Business Mathematics (BK/IBA) – Quantitative Research Methods I (EBE)
Tutorial 2 – Exercises

Instruction

In a tutorial session of 2 hours, we will obviously not be able to discuss all questions. Therefore, the following procedure applies:

- we expect students to prepare all exercises in advance;
- we will dicuss only a selection of exercises;
- exercises that were not discussed during class are nevertheless part of the course;
- students can indicate their wish list of exercises to be discussed during the session;
- teachers may invite students to answer questions, orally or on the blackboard.

!!! We further understand that your time is limited, and in particular that your time between lecture and tutorial may be limited. In case you have no time to prepare everything, we kindly advise you to give priority to the exercises that are indicated with the !!! sign.

Functions and equations

!!! Q1 (Sydsæter & Hammond, 4/E, 6.11.5.a)
Determine the domain of the function defined by: $y = \ln(x^2 - 1)$

Q2 (Sydsæter & Hammond, 4/E, 4.8.3.a)
Solve the following equation for x: $2^{2x} = 8$

!!! Q3 (Sydsæter & Hammond, 4/E, 4.8.3.c)
Solve the following equation for x: $10^{x^2-2x+2} = 100$

Q4 (Sydsæter & Hammond, 4/E, 4.9.7)
With $f(t) = Aa^t$, if $f(t + t^*) = 2f(t)$, prove that $a^{t^*} = 2$. (This shows that the doubling time t^* of the general exponential function is independent of the initial time t.)

Q5 If you have an amount of 1€, and you receive 100% interest at the end of the year, your amount will be 2€ after one year. If you instead receive 50% interest every half year, you will have $1.5^2 = 2.25$€. If you receive 25% interest every quarter, you will have $1.25^4 = 2.44$€.

(a) What amount will you have if you receive the interest n times per year?

(b) What do you find with $n = 12$ (monthly)?

(c) What do you find with $n = 365$ (daily)?

!!! Q6 (Sydsæter & Hammond, 4/E, 4.10.2.c)
Solve the following equation for x: $\ln(x^2 - 4x + 5) = 0$

Q7 (Sydsæter & Hammond, 4/E, 4.10.2.e)
Solve the following equation for x: $\frac{x\ln(x+3)}{x^2+1} = 0$

Q8 (Sydsæter & Hammond, 4/E, 4.10.3.a)
Solve the following equation for x: $3^x 4^{x+2} = 8$

Q9 (Sydsæter & Hammond, 4/E, 4.10.3.f)
Solve the following equation for x: $\log_3 x = -3$

!!! Q10 (Sydsæter & Hammond, 4/E, 4.10.7.c)
Simplify the following expression: $\exp[\ln(x^2) - 2\ln y]$

Q11 Solve: $e^{x+2} = 3$

Q12 Solve: $p^2 - 23p = 0$

Q13 Solve: $\frac{z+1}{z-1} = 0$ (with $z \neq 1$)

!!! Q14 Solve: $\alpha^2 + 8\alpha < -15$

Q15 Solve: $\begin{cases} a + 3b = 4 \\ -2a - b = -3 \end{cases}$

Q16 Solve: $x^3 - 4x^2 - 21x = 0$

Q17 Solve: $\sqrt{2-x} = \sqrt{2+x}$

Q18 Solve: $\sqrt{x-2} = \sqrt{x+2}$

Q19 Solve: $\sqrt{2-x} = -\sqrt{2+x}$

Q20 Solve: $\sum_{i=1}^{10}(i-b) = 60$

!!! Q21 Solve: $\sum_{i=0}^{n} 3 = 30$

Extreme values

Q1 (Sydsæter & Hammond, 4/E, 8.1.2.a)
Use non-calculus arguments in order to find the maximum or minimum points for the following function: $F(x) = \frac{-2}{2+x^2}$

Q2 (Sydsæter & Hammond, 4/E, 8.1.2.b)
Use non-calculus arguments in order to find the maximum or minimum points for the following function: $G(x) = 2 - \sqrt{1-x}$

!!! Q3 (Sydsæter & Hammond, 4/E, 8.1.2.c)
Use non-calculus arguments in order to find the maximum or minimum points for the following function: $H(x) = 100 - e^{-x^2}$

!!! Q4 (based on Sydsæter & Hammond, 4/E, 8.2.6)
Find possible extreme points for $f(x) = e^{3x} - 6e^x$, $x \in (-\infty, \infty)$, and investigate their nature

Q5 (based on Sydsæter & Hammond, 4/E, 8.3.4)
In an economic model, the proportion of families whose income is no more than x, and who have a home computer, is given by

$$p(x) = a + k(1 - e^{-cx}) \quad (a, k \text{, and } c \text{ are positive constants})$$

Determine $p'(x)$ and $p''(x)$. Does $p(x)$ have a maximum? Sketch the graph of p. Which restraints are needed for a and k to have $0 \leq p(x) \leq 1$?

!!! Q6 (Sydsæter & Hammond, 4/E, 8.6.6.a)
Find the extreme points of $f(x) = x^3 e^x$

Vectors

!!! Q1 (based on Sydsæter & Hammond, 4/E, 15.7.1)
Compute $\mathbf{a} - \mathbf{b}$ and $2\mathbf{a} + 3\mathbf{b}$ when $\mathbf{a} = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$.

Q2 (Sydsæter & Hammond, 4/E, 15.7.3)
If $3(x, y, z) + 5(-1,2,3) = (4,1,3)$, find x, y, and z.

Q3 (Sydsæter & Hammond, 4/E, 15.7.6)
Solve the vector equation $4\mathbf{x} - 7\mathbf{a} = 2\mathbf{x} + 8\mathbf{b} - \mathbf{a}$ for $\mathbf{x}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.

!!! Q4 (based on Sydsæter & Hammond, 4/E, 15.7.7)
If $\mathbf{a} = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$, find $\mathbf{a} \cdot \mathbf{a}$ and $\mathbf{a} \cdot (\mathbf{a} + \mathbf{b})$.

Q5 (Sydsæter & Hammond, 4/E, 15.7.8)
For what values of x is the inner product of $(x, x - 1,3)$ and $(x, x, 3x)$ equal to 0?

Q6 (based on Sydsæter & Hammond, 4/E, 15.7.11)
A firm produces the first of two different goods as its output, using the second good as its input. Its net output vector is $\begin{pmatrix} 2 \\ -1 \end{pmatrix}$. The price vector it faces is $\begin{pmatrix} 1 \\ 3 \end{pmatrix}$. Find the firm's

(a) input vector
(b) output vector
(c) costs
(d) revenue
(e) value of net output

Elasticities and approximations

!!! Q1 (Sydsæter & Hammond, 4/E, 7.4.1)
Prove that

$$\sqrt{1 + x} \approx 1 + \tfrac{1}{2}x$$

for x close to 0, and illustrate this approximation by drawing the graphs of $y = 1 + \frac{1}{2}x$ and $y = \sqrt{1 + x}$ in the same coordinate system.

Q2 (Sydsæter & Hammond, 4/E, 7.7.1.a)
Find the elasticity of the function given by the following formula: $3x^{-3}$

!!! Q3 (Sydsæter & Hammond, 4/E, 7.7.1.d)
Find the elasticity of the function given by the following formula: $\frac{A}{x\sqrt{x}}$ (A is constant)

Q4 (based on Sydsæter & Hammond, 4/E, 7.7.8)

Show that $\mathrm{El}_x\big(Af(x)\big) = \mathrm{El}_x f(x)$ (multiplicative constants vanish)