Business Mathematics (BK/IBA) – Quantitative Research Methods I (EBE)
Tutorial 1 – Exercises

Instruction

In a tutorial session of 2 hours, we will obviously not be able to discuss all questions. Therefore, the following procedure applies:

- we expect students to prepare all exercises in advance;
- we will dicuss only a selection of exercises;
- exercises that were not discussed during class are nevertheless part of the course;
- students can indicate their wish list of exercises to be discussed during the session;
- teachers may invite students to answer questions, orally or on the blackboard.

We further understand that your time is limited, and in particular that your time between lecture and tutorial may be limited. In case you have no time to prepare everything, we kindly advise you to give priority to the exercises that are indicated with the **!!!** sign.

Summation

Q1 (Sydsæter & Hammond, 4/E, 3.1.1.a)
Evaluate the following sum: $\sum_{i=1}^{10} i$

!!! Q2 (Sydsæter & Hammond, 4/E, 3.1.1.c)
Evaluate the following sum: $\sum_{m=0}^{5}(2m+1)$

Q3 (Sydsæter & Hammond, 4/E, 3.1.1.e)
Evaluate the following sum: $\sum_{i=1}^{10} 2$

Q4 (Sydsæter & Hammond, 4/E, 3.1.3.a)
Express this sum in summation notation: $4+8+12+16+\cdots+4n$

Q5 (Sydsæter & Hammond, 4/E, 3.1.3.d)
Express this sum in summation notation: $a_{i1}b_{1j}+a_{i2}b_{2j}+\cdots+a_{in}b_{nj}$

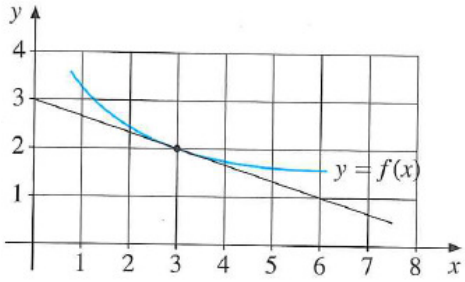
Q6 (Sydsæter & Hammond, 4/E, 3.1.3.e)
Express this sum in summation notation: $3x+9x^2+27x^3+81x^4+243x^5$

!!! Q7 (Sydsæter & Hammond, 4/E, 3.3.1.a)
Expand and compute the following double sum: $\sum_{i=1}^{3}\sum_{j=1}^{4} i\cdot 3^j$

Q8 (Sydsæter & Hammond, 4/E, 3.3.1.b)
Expand and compute the following double sum: $\sum_{s=0}^{2}\sum_{r=2}^{4}\left(\frac{rs}{r+s}\right)^2$

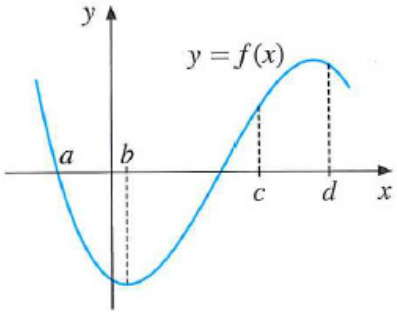
Derivatives

Q1 (Sydsæter & Hammond, 4/E, 6.1.1)



The figure shows the graph of a function f. Find the values of $f(3)$ and $f'(3)$.

!!! Q2 (Sydsæter & Hammond, 4/E, 6.2.7)



The figure shows the graph of a function f. Determine the sign of the derivative $f'(x)$ at each of the four points a, b, c, and d.

Q3 (Sydsæter & Hammond, 4/E, 6.4.1)
Let $C(x) = x^2 + 3x + 100$ be the cost function of a firm. Show that the average per unit rate of change when x is changed from 100 to $100 + h$ is

$$\frac{C(100+h) - C(100)}{h} = 203 + h \qquad (h \neq 0)$$

!!! Q4 (Sydsæter & Hammond, 4/E, 6.6.3.g)
Find the derivative of the following: $\frac{3}{\sqrt[3]{x}}$

Q5 (Sydsæter & Hammond, 4/E, 6.6.4.c)
Compute the following: $\frac{d}{dA}\left(\frac{1}{A^2\sqrt{A}}\right)$

Q6 (Sydsæter & Hammond, 4/E, 6.7.3.e)
Differentiate the function defined by the formula: $\frac{x+1}{x^5}$

!!! Q7 (Sydsæter & Hammond, 4/E, 6.7.7.b)
Find the equations for the tangents to the graph of the following function at the specified point: $y = \frac{x^2-1}{x^2+1}$ at $x = 1$

!!! Q8 If $D(P)$ denotes the demand for a product when the price per unit is P, then the revenue function $R(P)$ is given by $R(P) = P \cdot D(P)$. Find an expression for $R'(P)$.

Q9 (Sydsæter & Hammond, 4/E, 6.8.3.a)

Find the derivative of the following function: $y = \frac{1}{(x^2+x+1)^5}$

Q10 (Sydsæter & Hammond, 4/E, 6.8.6)
Compute dx/dp for the demand function $x = b - \sqrt{ap - c}$, where a, b, and c are positive constants, while x is the number of units demanded, and p is the price per unit, with $p > c/a$.

Q11 (Sydsæter & Hammond, 4/E, 6.8.9)
Suppose that $C = 20q - 4q\left(25 - \frac{1}{2}x\right)^{1/2}$, where q is a constant and $x < 50$. Find dC/dx.

!!! Q12 (Sydsæter & Hammond, 4/E, 6.10.1.f)
Find the first-order derivative of: $y = x^3 e^x$

Q13 (Sydsæter & Hammond, 4/E, 6.10.3.d)
Find the first and second-order derivatives of: $y = 5e^{2x^2-3x+1}$

Q14 (Sydsæter & Hammond, 4/E, 6.10.7.b)
Differentiate: $y = x2^x$

!!! Q15 (Sydsæter & Hammond, 4/E, 6.11.1.b)
Compute the first and second-order derivatives of: $y = x^2 - 2\ln x$

Indexing

!!! Q1 Suppose we develop a model for regional unemployment in The Netherlands, distinguishing 12 provinces (Groningen, Friesland, etc.). How would you indicate the unemployment numbers in these provinces in an efficient way?

Q2 A consultant proposes to use the symbols g for unemployment in Groningen, f for unemployment in Friesland, etc. Why is this not a good idea?

Q3 Because we are interested in the phenomenon of "brain drain", we wish to subdivide the model into 3 different labour skills: high, medium, and low. What symbols and indices do you propose?

Descriptive statistics

Q1 Compute the mean of the data set $(5,6,4,3,0,-4)$.

!!! Q2 Compute the variance of the above data set.

!!! Q3 Given a data set of size $n = 12$ with mean $\bar{x} = 8$. What is the sum of the data?

Q4 A data set of a variable measured in euro has a mean of 23 and a variance of 4. What is the standard deviation if we measure the data in cent?

Q5 In which cases does it make sense to compute a correlation coefficient of two data sets x and y?
(a) if x is the income of a random sample of married men and y the income of a random sample of married women.
(b) if in a random sample of married couples, x is the income of the man and y the income of the woman.

(c) if x denotes the stock values of 300 companies in 2012 and y the stock value of the same 300 companies in 2013.
(d) if x denotes the stock values of 300 Dutch companies and y the stock value of 300 German companies.
There may be 1 correct answer, or more, or less.

Q6 Prove that the mean deviation, defined by $\frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})$ always is equal to 0, and therefore does not convey information on the data.

!!! Q7 A data set has a variance of 5 m^2 and a coefficient of variation of 1.2. What is the mean?

Q8 Given are three data pairs (p, q): $(3, -7)$, $(5,2)$, and $(-12,0)$. Calculate the correlation coefficient.

Q9 Two paired data vectors q and h have $n = 82$ has $s_q^2 = 23$, $s_h^2 = 0.45$ and $r_{q,h} = 0.19$. Calculate the covariance.

!!! Q10 A table with prices and some other data is given below.

Commodity		28-Dec-01	11-Jul-08	Price of Commodity on 11-Jul-08 If the USD/ EURO exchange rate remained at 0.8912 (28-Dec-01)	Exchange-rate Contribution to the Total Change in Commodity Price	Direction of Real Supply-Demand Fundamentals
Rough Rice	(cents/cwt.)	369.00	1790.00	1,000.91	55.53%	+
Soybeans	(cents/bushel)	421.00	1615.50	903.33	59.62%	+
Corn	(cents/bushel)	209.00	691.00	386.38	63.20%	+
Coffee	(cents/pound)	46.20	142.25	79.54	65.29%	+
Wheat	(cents/bushel)	289.00	830.75	464.53	67.60%	+
Oats	(cents/bushel)	195.75	449.50	251.35	78.09%	+
Cocoa	(USD/mt.)	1,310.00	2912.00	1,628.29	80.13%	+
Sugar #11	(cents/pound)	7.39	13.99	7.82	93.44%	+
Live Cattle	(cents/pound)	68.17	101.20	56.59	135.07%	–
Orange Juice	(cents/pound)	89.10	123.05	68.81	159.78%	–
Lean Hogs	(cents/pound)	57.05	74.65	41.74	186.98%	–
Gold	(USD/troy oz.)	279.00	960.40	537.02	62.13%	+
Crude Oil	(USD/barrel)	19.84	145.66	81.45	51.03%	+

Suppose we have converted these data into a data matrix **X**, ignoring qualitative data, so that, for example, the number in row 1 column 1 is 369.00.
Give a formula to calculate the average price at 11-Jul-08. You don't need to do the calculation.

Q11 Use the same data as above. Give a formula for the correlation coefficient between the prices at 28-Dec-01 and 11-Jul-08.

Q12 We have seen in the lectures that the covariance of a data vector **x** and itself is equal to the variance of **x**. What about the correlation coefficient of a data vector **x** and itself?

!!! Q13 Which statement(s) is/are true?

(a) The mean of a data set is never 0.
(b) The variance of a data set is always positive.
(c) The variance of a data set is never negative.
(d) A large value of the covariance means that all data points fall almost on a straight line.
(e) A large value of the correlation coefficient means that all data points fall almost on a straight line.
(f) The coefficient of variation always falls between 0 and 1.
(g) When the mean of a data set is 0, the coefficient of variation is 0 as well.
(h) When you change measurment units (e.g., from euro to dollar), the coefficient of variation is unaffected.