

Business Mathematics (BK/IBA) – Quantitative Research Methods I (EBE) Computer tutorial 3

Introduction

In this tutorial session, we will continue to work on using Microsoft Excel for quantitative modelling. The part with Microsoft's equation editor is over, we hope you enjoyed it and learned a lot. As a final exercise, here are some formulas to rehearse your skills:

Exercise 1.

$$\frac{df(x)}{dx} \equiv \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

Exercise 2.

$$\int_a^b f(x) dx = F(b) - F(a)$$

Exercise 3.

$$\left(\begin{array}{cc|c} 3 & 6 & 0 \\ 4 & -2 & 5 \end{array} \right)$$

Exercise 4.

$$x^n \cdot x^m = \underbrace{x \cdot x \cdots x}_{n \text{ terms}} \cdot \underbrace{x \cdot x \cdots x}_{m \text{ terms}} = \underbrace{x \cdot x \cdots x}_{n+m \text{ terms}} = x^{n+m}$$

Exercise 5.

$$\boxed{|\mathbf{a} \cdot \mathbf{b}| \leq \|\mathbf{a}\| \cdot \|\mathbf{b}\|}$$

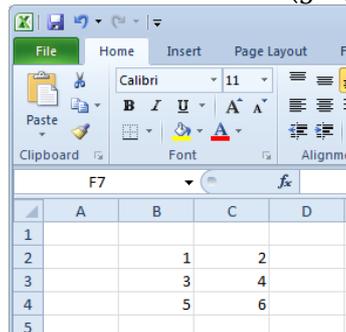
Exercise 6.

$$\boxed{\forall \xi \in \mathbb{R}: \xi^2 \geq 0 \Rightarrow (\xi + 1)^2 \geq 0}$$

Mathematical modelling in Excel – vectors and matrices

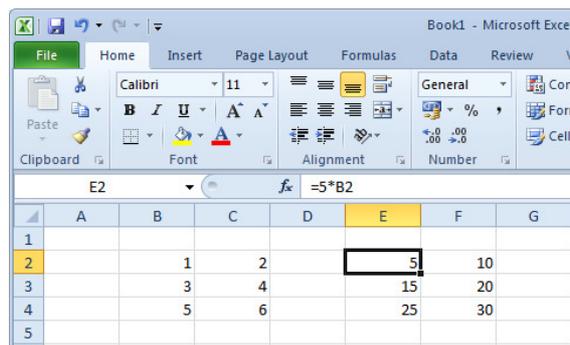
So far, we have often used Excel for column-oriented operations, organizing our data as in a data matrix. However, we can also use Excel for matrix-based calculations, including the multiplication, transposition, and inversion of matrices.

Of course, we can use Excel to type a matrix. Here, we typed $\begin{pmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{pmatrix}$:



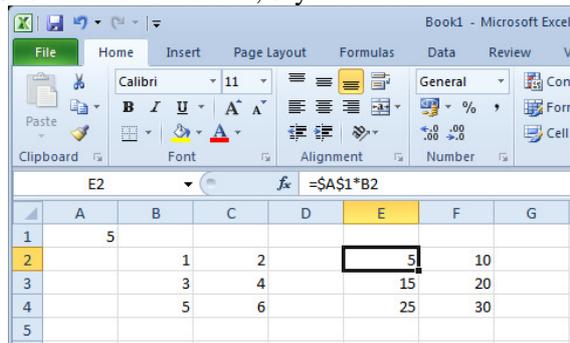
	A	B	C	D
1				
2		1	2	
3		3	4	
4		5	6	
5				

But the real action is of course using matrix algebra. Start easy: referring to the previous matrix as **A**, can we calculate $5A$?



	A	B	C	D	E	F	G
1							
2		1	2		5	10	
3		3	4		15	20	
4		5	6		25	30	
5							

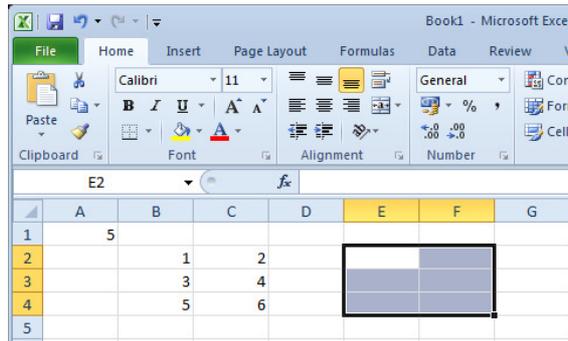
And what about cA , where c is a number in Excel, say in cell A1?



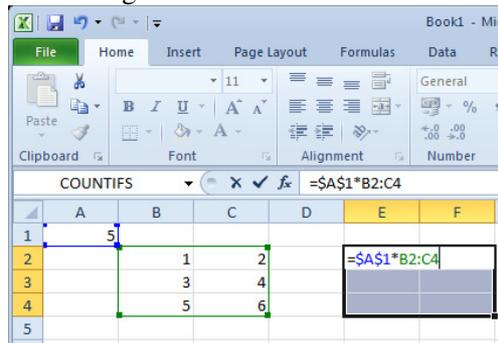
	A	B	C	D	E	F	G
1	5						
2		1	2		5	10	
3		3	4		15	20	
4		5	6		25	30	
5							

There is, however, a special way of typing a matrix equation.

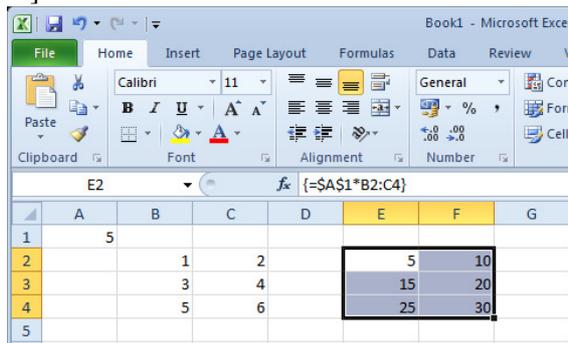
Step 1: first demarcate the area where the new matrix is supposed to be placed



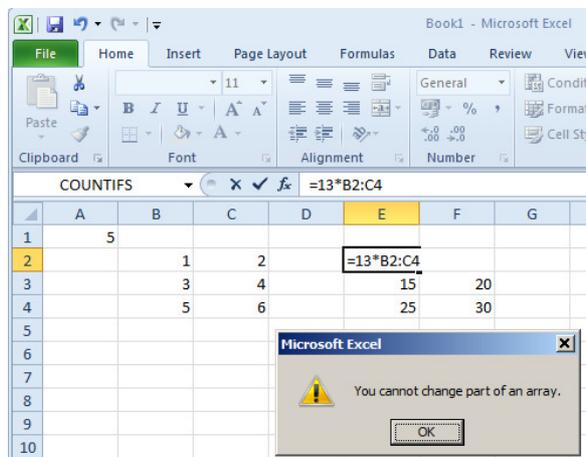
Step 2: type the formula, with the full range of cells. Here: “=5*B2:C4” or “=\$A\$1*B2:C4”



Step 3: press [ctrl][shift][enter]:



That’s all. You will see that the correct numbers show up. The big advantage of this approach is that the new matrix is – so to speak – protected. You cannot change the formula of an individual cell of the new matrix.



Another big example is that the less straightforward matrix functions (multiplication, transposition, inversion) have been implemented in Excel as well. We will show how to transposition. We want to have the transpose of the matrix $\begin{pmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{pmatrix}$, which is in the range B2:C4. Of course we could enter in E2 “=B2”, but how to get in in F2 “=B3”? There is no dollar-trick for this. There is an option to paste a matrix transposed in the menu command Paste Special, but it will give a “hard copy”: the transposed result will not change if you change the original matrix. The function “=TRANSPOSE(B2:C4)” will do the trick, provided we use the same precautions as before: copy to the full range, and use [ctrl][shift][enter].

Exercise 1. Make the transpose in the range E2:G3.

Exercise 2. Use the function MMULT to multiply the original matrix and the transposed matrix. As a

check: this should give $\begin{pmatrix} 5 & 11 & 17 \\ 11 & 25 & 39 \\ 17 & 39 & 61 \end{pmatrix}$.

Exercise 3. Use the function MINVERSE to find the inverse of this matrix. As a check: this should give an error message. Any clue?

Exercise 4. Also try switching the role of the two matrices in exercise 2 and 3, so multiply E2:G3 with B2:C4, find the transpose (which, as a check, should be $\begin{pmatrix} 35 & 44 \\ 44 & 56 \end{pmatrix}$), and calculates its inverse (which, as a check, should be $\begin{pmatrix} 2.33 \dots & -1.83 \dots \\ -1.83 \dots & 1.45 \dots \end{pmatrix}$).

Exercise 5. Change in the original matrix one cell value, and watch that the numbers in the other matrices are updated. Probably, the error message disappears as well.

To facilitate working with matrices, we may employ naming the cells, also for matrices. So we can put B2:C4 in a block and type “A” in the name box. The formula “=TRANSPOSE(B2:C4)” is not changed, but you could have typed “=TRANSPOSE(A)”

An important area of application of matrices is solving a system of equations. When, for scalars, we have $ax = b$, we may find x by calculating $\frac{b}{a}$ (provided $a \neq 0$). Of course, Excel can do that easily through “=B/A”. In the case of a system of equations, we have $\begin{cases} ax + by = c \\ dx + ey = f \end{cases}$ and solving is more difficult. Using matrix algebra helps a lot: by creating the matrix $\mathbf{A} = \begin{pmatrix} a & b \\ d & e \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} c \\ f \end{pmatrix}$, and $\mathbf{x} = \begin{pmatrix} x \\ y \end{pmatrix}$, we can write the system of equations as $\mathbf{Ax} = \mathbf{b}$, and solve it through $\mathbf{x} = \mathbf{A}^{-1}\mathbf{b}$ (provided \mathbf{A} is invertible). Let’s do this in Excel, and try to solve $\begin{cases} 5x - 3y = 11 \\ -3x - 2y = -18 \end{cases}$.

Step 1: Create in Excel a matrix with the coefficients $\begin{pmatrix} 5 & -3 \\ -3 & -2 \end{pmatrix}$ and a vector with the coefficients $\begin{pmatrix} 11 \\ -18 \end{pmatrix}$:

	A	B	C	D	E
1					
2		5	-3		11
3		-3	-2		-18

Step 2: Assign the name “A” to B2:C3 and “b” to E2:E3.

Step 3: Create a matrix “Ainv” (here in G2:H3) by means of “=MINVERSE(A)”

	A	B	C	D	E	F	G	H
1								
2		5	-3		11		0.105263	-0.15789
3		-3	-2		-18		-0.15789	-0.26316

Step 4: multiply “Ainv” and “b” into “x” (here J2:J3):

	A	B	C	D	E	F	G	H	I	J
1										
2		5	-3		11		0.105263	-0.15789		4
3		-3	-2		-18		-0.15789	-0.26316		3

So, we have found that $\mathbf{x} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$, or phrased in original variables, that $x = 4$ and $y = 3$.

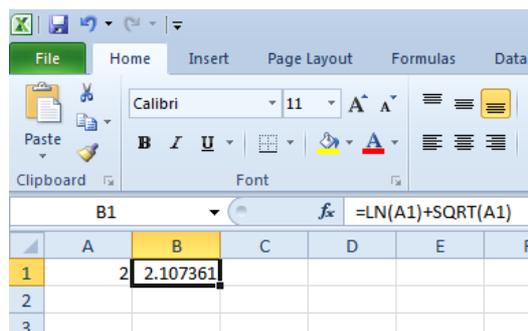
- | | |
|--|--|
| Exercise 6. Use Excel to solve the system of equations | $\begin{cases} 2x - z = 7 \\ -y + 5z = 11 \\ 3x - y + 4z = 23 \end{cases}$ |
| Exercise 7. Use Excel to solve the system of equations | $\begin{cases} 3x - 4y - 4 = 0 \\ -2x + 2y + 6 = 0 \end{cases}$ |
| Exercise 8. Use Excel to solve the system of equations | $\begin{cases} 2x - 4y = 3 \\ -6x + 12y = -6 \end{cases}$ |

Mathematical modelling in Excel – using the Solver

Excel has an add-in that is known as the Solver. Perhaps it must be turned on first, using the Options - Add-Ins. The task of the Solver is – to solve.

Let us take an example, solve for x : $\ln x + \sqrt{x} = 10$. This is not so easy by using algebra. But the Solver can find an approximate answer.

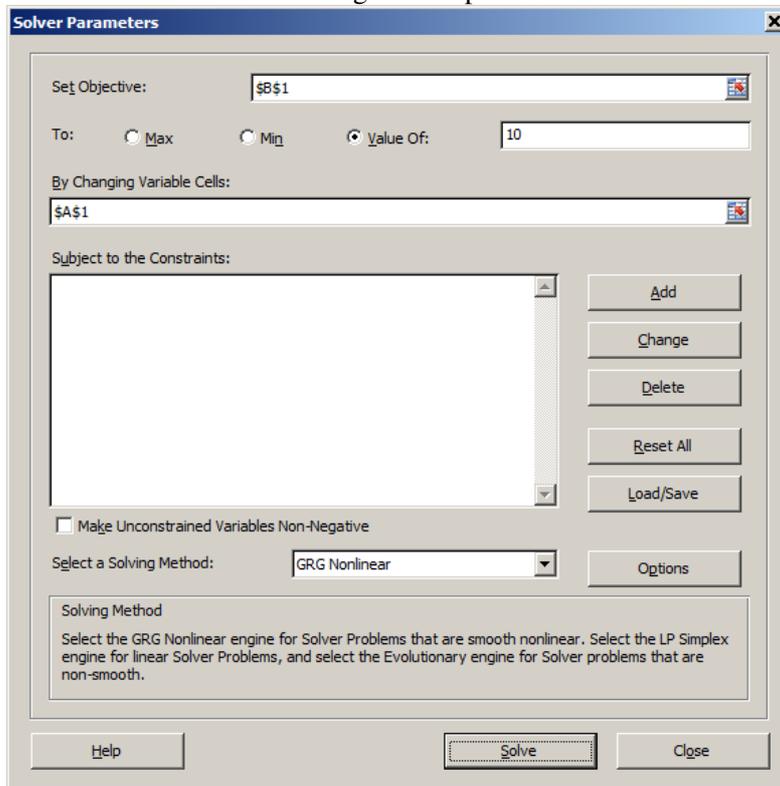
We start a new worksheet to prepare the problem Type just a trial value for x in cell A1. With a trial value, we mean a value that does not give an error (such as $x = -1$), but that is most likely not the correct answer. Let’s take $x = 2$, although we might also choose $x = 20$ or $x = 200$. Now, type the formula for $\ln x + \sqrt{x}$ in cell B1. This is done through “=LN(A1)+SQRT(A1)”. The value in B1 will be evaluated, it is something like “2.107”, certainly not 10, as required. So, indeed our trial value is wrong and should be modified.



The Solver actually will use a clever way to quickly try other trial values repeatedly, until it finds a value of x that will generate a value of y that is very close to 10. This is then the approximate solution to the problem.

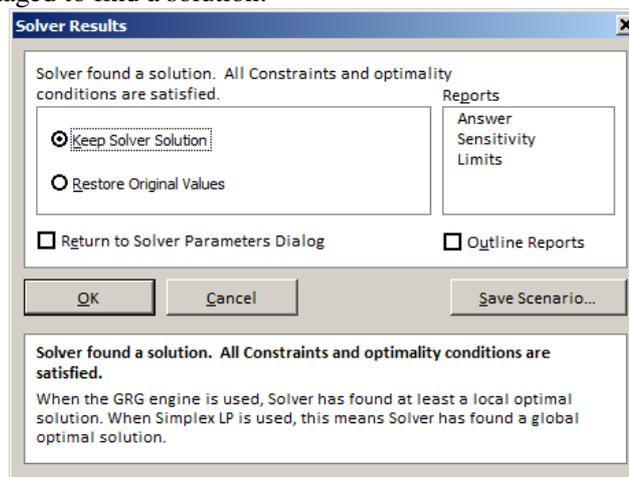
Start the Solver. You can find it in the menu on the tab Data - Analysis. Click Solver, and the Solver interface will appear. There are several fields of interest:

- “Set Objective” – here we define which cell value we want to contain a certain value (here: y);
- “To/Value Of” – here we can type which value the cell should have (here: 10);
- “By Changing Variable Cells” – here we define which cell value is to be modified (here: x);
- “Make Unconstrained Variables Non-Negative” – please turn this off.

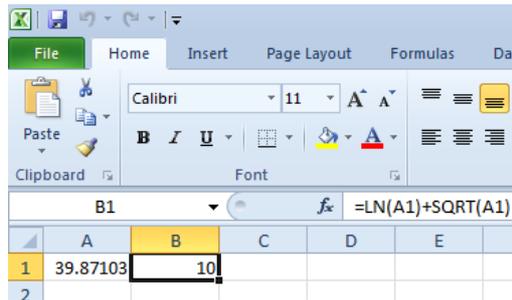


So, at “Set Objective” we add a reference to the cell that contains the formula for y (here: B1), while at “By Changing Variable Cells” we add a reference to the cell that contains the trial value of x (here: A1). Finally, at “To/Value Of” we select “Value Of” and enter a value of 10.

Now we click “Solve”, after 1 second or so, the Solver will give a message. In this case, it is a positive message: the Solver managed to find a solution:



Click OK, and you will return to the worksheet:



Note that the value in B2 has changed to 10. This means that the Solver has changed the value of A1 in such a way that B2 yields 10. So indeed, the x -value in A1 satisfies $\ln x + \sqrt{x} = 10$. Apparently, the solution to this equation is 38.87103. Now, you will understand why this add-in is called the Solver!

Mathematical modelling in Excel –multiple, no, or incorrect solutions

Consider the equation $x^2 - 5x + 6 = 0$. Elementary algebra tells us that we may complete the squares and rewrite as $(x - 2)(x - 3) = 0$, which gives $x = 2$ and $x = 3$. Two solutions! What will the Solver do?

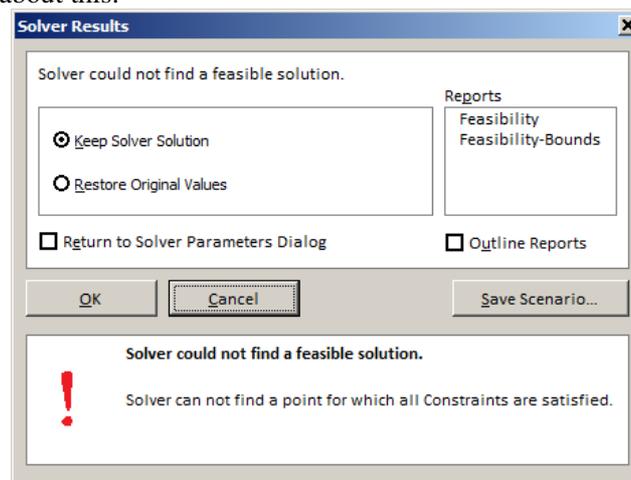
Exercise 9. Use the Excel Solver to solve $x^2 - 5x + 6 = 0$. Take a start value of $x = 1$.

Exercise 10. Use the Excel Solver to solve $x^2 - 5x + 6 = 0$. Take a start value of $x = 4$.

Clearly, the Solver finds *a* solution, not *all* solutions. Which solution it finds depends, amongst others, on where you start. This is an important limitation to keep in mind.

Exercise 11. Use the Excel Solver to solve $x^2 + 6 = 0$.

The quadratic equation $x^2 + 6 = 0$ has no solution, as can be seen by inspecting the discriminant: -24 . The Solver informs you about this:



Finally, do the next exercise:

Exercise 12. Use the Excel Solver to solve $\frac{1}{x^2} = 0$.

Now, the Solver will just find a solution, for instance $x = 3278.8$, with an associated function value of $9.3 \cdot 10^{-8}$. Somehow, the Solver considers the function value $9.3 \cdot 10^{-8}$ close enough to 0, and decides to stop.

So, what is the take-home message? The Solver can in many cases find a quite good approximation to the answer of an equation. However, it may give a misleading answer in some cases, especially when there is more than one answer or when there is no answer. Use of the Solver is therefore risky and should always be accompanied with a closer analysis. Examples of such analyses include:

- insert the solution in the equation to see if it indeed solves the equation;
- use different initial solution to start with;
- use algebra as a complement;
- make a graph as a complement.

Now finalize this part with two final exercises:

Exercise 13. Use the Excel Solver to solve $3x^2 = e^x$. Is the solution OK? How many solutions would you expect?

Exercise 14. Use the Excel Solver to solve $\frac{e^x}{x-3} = 0$. Is the solution OK? How many solutions would you expect?