Business Mathematics (BK/IBA) – Quantitative Research Methods I (EBE)
Tutorial 2 – Full solutions

Instruction

In a tutorial session of 2 hours, we will obviously not be able to discuss all questions. Therefore, the following procedure applies:

- we expect students to prepare all exercises in advance;
- we will dicuss only a selection of exercises;
- exercises that were not discussed during class are nevertheless part of the course;
- students can indicate their wish list of exercises to be discussed during the session;
- teachers may invite students to answer questions, orally or on the blackboard.

!!! We further understand that your time is limited, and in particular that your time between lecture and tutorial may be limited. In case you have no time to prepare everything, we kindly advise you to give priority to the exercises that are indicated with the !!! sign.

Functions and equations

!!! Q1 (Sydsæter & Hammond, 4/E, 6.11.5.a)
Determine the domain of the function defined by: $y = \ln(x^2 - 1)$

A1 $(-\infty, -1) \cup (1, \infty)$

Sol $y = f(x) = \ln(x^2 - 1)$
$\Rightarrow \text{domain}: x^2 - 1 > 0 \Rightarrow x^2 > 1 \Rightarrow (-\infty, -1) \cup (1, \infty)$

Extra Alternative ways to write: $(x < -1 \text{ or } x > 1)$ and $|x| > 1$. Please draw attention to different ways of indicating a domain.

Q2 (Sydsæter & Hammond, 4/E, 4.8.3.a)
Solve the following equation for x: $2^{2x} = 8$

A2 $1\frac{1}{2}$

Sol $2^{2x} = 8 \Rightarrow 2^{2x} = 2^3 \Rightarrow 2x = 3 \Rightarrow x = 1\frac{1}{2}$

!!! Q3 (Sydsæter & Hammond, 4/E, 4.8.3.c)
Solve the following equation for x: $10^{x^2-2x+2} = 100$

A3 $x = 0 \vee x = 2$

Sol $10^{x^2-2x+2} = 100 \Rightarrow 10^{x^2-2x+2} = 10^2 \Rightarrow x^2 - 2x + 2 = 2 \Rightarrow x^2 - 2x = 0 \Rightarrow x(x-2) = 0 \Rightarrow$
$x = 0 \vee x = 2$

Extra Mind that we prefer to solve $x^2 - 2x = 0$ not the quadratic equation if we do not really have to. However, it is allowed of course.

Q4 (Sydsæter & Hammond, 4/E, 4.9.7)
With $f(t) = Aa^t$, if $f(t + t^*) = 2f(t)$, prove that $a^{t^*} = 2$. (This shows that the doubling time t^* of the general exponential function is independent of the initial time t.)

Sol Combine $f(t + t^*) = 2f(t)$ and $f(t) = Aa^t$: $Aa^{t+t^*} = 2Aa^t \Rightarrow Aa^t a^{t^*} = 2Aa^t \Rightarrow a^{t^*} = 2$
QED

Extra Observe the meaning of this. If your savings double in 20 years, they will double again in the next 20 years, so the doubling is independent of whether the start date of the analysis is 20 years ago, now, or 20 years from now.

Also observe the use of t^*. In some cases, an ornament means something special, like in f' or in $\bar{x}$. But there are also many cases where we define a symbol which possesse an ornament. For instance, we might define the equilibrium price as $\hat{p}$ instead of p_{eq}. Here we have defined t^* as the doubling time. We might of course also have used a symbol like t_d or t_2 or T etc.

Q5 If you have an amount of 1€, and you receive 100% interest at the end of the year, your amount will be 2€ after one year. If you instead receive 50% interest every half year, you will have $1.5^2 = 2.25$€. If you receive 25% interest every quarter, you will have $1.25^4 = 2.44$€.

(a) What amount will you have if you receive the interest n times per year?

(b) What do you find with $n = 12$ (monthly)?

(c) What do you find with $n = 365$ (daily)?

A5 (a) $A_2 = \left(1 + \frac{1}{n}\right)^n A_1$ (b) 2.61 (c) 2.71

Sol (a) Let A_1 denote the amount at time 1 and A_2 the amount at time 2. Consider $n = 1$: $A_2 = 2A_1$; $n = 2$: $A_2 = \left(1\frac{1}{2}\right)^2 A_1$; $n = 4$: $A_2 = \left(1\frac{1}{4}\right)^4 A_1$; and general n: $A_2 = \left(1 + \frac{1}{n}\right)^n A_1$.

(b) with $n = 12$, this gives 2.61.

(c) with $n = 365$, it gives 2.71.

Extra In the limit $n \to \infty$, this gives $2.718282 \cdots$, also known as e. The Swiss mathematician Jacob Bernoulli was the first person to derive this formula in the seventeenth century. This is just one of the places where the mysterious number e suddenly pops up. It also pops up in differentiation (ce^x is the only function that has itself as derivative), and in integration (the indefinte integral of $\frac{1}{x}$ is the logarithm of $|x|$ with e as the base of the logarithm).

!!! Q6 (Sydsæter & Hammond, 4/E, 4.10.2.c)

Solve the following equation for x: $\ln(x^2 - 4x + 5) = 0$

A6 $x = 2$

Sol $\ln(x^2 - 4x + 5) = 0 \Rightarrow x^2 - 4x + 5 = 1 \Rightarrow x^2 - 4x + 4 = 0 \Rightarrow (x-2)^2 = 0 \Rightarrow x = 2$

Q7 (Sydsæter & Hammond, 4/E, 4.10.2.e)

Solve the following equation for x: $\frac{x\ln(x+3)}{x^2+1} = 0$

A7 $x = 0 \lor x = -2$

Sol $\frac{x\ln(x+3)}{x^2+1} = 0 \Rightarrow x\ln(x+3) = 0 \Rightarrow x = 0 \lor \ln(x+3) = 0 \Rightarrow x = 0 \lor x + 3 = 1 \Rightarrow x = 0 \lor x = -2$. Do not forget to check if these solutions are admissibale given that the denominator $x^2 + 1$ should not be 0.

Q8 (Sydsæter & Hammond, 4/E, 4.10.3.a)

Solve the following equation for x: $3^x 4^{x+2} = 8$

A8 $-\frac{\ln 2}{\ln 12}$

Sol $3^x 4^{x+2} = 8 \Rightarrow 3^x 4^x 4^2 = 8 \Rightarrow 12^x = \frac{8}{16} = \frac{1}{2} \Rightarrow x = \log_{12}\frac{1}{2} = -\log_{12} 2 = -\frac{\ln 2}{\ln 12}$

Q9 (Sydsæter & Hammond, 4/E, 4.10.3.f)

Solve the following equation for x: $\log_3 x = -3$

A9 $\frac{1}{27}$

Sol $\log_3 x = -3 \Rightarrow x = 3^{-3} = \frac{1}{27}$

!!! Q10 (Sydsæter & Hammond, 4/E, 4.10.7.c)

	Simplify the following expression: $\exp[\ln(x^2) - 2\ln y]$
A10	$\frac{x^2}{y^2}$
Sol	$\exp[\ln(x^2) - 2\ln y] = \exp[\ln(x^2)]\exp[-2\ln y] = x^2 y^{-2} = \frac{x^2}{y^2}$
Extra	Observe the use of $\exp z$ instead of e^z. The two forms are equivalent, but $e^{[\ln(x^2) - 2\ln y]}$ is less clear to read.

Q11	Solve: $e^{x+2} = 3$
A11	$x = \ln 3 - 2$
Sol	$e^{x+2} = 3 \Rightarrow e^x e^2 = 3 \Rightarrow e^x = 3e^{-2} \Rightarrow x = \ln 3e^{-2} = \ln 3 + \ln e^{-2} = \ln 3 - 2$

Q12	Solve: $p^2 - 23p = 0$
A12	$p = 0 \vee p = 23$
Sol	$p^2 - 23p = 0 \Rightarrow p(p - 23) = 0 \Rightarrow p = 0 \vee p = 23$
Extra	Not everyone knows the notation with "∨", meaning "or". You may also write "or". But we prefer you to at least understand "∨".

Q13	Solve: $\frac{z+1}{z-1} = 0$ (with $z \neq 1$)
A13	$z = -1$
Sol	$\frac{z+1}{z-1} = 0 \Rightarrow z + 1 = 0 \Rightarrow z = -1$
Extra	Note the importance of checking the fractions's validity (denominator not 0, so $z \neq 1$ indeed).

!!!	Q14	Solve: $\alpha^2 + 8\alpha < -15$
	A14	$\alpha \in (-5, -3)$
	Sol	$\alpha^2 + 8\alpha < -15 \rightarrow \alpha^2 + 8\alpha + 15 < 0 \Rightarrow (\alpha + 5)(\alpha + 3) < 0$. Now consider $(\alpha + 5)(\alpha + 3) = 0 \Rightarrow \alpha = -5 \vee \alpha = -3$. Observe that we have three parts: $\alpha < -5$, where $(\alpha + 5)(\alpha + 3) > 0$, $-5 < \alpha < -3$, where $(\alpha + 5)(\alpha + 3) < 0$, and $\alpha > -3$, where $(\alpha + 5)(\alpha + 3) > 0$. So $\alpha \in (-5, -3)$
	Extra	A sign diagram may be useful. Further, many students typically do not know symbols like "∈", but they do know the notation $(-5, -3)$, although perhaps as $\langle -5, -3 \rangle$.

Q15	Solve: $\begin{cases} a + 3b = 4 \\ -2a - b = -3 \end{cases}$
A15	$a = 1 \wedge b = 1$
Sol	Multiply first equation by 2: $\begin{cases} 2a + 6b = 8 \\ -2a - b = -3 \end{cases}$ Then add the two equations: $5b = 5 \Rightarrow b = 1$. Insert in first original equations: $a = 4 - 3b = 4 - 3 = 1$.
Extra	The word "and" instead of "∧" is also OK.

Q16	Solve: $x^3 - 4x^2 - 21x = 0$
A16	$x = 0 \vee x = 7 \vee x = -3$
Sol	$x^3 - 4x^2 - 21x = 0 \Rightarrow x(x^2 - 4x - 21) = 0 \Rightarrow x(x - 7)(x + 3) = 0 \Rightarrow x = 0 \vee x = 7 \vee x = -3$
Extra	Make sure you don't "divide out" the term with x, in which case you would lose $x = 0$.

Q17	Solve: $\sqrt{2 - x} = \sqrt{2 + x}$
A17	$x = 0$

Sol $\sqrt{2-x} = \sqrt{2+x} \Rightarrow 2-x = 2+x \Rightarrow 0 = 2x \Rightarrow x = 0$.
Extra See also Q19 below.

Q18 Solve: $\sqrt{x-2} = \sqrt{x+2}$
A18 There is no solution x.
Sol $\sqrt{x-2} = \sqrt{x+2} \Rightarrow x-2 = x+2 \Rightarrow -2 = 2$. This is never true, so there is no value of x for which the equation is true. So there is no solution x.

Q19 Solve: $\sqrt{2-x} = -\sqrt{2+x}$
A19 There is no solution x.
Sol $\sqrt{2-x} = -\sqrt{2+x} \Rightarrow 2-x = 2+x \Rightarrow 0 = 2x \Rightarrow x = 0$. However, this is not correct, as we can see by inserting the candidate solution $x = 0$ into the equation: $\sqrt{2} = -\sqrt{2}$.
Extra You can also start by observing that $\sqrt{a} = -\sqrt{b}$ implies $a = b = 0$. In this case: $2 - x = 2 + x = 0$, which would yield $x = 2 \wedge x = -2$, hence an inconsistentcy.

Q20 Solve: $\sum_{i=1}^{10}(i-b) = 60$
A20 $b = -\frac{1}{2}$
Sol Solve: $\sum_{i=1}^{10}(i-b) = 60 \Rightarrow \sum_{i=1}^{10} i - \sum_{i=1}^{10} b = 60 \Rightarrow 55 - 10b = 60 \Rightarrow b = -\frac{1}{2}$
Extra Stress that when there is only one variable, the phrase "solve" is unambiguous, and need not be extended into "solve for b".

!!! Q21 Solve: $\sum_{i=0}^{n} 3 = 30$
A21 $n = 9$
Sol $\sum_{i=0}^{n} 3 = 30 \Rightarrow 3(n+1) = 30 \Rightarrow n+1 = 10 \Rightarrow n = 9$

Extreme values

Q1 (Sydsæter & Hammond, 4/E, 8.1.2.a)
Use non-calculus arguments in order to find the maximum or minimum points for the following function: $F(x) = \frac{-2}{2+x^2}$
Sol $F(x) = \frac{-2}{2+x^2}$ has minimum -1 at $x = 0$ because: $2 + x^2 \geq 2$ and $2 + x^2 = 2$ at $x = 0$; $F(0) = \frac{-2}{2} = -1$. There is no maximum!

Q2 (Sydsæter & Hammond, 4/E, 8.1.2.b)
Use non-calculus arguments in order to find the maximum or minimum points for the following function: $G(x) = 2 - \sqrt{1-x}$
Sol $G(x) = 2 - \sqrt{1-x}$
Note 1: $1 - x \geq 0 \Rightarrow x \leq 1$
Note 2: $\sqrt{1-x} \geq 0$ and decreasing for increasing x
so: $G(x)$ has a maximum at $x = 1$; $G(1) = 2$; there is no minimum!

!!! Q3 (Sydsæter & Hammond, 4/E, 8.1.2.c)
Use non-calculus arguments in order to find the maximum or minimum points for the following function: $H(x) = 100 - e^{-x^2}$
Sol $H(x) = 100 - e^{-x^2}$
Note 1: when $x \to \pm\infty$ then $e^{-x^2} \to 0$ so $H(x) \to 100$

Note 2: x^2 has a minimum at $x = 0 \Rightarrow -x^2$ has a maximum at $x = 0 \Rightarrow e^{-x^2}$ has a maximum at $x = 0 \Rightarrow -e^{-x^2}$ has a minimum at $x = 0 \Rightarrow H(x) = 100 - e^{-x^2}$ has a minimum at $x = 0$.
The minimum is $H(0) = 100 - e^0 = 100 - 1 = 99$; there is no maximum!

!!! Q4 (based on Sydsæter & Hammond, 4/E, 8.2.6)
Find possible extreme points for $f(x) = e^{3x} - 6e^x$, $x \in (-\infty, \infty)$, and investigate their nature

A4 $f(x)$ has a minimum at $x = \frac{1}{2}\ln 2$ and no maximum.
The minimum is $f\left(\frac{1}{2}\ln 2\right) = -4\sqrt{2}$

Sol $f(x) = e^{3x} - 6e^x$ $(x \in \mathbb{R})$
$f'(x) = 3e^{3x} - 6e^x = 3e^x(e^{2x} - 2) \Rightarrow e^{2x} - 2 = 0 \Rightarrow e^{2x} = 2 \Rightarrow 2x = \ln 2 \Rightarrow x = \frac{1}{2}\ln 2$
Make a "sign diagram" for the function $f'(x)$:

x		$\frac{1}{2}\ln 2$	
$f'(x)$	$- - -$	0	$+ + +$
$f(x)$	decreasing		increasing

$\Rightarrow f(x)$ has a minimum at $x = \frac{1}{2}\ln 2$ and no maximum.
The minimum is $f\left(\frac{1}{2}\ln 2\right) = e^{\frac{3}{2}\ln 2} - 6e^{\frac{1}{2}\ln 2} = e^{\ln 2^{\frac{3}{2}}} - 6e^{\ln 2^{\frac{1}{2}}} = 2^{\frac{3}{2}} - 6 \cdot 2^{\frac{1}{2}} = 2\sqrt{2} - 6\sqrt{2} = -4\sqrt{2}$
Instead of the sign diagram, we can work with the second-order derivative. $f''(x) = 9e^{3x} - 6e^x = 3e^x(3e^{2x} - 2)$. At $x = \frac{1}{2}\ln 2$, we have $f''\left(\frac{1}{2}\ln 2\right) = 3e^{\frac{1}{2}\ln 2}\left(3e^{2\cdot\frac{1}{2}\ln 2} - 2\right) = 3\sqrt{2}(6 - 2) = 12\sqrt{2} > 0$, so the stationary point is a minimum.

Extra Working out the details with the e-to-the-power-ln is pretty important to do in detail. Also the idea of functions that "tend to" is important to understand.

Q5 (based on Sydsæter & Hammond, 4/E, 8.3.4)
In an economic model, the proportion of families whose income is no more than x, and who have a home computer, is given by

$$p(x) = a + k(1 - e^{-cx}) \quad (a, k \text{, and } c \text{ are positive constants})$$

Determine $p'(x)$ and $p''(x)$. Does $p(x)$ have a maximum? Sketch the graph of p. Which restraints are needed for a and k to have $0 \leq p(x) \leq 1$?

A5 $p'(x) = +cke^{-cx} > 0$
$p''(x) = -c^2ke^{-cx}$
there exists no maximum!
See figure A8.3.4 in the book, p.684

Sol $p(x) = a + k(1 - e^{-cx})$
$p'(x) = +cke^{-cx}$
$p''(x) = -c^2ke^{-cx}$
Necessary for an extremum: $p'(x) = +cke^{-cx} = 0$ and that is not possible, so there exists no maximum!
Note 1: $p'(x) > 0$ for all x; this implies: the function $p(x)$ is increasing
Note 2: when $x \to +\infty$ then $e^{-cx} \to 0$ so $p(x) \to a + k$
See figure A8.3.4 in the book (p.684)

!!! Q6 (Sydsæter & Hammond, 4/E, 8.6.6.a)
Find the extreme points of $f(x) = x^3e^x$

A6 there is global minimum at $x = -3$ and no maximum

Sol $f(x) = x^3e^x \Rightarrow f'(x) = 3x^2e^x + x^3e^x = x^2e^x(3 + x) = 0 \Rightarrow x = 0$ or $x = -3$

Make a “sign diagram” for the function $f'(x)$:

x		-3		0	
$f'(x)$	$---$	0	$+++$	0	$+++$
$f(x)$	decreasing		increasing		increasing

so there is global minimum at $x = -3$ and no maximum.
Alternatively, the second-order test gives $f''(x) = 6xe^x + 3x^2e^x + 3x^2e^x + x^3e^x = xe^x(6 + 6x + x^2)$. At $x = -3$, $f''(x) = -3e^{-3}(6 - 18 + 9) = 9e^{-3} > 0$, so the stationary point is a minimum.

Vectors

!!! Q1 (based on Sydsæter & Hammond, 4/E, 15.7.1)

Compute $\mathbf{a} - \mathbf{b}$ and $2\mathbf{a} + 3\mathbf{b}$ when $\mathbf{a} = \binom{2}{-1}$ and $\mathbf{b} = \binom{3}{4}$.

A1 $\binom{-1}{-5}$ and $\binom{13}{10}$

Sol $\mathbf{a} - \mathbf{b} = \binom{2}{-1} - \binom{3}{4} = \binom{2-3}{-1-4} = \binom{-1}{-5}$

$2\mathbf{a} + 3\mathbf{b} = 2\binom{2}{-1} + 3\binom{3}{4} = \binom{4}{-2} + \binom{9}{12} = \binom{13}{10}$.

Q2 (Sydsæter & Hammond, 4/E, 15.7.3)

If $3(x, y, z) + 5(-1,2,3) = (4,1,3)$, find x, y, and z.

A2 $x = 3 \wedge y = -3 \wedge z = -4$

Sol $3(x, y, z) + 5(-1,2,3) = (4,1,3) \Rightarrow (3x, 3y, 3z) + (-5,10,15) = (4,1,3) \Rightarrow (3x - 5, 3y + 10, 3z + 15) = (4,1,3) \Rightarrow 3x - 5 = 4 \wedge 3y + 10 = 1 \wedge 3z + 15 = 3 \Rightarrow x = 3 \wedge y = -3 \wedge z = -4$

Q3 (Sydsæter & Hammond, 4/E, 15.7.6)

Solve the vector equation $4\mathbf{x} - 7\mathbf{a} = 2\mathbf{x} + 8\mathbf{b} - \mathbf{a}$ for $\mathbf{x}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.

A3 $\mathbf{x} = 3\mathbf{a} + 4\mathbf{b}$

Sol $4\mathbf{x} - 7\mathbf{a} = 2\mathbf{x} + 8\mathbf{b} - \mathbf{a} \Rightarrow 2\mathbf{x} = 6\mathbf{a} + 8\mathbf{b} \Rightarrow \mathbf{x} = 3\mathbf{a} + 4\mathbf{b}$.

!!! Q4 (based on Sydsæter & Hammond, 4/E, 15.7.7)

If $\mathbf{a} = \binom{2}{-1}$ and $\mathbf{b} = \binom{3}{4}$, find $\mathbf{a} \cdot \mathbf{a}$ and $\mathbf{a} \cdot (\mathbf{a} + \mathbf{b})$.

A4 5 and 7

Sol $\mathbf{a} \cdot \mathbf{a} = \binom{2}{-1} \cdot \binom{2}{-1} = 2 \cdot 2 + -1 \cdot -1 = 4 + 1 = 5$

$\mathbf{a} \cdot (\mathbf{a} + \mathbf{b}) = \binom{2}{-1} \cdot \left(\binom{2}{-1} + \binom{3}{4}\right) = \binom{2}{-1} \cdot \binom{2+3}{-1+4} = \binom{2}{-1} \cdot \binom{5}{3} = 2 \cdot 5 + -1 \cdot 3 = 10 - 3 = 7$

Q5 (Sydsæter & Hammond, 4/E, 15.7.8)

For what values of x is the inner product of $(x, x - 1, 3)$ and $(x, x, 3x)$ equal to 0?

A5 $x = 0 \vee x = -4$

Sol $(x, x - 1, 3) \cdot (x, x, 3x) = x \cdot x + (x - 1) \cdot x + 3 \cdot 3x = x^2 + x^2 - x + 9x = 2x^2 + 8x$. Now $2x^2 + 8x = 0 \Rightarrow 2x(x + 4) = 0 \Rightarrow x = 0 \vee x = -4$.

Q6 (based on Sydsæter & Hammond, 4/E, 15.7.11)

A firm produces the first of two different goods as its output, using the second good as its input. Its net output vector is $\binom{2}{-1}$. The price vector it faces is $\binom{1}{3}$. Find the firm's

(a) input vector
(b) output vector
(c) costs
(d) revenue
(e) value of net output

A6 $\binom{0}{1}, \binom{2}{0}, 3, 2, -1$

Sol Observe that the output is apparently indicated by a positive number, and the input by a negative number.

(a) $-\binom{0}{-1} = \binom{0}{1}$

(b) $\binom{2}{0}$

(c) $-\binom{0}{-1} \cdot \binom{1}{3} = -0 \cdot 1 + 1 \cdot 3 = 0 + 3 = 3$

(d) $\binom{2}{0} \cdot \binom{1}{3} = 2 \cdot 1 + 0 \cdot 3 = 2 + 0 = 2$

(e) $\binom{2}{-1} \cdot \binom{1}{3} = 2 \cdot 1 + -1 \cdot 3 = 2 + 3 = -1$

Extra The original version in the book uses vector multiplication instead of the inner product. Also fine, but it requires to distinguish a column vector and a row vector. Moreover, vector multiplication has not yet been introduced by us.

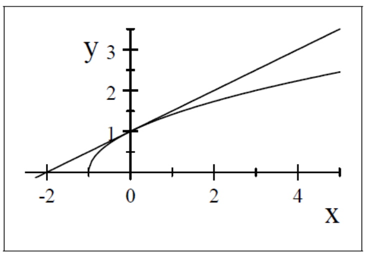
Elasticities and approximations

!!! Q1 (Sydsæter & Hammond, 4/E, 7.4.1)
Prove that

$$\sqrt{1+x} \approx 1 + \tfrac{1}{2}x$$

for x close to 0, and illustrate this approximation by drawing the graphs of $y = 1 + \frac{1}{2}x$ and $y = \sqrt{1+x}$ in the same coordinate system.

A1



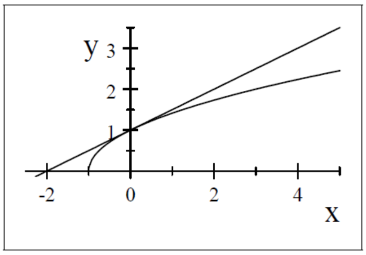
Sol $f(x) = \sqrt{1+x} = (1+x)^{\frac{1}{2}}$

$f(x) \approx f(0) + f'(0)\frac{(x-0)}{1!} = f(0) + f'(0)x$

$f(0) = \sqrt{1+0} = \sqrt{1} = 1$

$f'(x) = \frac{1}{2}(1+x)^{-\frac{1}{2}} = \frac{1}{2\sqrt{1+x}} \Rightarrow f'(0) = \frac{1}{2}$

$\Rightarrow f(x) \approx f(0) + f'(0)x = 1 + \frac{1}{2}x$



Q2 (Sydsæter & Hammond, 4/E, 7.7.1.a)
Find the elasticity of the function given by the following formula: $3x^{-3}$

A2 -3

Sol $\mathrm{El}_x f(x) = \frac{x}{f(x)} \frac{df(x)}{dx} = \frac{x}{3x^{-3}} \cdot -3 \cdot 3x^{-4} = -3$

Extra Note that the elasticity is constant

!!! Q3 (Sydsæter & Hammond, 4/E, 7.7.1.d)
Find the elasticity of the function given by the following formula: $\frac{A}{x\sqrt{x}}$ (A is constant)

A3 $-\frac{3}{2}$

Sol Observe that $\frac{A}{x\sqrt{x}} = \frac{A}{x^{3/2}} = Ax^{-3/2}$. $\mathrm{El}_x f(x) = \frac{x}{f(x)} \frac{df(x)}{dx} = \frac{x}{Ax^{-3/2}} A \cdot -\frac{3}{2} x^{-5/2} = -\frac{3}{2}$

Extra Note that the elasticity is constant

Q4 (based on Sydsæter & Hammond, 4/E, 7.7.8)
Show that $\mathrm{El}_x\big(Af(x)\big) = \mathrm{El}_x f(x)$ (multiplicative constants vanish)

A4 $\mathrm{El}_x\big(Af(x)\big) = \frac{x}{Af(x)} \frac{d\big(Af(x)\big)}{dx} = \frac{x}{Af(x)} \frac{Adf(x)}{dx} = \frac{x}{f(x)} \frac{df(x)}{dx} = \mathrm{El}_x f(x)$

Extra Stress the simplicity and elegance of such proofs. Proving basic properties may be quite simple if you use the right strategy.