Business Mathematics (BK/IBA) – Quantitative Research Methods I (EBE)
Tutorial 5 – Exercises

Instruction

In a tutorial session of 2 hours, we will obviously not be able to discuss all questions. Therefore, the following procedure applies:

- we expect students to prepare all exercises in advance;
- we will dicuss only a selection of exercises;
- exercises that were not discussed during class are nevertheless part of the course;
- students can indicate their wish list of exercises to be discussed during the session;
- teachers may invite students to answer questions, orally or on the blackboard.

!!! We further understand that your time is limited, and in particular that your time between lecture and tutorial may be limited. In case you have no time to prepare everything, we kindly advise you to give priority to the exercises that are indicated with the !!! sign.

Implicit differentiation

!!! Q1 (Sydsæter & Hammond, 4/E, 7.1.2)
For the equation $x^2y = 1$, find dy/dx and d^2y/dx^2 by implicit differentiation. Check by solving the equation for y and then differentiating.

Example: $x^5y = 3$

Q2 (Sydsæter & Hammond, 4/E, 7.1.5)
Suppose that y is a differentiable function of x that satisfies the equation

$$2x^2 + 6xy + y^2 = 18$$

Find y' and y''at the point $(x, y) = (1,2)$.

!!! Q3 (based on Sydsæter & Hammond, 4/E, 7.4.8)

(a) The equation

$$2x^2 + 6xy + y^2 = 18$$

defines y as a differentiable function of x about the point $(x, y) = (1,2)$. Find the slope of the graph at this point by implicit differentiation.

(b) What is the linear approximation about $x = 1$?

Example: $x^2y + xy + y^2 = 9$

Numbers and units

!!! Q1 You combine data x and y from different sources, using different notations. State the value of $z = xy$.

(a) $x = 4$ million, $y = 3\text{E}7$.
(b) $x = 3 \times 10^{-5}$, $y = 2\text{E}6$.
(c) $x = 1{,}200$, $y = 0.000{,}1$.
(d) $x = 4\text{E} - 5$, $y = 3$ thousandth

Q2 You combine data with different units. State the value and unit of z.

(a) $x = 5$ km, $y = 3$ hr, $z = xy$.

(b) $x = 8$ gallon, $y = 2$ inch^2, $z = \sqrt{\frac{x}{y}}$

(c) $x = -3$ euro, $y = 4$ kg, $z = x + y$

(d) $K = 4$ B\$, $L = 3$ B\$, $z = K^{0.2}L^{0.8}$

!!! Q3 You combine data with different units. State the value and unit of z.

(a) $x = 300$ mm, $y = 0.5$ cm, $z = x + y$

(b) $x = 12\text{E}6$ s, $y = 6$ month, $z = x + y$

(c) $x = 50$ keuro, $w = 50$ \$/day, $T = 1$ yr, 1 \$ = 0.8 euro, $z = x + wT$

Constrained optimization

!!! Q1 (Sydsæter & Hammond, 4/E, 14.1.1.a)
Use Lagrange's method to find the only possible solution to the problem:
max xy subject to $x + 3y = 24$.

Q2 (Sydsæter & Hammond, 4/E, 14.3.1.b)
max (min) $x + y$ subject to $x^2 + 3xy + 3y^2 = 3$

Q3 (Sydsæter & Hammond, 4/E, 14.3.1.a)
max (min) $3xy$ subject to $x^2 + y^2 = 8$

Example: min $x^2 + y^2$ s.t. $x + 2y = 4$

Example: min $2x + y$ s.t. $x^2 + 2y = 44$

Example: min $x^2 + y^2$ s.t. $x^2 + xy + y^2 = 3$

Example: max $q = AK^{0.4}L^{0.7}$ s.t. $25K + 10L = m$

!!! Q4 A manufacturer wants to produce cylindrical cans with a content of exactly 1 liter using as few tin as possible. So the total outside surface of a can has to be minimal. Determine the height h and the radius r of upper and bottom surface of such a can (in centimeters).
Note: The surface of the top side of the can (and the bottom of the can) can be calculated by πr^2 squared centimeters (cm^2); calculation of the surface of the side of the can gives $2\pi rh$ cm^2. The content of the can has to be $\pi r^2 h = 1000$ cm^3.

Applications of integrals

!!! Q1 (Sydsæter & Hammond, 4/E, 9.2.2.d)
Compute the area bounded by the graph of the function over the indicated interval: $f(x) = 1/x^2$ in $[1,10]$

Q2 (Sydsæter & Hammond, 4/E, 9.2.4)
Compute the area A bounded by the graph of $f(x) = \frac{1}{2}(e^x + e^{-x})$, the x-axis, and the lines $x = -1$ and $x = 1$.

!!! Q3 (Sydsæter & Hammond, 4/E, 9.4.7)

Suppose the demand and supply curves are $P = f(Q) = \frac{6000}{Q+50}$ and $P = g(Q) = Q + 10$. Find the equilibrium price and quantity, and compute the consumer and producer surplus.