Business Mathematics (BK/IBA) – Quantitative Research Methods I (EBE)
Tutorial 6 – Exercises

Instruction

In a tutorial session of 2 hours, we will obviously not be able to discuss all questions. Therefore, the following procedure applies:

- we expect students to prepare all exercises in advance;
- we will dicuss only a selection of exercises;
- exercises that were not discussed during class are nevertheless part of the course;
- students can indicate their wish list of exercises to be discussed during the session;
- teachers may invite students to answer questions, orally or on the blackboard.

!!! We further understand that your time is limited, and in particular that your time between lecture and tutorial may be limited. In case you have no time to prepare everything, we kindly advise you to give priority to the exercises that are indicated with the !!! sign.

Multiple constrained optimization

!!! Q1 (Sydsæter & Hammond, 4/E, 14.6.7)

Solve the problem: max (min) $x + y$ subject to $\begin{cases} x^2 + 2y^2 + z^2 = 1 \\ x + y + z = 1 \end{cases}$

!!! Q2 (Sydsæter & Hammond, 4/E, 14.2.1)

Verify that $\frac{df^*(m)}{dm} = \lambda(m)$ holds for the problem max x^3y subject to $2x + 3y = m$.

Q3 (Sydsæter & Hammond, 4/E, 14.3.3)

(a) Find the solutions to the necessary conditions for the problem max(min) $f(x,y) = x + y$ subject to $g(x,y) = x^2 + y = 1$.

(b) Explain the solution geometrically by drawing appropriate level curves for $f(x,y)$ together with the graph of the parabola $x^2 + y = 1$. Does the associated minimization problem have a solution?

(c) Replace the constraint by $x^2 + y = 1.1$, and solve the problem in this case. Find the corresponding change in the optimal value of $f(x,y) = x + y$, and check to see if this change is approximately equal to $\lambda \times 0.1$, as suggested by the theory.

Q4 (Sydsæter & Hammond, 4/E, 14.2.4)

(a) Solve the utility maximization problem max $U(x,y) = \sqrt{x} + y$ subject to $x + 4y = 100$

(b) Suppose income increases from 100 to 101. What is the exact increase in the optimal value of $U(x,y)$? Compare with the value found in (a) for the Lagrange multiplier.

(c) Suppose we change the budget constraint to $px + qy = m$, but keep the same utility function. Derive the quantities demanded of the two goods if $m > q^2/4p$.

!!! Q5 (based on Sydsæter & Hammond, 4/E, 13.2.3)

Two weeks ago, a question was posed : "Solve the utility-maximizing problem max $U = xyz$ subject to $x + 3y + 4z = 108$, by making U a function of y and z by eliminating the variable x. Assume: $x > 0$, $y > 0$, and $z > 0$." At that time, it was solved using substitution. How would you proceed solving this question with Lagrange methode?

Linear programming

!!! Q1 (Sydsæter & Hammond, 4/E, 17.1.1.a)

Use the graphical method to solve the following LP problem:

$$\max 3x_1 + 4x_2 \text{ s.t. } \begin{cases} 3x_1 + 2x_2 \leq 6 \\ x_1 + 4x_2 \leq 4 \end{cases} \quad x_1 \geq 0, x_2 \geq 0$$

Q2 (Sydsæter & Hammond, 4/E, 17.1.4.a)

Is there a solution to the following problem?

$$\max x_1 + x_2 \text{ s.t. } \begin{cases} -x_1 + x_2 \leq -1 \\ -x_1 + 3x_2 \leq 3 \end{cases} \quad x_1 \geq 0, x_2 \geq 0$$

Mathematical modeling & Miscellaneous topics

The best way to practice is by taking an old exam. We now put the exam of 22 October 2014 on the agenda. See BlackBoard for questions and solutions.