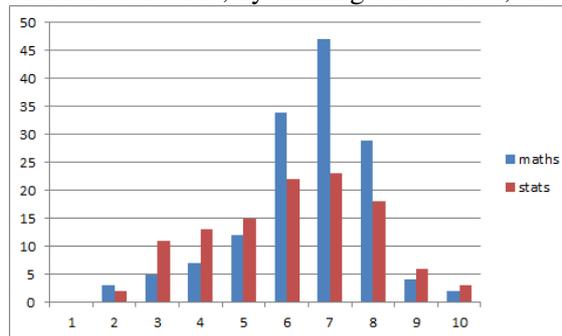
In addition, we may wish to give the series a name. Because the data represent the grades for maths, not for stats, a natural name is "maths". We may indeed type in that name, but we may also refer to the content of cell B1, so that updating this content into, for instance, "mathematics" will be reflected automatically.



Now click OK, and another OK, and you will find your first bar graph!



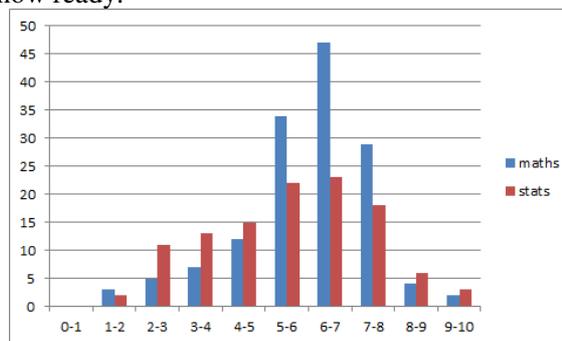
Now, can we also add the grades for stats? Yes, by clicking Select Data, and adding a second series:



Basically, we're done. You have a bar chart with the two sets of values. But one thing is still wrong: the labels at the horizontal axis. Now they are "1" to "10", but we would like to change this into "0-1" to "9-10". Go back to Select Data, but now click Edit in the right pane (Horizontal (Category) Axis Labels), and add a reference to the data in column A.



Click OK, and the graph is now ready:

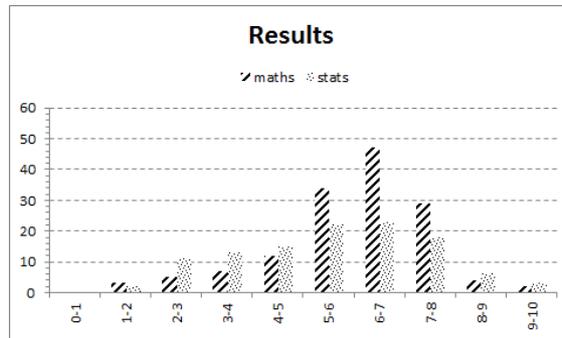


5 Mathematical modelling in Excel – finetuning graphs

Next you might wish to finetune the graph. The toolbar with Chart Tools provide many options for doing so. Many of these options can also be accessed by rightclicking in a specific part of the graph. Options include:

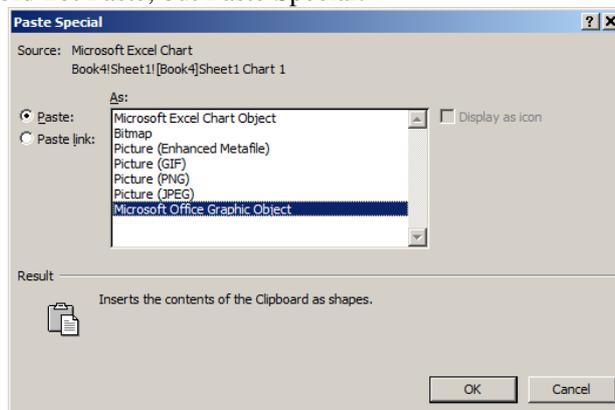
- choosing colours and/or patterns of the bars;
- changing how the axes are shown;
- setting gridlines and/or tickmarks;
- changing or moving the legend and/or title.

Below is an example where many such changes have been introduced. Try to reconstruct the graph below!



Exercise 1: Type the data on grades, and try to reproduce the graph above as exactly as possible. Look carefully, we changed many features!

Finally, when our graph is ready, we of course wish to copy it to the Word document which describes the entire research. This is less straightforward than you might think. You can just use the copy icon or keyboard shortcut (ctrl-c) when you have selected the graph, switch to Word, and use the paste icon or keyboard shortcut (ctrl-v). It works, but this is very dangerous! Change one number in your Excel file, and you will see that the graph in Excel will automatically update. That is a good thing. But now watch what has happened in your Word file: the copied graph has also updated! In other words, the copy to Word is a “soft” copy. With a soft graph, you risk that your beautifully composed graph suddenly disappears if you close Excel without saving (after all, you don’t need to hand in the Excel file at the test). To prevent this, use in Word not Paste, but Paste Special.



It will bring an extra form that allows you to control the way of copying: as a hard copy (picture or bitmap), or as a soft copy.

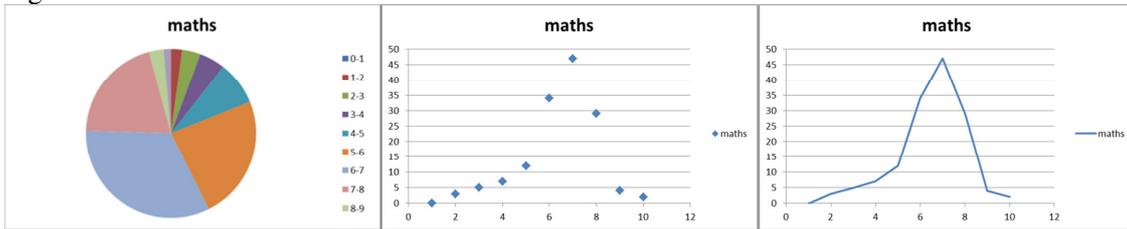
Exercise 2: Copy the graph from Excel to Word, and explore which copy types are “safe”.

6 Mathematical modelling in Excel – more types of graphs

Besides the bar chart, Excel offers many other type of graphs. We mention those that will be part of our course:

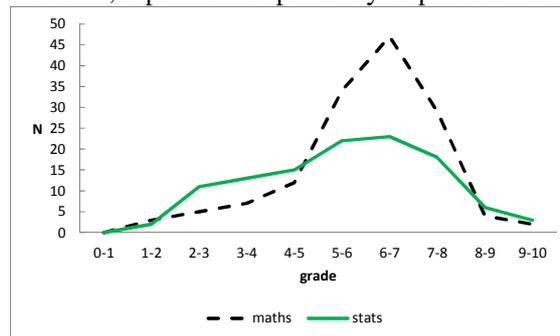
- line charts;
- dot charts;
- pie charts.

Notice that Excel in some cases uses different terms, and on top offers additional graph types. See our catalogue below:



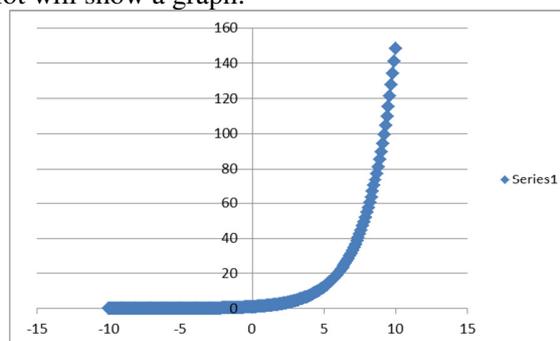
The basic principles of making such graphs are the same as those for bar charts discussed above.

Exercise 3: Using the previous data, reproduce as precisely as possible the graph below.

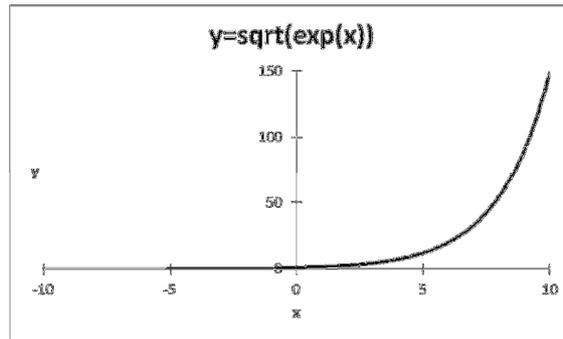


7 Mathematical modelling in Excel – function graphs

Sometimes, we want to explore the shape of a mathematical function. For example, how does the graph of $f(x) = \sqrt{e^x}$ look like? Excel provides an excellent way to make a quick-and-dirty plot. We define a column (e.g., A) that contains the number $-10.0, -9.9, -9.8$, etc. until 9.9 and 10.0 . Next we define another column (e.g., B) that contains the formula “=SQRT(EXP(A1))”, copied dozens of times to below. Next, a simple dot or line plot will show a graph:

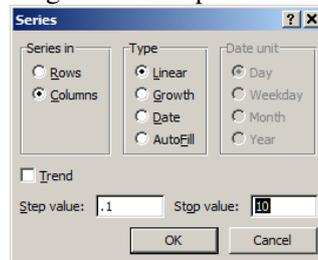


Of course, we may further refine the graph, e.g., into



This sounds great, but there is one issue: we need to type 201 numbers, from -10.0 to 10.0 . That is a dull and time-consuming activity. Fortunately, there are ways to speed this up. You can do in various ways:

- by typing -10 in cell A1, typing “=A1+0.1” in cell A2, and copying this cell down;
- by typing -10 in cell A1, typing -9.9 in cell A2, making a block of the two cells, placing the mouse cursor on the right bottom corner of cell A2, and dragging down;
- by typing -10 in cell A1 and using the menu option Fill - Series:



Exercise 4: Make a function plot of the function $f(x) = \ln x^2$ from $x \in [-10,10]$. Take particular care about what happens in the middle of the series.

8 Mathematical modelling in Excel – function graphs for functions of two variables

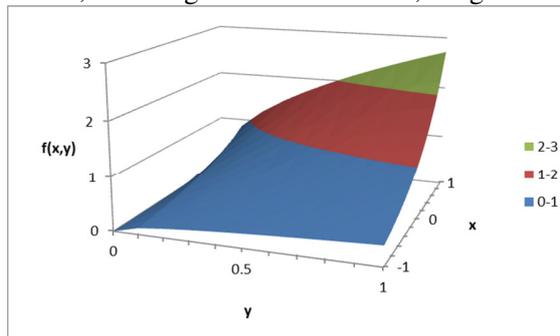
Functions of two variables play an important role in the course. They can be put in graphical form using $z = f(x, y)$. The coordinates for x and y then define a plane, and the z -coordinate the “height”. For example, consider the function $f(x, y) = e^x \sqrt{y}$, on a domain $[-1,1] \times [0,1]$. First we make a spreadsheet with a range of values for x and y :

	A	B	C	D	E	F	G	H	I	J	K	L
1		0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
2	-1											
3	-0.8											
4	-0.6											
5	-0.4											
6	-0.2											
7	0											
8	0.2											
9	0.4											
10	0.6											
11	0.8											
12	1											

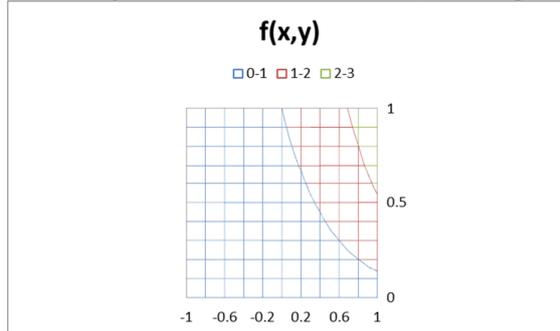
Now, we fill the blank area by the function values:

	A	B	C	D	E	F	G	H	I	J	K	L
1		0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
2	-1	0	0.116334	0.164521	0.201496	0.232667	0.26013	0.284958	0.30779	0.329041	0.349001	0.367879
3	-0.8	0	0.14209	0.200946	0.246108	0.284181	0.317724	0.348049	0.375936	0.401892	0.426271	0.449329
4	-0.6	0	0.173549	0.245436	0.300597	0.347099	0.388068	0.425108	0.459169	0.490872	0.520648	0.548812
5	-0.4	0	0.211974	0.299776	0.367149	0.423948	0.473988	0.519228	0.56083	0.599552	0.635921	0.67032
6	-0.2	0	0.258905	0.366148	0.448437	0.517811	0.57893	0.634186	0.684999	0.732295	0.776716	0.818731
7	0	0	0.316228	0.447214	0.547723	0.632456	0.707107	0.774597	0.83666	0.894427	0.948683	1
8	0.2	0	0.386241	0.546228	0.668899	0.772483	0.863662	0.946095	1.021899	1.092456	1.158724	1.221403
9	0.4	0	0.471756	0.667164	0.817106	0.943513	1.054879	1.155562	1.24815	1.334329	1.415269	1.491825
10	0.6	0	0.576205	0.814876	0.998016	1.152409	1.288433	1.411407	1.524494	1.629753	1.728614	1.822119
11	0.8	0	0.703778	0.995292	1.218979	1.407556	1.573695	1.723897	1.862021	1.990584	2.111334	2.225541
12	1	0	0.859596	1.215653	1.488864	1.719192	1.922116	2.105572	2.274278	2.431305	2.578789	2.718282

Of course, we do this by typing the formula once, and then copying. You need to work in a clever way with the \$-sign to do this. Finally, use Insert Chart - Surface. Unfortunately, Excel requires you to do this while selecting the data. In the end, tweaking a bit with the axes, we get a nice result:



Instead of a projected 3D plot, we may also make a level curve (contour plot).



Exercise 5: Reproduce the two function plots above, representing $f(x, y) = e^x \sqrt{y}$.