Business Mathematics (BK/IBA) – Quantitative Research Methods I (EBE)
Tutorial 4 – Answers

Extreme values in two dimensions

A1 $(1,2)$

A2 $(2.8,5.75)$

A3 $x = 36, y = 12, z = 9, U = 3888$

A4 $\left(\frac{1}{3}(2p - q - 1), \frac{1}{3}(2q - p - 1)\right)$

A5 $(3,2)$ is a maximum point

A6 $(1,1)$ is a saddle point, and $\left(-\frac{5}{3}, -\frac{5}{3}\right)$ is a maximum point.

A7 For $a \neq 0$, $(1,0)$ is a minimum point and $(1 - a^3, a^2)$ is a saddle point. For $a = 0$, $(1,0)$ is a saddle point.

Systems of linear equations

A3 $\mathbf{A}^{-1} = \frac{1}{2}\begin{pmatrix} -1 & \sqrt{3} \\ -\sqrt{3} & -1 \end{pmatrix}$

A4 $\mathbf{X} = \begin{pmatrix} 2 & -2 \\ -3 & 8 \end{pmatrix}$

A5 $b = 2$

A6 $\mathbf{x} = \begin{pmatrix} \frac{20}{9} \\ -\frac{1}{3} \\ \frac{22}{9} \end{pmatrix}$

Curve fitting

A1 $y = 2.6 + 1.8x$

A2 (a) a and b scale down by a factor 1000 (b) a remains the same, and b scales down by a factor 1,000,000 (c) a scales up by a factor β, and b scales up by a factor $\frac{\beta}{\alpha}$.

A3 (B) is true.

A4 No, not when the regression line is vertical.

A5 $\begin{pmatrix} 8 \\ 10 \\ 11 \\ 14 \\ 15 \end{pmatrix} = \begin{pmatrix} 1 & 3 \\ 1 & 4 \\ 1 & 5 \\ 1 & 6 \\ 1 & 7 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} + \begin{pmatrix} e_1 \\ e_1 \\ e_3 \\ e_4 \\ e_5 \end{pmatrix}$

Gaussian elimination

A1 $\begin{cases} x_1 = \frac{20}{9} \\ x_2 = -\frac{1}{3} \\ x_3 = \frac{22}{9} \end{cases}$

A2 $\left\{ \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} \frac{2}{5}\lambda \\ \frac{3}{5}\lambda \\ \lambda \end{pmatrix} : \lambda \in \mathbb{R} \right\}$

A3 1 solution: $a \neq -\frac{5}{2}$; no solutions: $a = -\frac{5}{2}$ and $b \neq \frac{1}{2}$; more solutions: $a = -\frac{5}{2}$ and $b = \frac{1}{2}$

A4 $a = 1$: $\left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3\lambda - 4 \\ 4 - 5\lambda \\ \lambda \end{pmatrix} : \lambda \in \mathbb{R} \right\}$; $a = 0$: no solution; $a \neq 1$ and $a \neq 0$: one solution: $x = -\frac{8}{3}$, $y = \frac{16}{9a}$, $z = \frac{4}{9}$.

A5 $\mathbf{A}^{-1} = \begin{pmatrix} -2 & 1 \\ 1\frac{1}{2} & -\frac{1}{2} \end{pmatrix}$

A6 $a = 12$