vrije Universiteit *amsterdam*

Faculty of Economics and Business Administration

Exam: Business Mathematics / Quantitative Research Methods I

Code: E_BK1_BUSM / E_IBA1_BUSM / E_EBE1_QRM1

Examinator: dr. R. Heijungs
Co-reader: dr. G.J. Franx

Date: 23 April, 2015

Time: 18:00

Duration: 2 hours

Calculator allowed: Yes

Graphical calculator allowed: No

Number of questions: 3

Type of questions: Open / multiple choice

Answer in: Dutch or English (BK, EBE) / English (IBA)

Remarks:
(1) You will receive a special answer sheet for question 1
(2) You will receive normal empty paper for questions 2 and 3
(3) Please write your name and student number (7 digits) on paper (1) and (2)
(4) You may keep the questions

Credit score:

Start	Question 1	Question 2	Question 3
10	42	25	23

Grades: 11 May, 2015.

Inspection: Will be announced on BlackBoard.

Number of pages: (7 (including front page and formula sheet))

Good luck!

Question 1 (42 points)

Question 1 consists of 14 short subquestions. Each subquestion counts for 3 points. **You must give an answer only**, on a separate special answer sheet. Note the following in answering the subquestions:

- The indication "exact" means that you have to fill in an exact number, such as $12, \frac{2}{3}$ and $\sqrt{3}e^{-2}$.
- The indication "1 decimal" means you have to fill in a number at the specified accuracy, such as "-23.0". In addition, you may have to specify additional text, such as "euro".
- The indication "2 significant digits" means you have to fill in a number at the specified accuracy, such as "$1.2 \cdot 10^3$". In addition, you may have to specify additional text, such as "euro".
- The indication "text", means you have to supply a phrase, such as "There is no stationary point".
- The indication "formula" means that you have to fill in a mathematical expression, such as "$\sqrt{a^2+1}$".
- The indication "choose one" means that you have to choose one option, such as "(B)".
- The indication "choose one or more" means that you have to choose one or more options, such as "(B) and (D) and (F)".

(a) Find the derivative $\frac{dy}{dx}$ of the curve $x^2 - 4xy - 2y^2 = 6$. (formula)

(b) Solve $\mathbf{X}$ from the following matrix equation: $\mathbf{A}^{-1}\mathbf{X}' - \mathbf{B} = \mathbf{C}$, where $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$ are square invertible matrices of an equal order. Simplify as much as possible. (formula)

(c) Which statement is always true for the coefficient of variation, CV? (choose one or more)

(A) $|CV| > 0$ (C) $|CV| < 1$ (E) $|CV| < 100$

(B) $|CV| \neq 0$ (D) $|CV| \leq 1$ (F) $|CV| \leq 100$

(G) None of the above is correct.

(d) Evaluate the following integral: $\int \left(\frac{1}{x^2} + \frac{1}{x}\right) dx$. (formula)

(e) For two functions $f(x)$ and $g(x)$, it is given that $\frac{df}{dx} = \frac{dg}{dx}$ for all $x \in \mathbb{R}$. What can you conclude? (choose one or more)

(A) $f(x) = g(x)$ (C) $f(x) = g(x) + constant$ (E) $f(x)g(x) = constant$

(B) $f(x) = constant$ (D) $f(x) = g(x) \times constant$ (F) $f(x) = e^x$

(G) None of the above is correct.

(f) Solve: $\frac{2}{x-2} > 1$. (exact)

(g) Solve $\mathbf{x}$ from the following matrix equation: $\begin{pmatrix} -1 & 2 \\ 0 & 3 \end{pmatrix} \mathbf{x} = \begin{pmatrix} -4 \\ 12 \end{pmatrix}$. (exact)

(h) Find $\frac{\partial g}{\partial y}$ when $g(y, z) = \frac{z}{y-z}$. (formula)

(i) The cost of flying a certain airplane is known to be $C = \phi d + \gamma d^2$, where C is costs in euros, d is distance in km, and ϕ and γ are constants. What is the unit of γ? (text)

(k) Simplify: $\sum_{i=1}^{10} x_i - \sum_{i=0}^{9} x_i$. (formula)

(l) Given is the function $f(x,y) = x^2 + 2x^2y^2 + y^4 - 7$, with $x \in \mathbb{R}$ and $y \in \mathbb{R}$. For which values of c can we draw level curves $f(x,y) = c$ of this function? (formula)

(m) Given is $f(x) = \sqrt{\sqrt{x}}$. Evaluate $f'(16)$. (exact)

(n) The system of equations $\begin{cases} xy = 39 \\ x + y = -16 \end{cases}$ has two pairs of solutions (x, y). Find one such pair. (exact)

Question 2 (25 points)

Question 2 must be answered on the empty exam sheets. Please start at the top of a page. You must **specify all steps** you take and **use good notation principles**.

Electricity supply in the Netherlands is very reliable. However, when a power failure occurs, many companies have problems.

(a) A business analyst has calculated that nation-wide, the costs C (in euro) of a failure are related to the duration of the interrupt t (in minutes) as $C = a\left(e^{bt} - 1\right)$, where a and b are constants. We only consider $t \geq 0$. This model should satisfy the following conditions:

1) it should not predict negative costs (so $C \geq 0$)

2) the costs should increase with increasing t (so $\frac{dC}{dt} > 0$)

3) the costs should increase with increasing t at an increasing speed (so $\frac{d^2C}{dt^2} > 0$)

For which values of a and b are these three conditions satisfied? (5 points)

(b) For a specific supermarket, the formula is $C = 5.3(e^{12.1t} - 1)$. The manager considers leasing a diesel generator for backup power supply. The costs of this device are K (euro). Give a formula for the duration of the power interrupt t such that break-even ($C = K$) occurs. (5 points)

(c) The risk of a power interruption can be decreased by several technical devices. We can install diesel aggregates with a total power capacity $D \in \mathbb{R}$ or storage batteries with a total capacity $B \in \mathbb{R}$. By doing this, the risk reduces to $R = 2D^2 - 4BD + B^4 + 2$. Find all stationary points of $R(D, B)$ and determine the nature of all those stationary points for which D and B are positive. (10 points)

(d) There exist urban legends on the relation between power failures and baby booms 9 months later. The government keeps a daily record of the length of the power interruption as a vector $\mathbf{p}' = (p_1, p_2, p_3, \ldots)$. On most days it will be zero, but on some days it will be non-zero. The number of births is given by $\mathbf{b}' = (b_1, b_2, b_3, \ldots)$. At this moment, both records have been kept for 3000 consecutive days. One way of studying the relation between $\mathbf{p}$ and $\mathbf{b}$ is by computing the correlation coefficient $r_{\mathbf{p},\mathbf{b}}$, but in a "delayed" form. Give an expression for the formula to use, taking into account the 9 months "delay". (5 points)

Question 3 (23 points)

Question 3 must be answered on the empty exam sheets. Please start at the top of a page. You must **specify all steps** you take and **use good notation principles**.

(a) The grade (g) for the mathematics exam is the result of two factors: number of hours studied at home (h), and number of hours studied at university (u). For a certain student, it is known that for positive h and u, $g(h,u) = h^{0.3}u^{0.2}$. Show by using calculus that g increases when h increases, u being fixed. (6 points)

(b) How should the student divide his time budget of 150 hours over h and u in order to maximize his grade, and what is his grade (in 2 significant digits) when he allocates his time in this way? (10 points)

(c) Suppose the student attends all classes at university at the maximum level, i.e. at $u = 48$ hours, and spends another $h = 120$ hours at home. This yields a grade $g = 9.1$. (You don't have to check this). The student considers working a bit more at home. Find a linear approximation around $(h,u) = (120,48)$ and use this to approximate the grade when the student works one hour extra at home. (7 points)

Business Mathematics/Quantitative Research Methods 1
Formula sheet (September 2014)

Descriptive statistics

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

$$s_x^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$$

$$s_{x,y} = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})$$

$$CV_x = \frac{s_x}{\bar{x}} \quad r_{x,y} = \frac{s_{x,y}}{s_x s_y}$$

Derivatives

$$\frac{df(x)}{dx} = f'(x) = \lim_{h\to 0}\frac{f(x+h) - f(x)}{h}$$

$$\frac{d^2 f(x)}{dx^2} = f''(x) = \frac{d}{dx}\left(\frac{df(x)}{dx}\right)$$

$$\frac{\partial f(x,y)}{\partial x} = \lim_{h\to 0}\frac{f(x+h,y) - f(x,y)}{h}$$

Function	Derivative	Remark
A	0	constant function
$Af(x)$	$Af'(x)$	$A \in \mathbb{R}$
x^a	ax^{a-1}	$a \neq 0$
$f(x) + g(x)$	$f'(x) + g'(x)$	sum rule
$f(x) \times g(x)$	$f'(x)g(x) + f(x)g'(x)$	product rule
$\frac{f(x)}{g(x)}$	$\frac{f'(x)g(x) - f(x)g'(x)}{\left(g(x)\right)^2}$	quotient rule
$f\left(g(x)\right)$	$f'\left(g(x)\right) \times g'(x)$	chain rule
e^x	e^x	exponential function
$\ln\lvert x\rvert$	$\frac{1}{x}$	$x \neq 0$, logarithmic function

Summation

$$\sum_{i=1}^{n} x_i = x_1 + x_2 + \cdots + x_n$$

$$\sum_{i=1}^{n}\sum_{j=1}^{m} x_{i,j} = \sum_{i=1}^{n}\left(\sum_{j=1}^{m} x_{i,j}\right) = \sum_{j=1}^{m}\left(\sum_{i=1}^{n} x_{i,j}\right)$$

Equations

$$ax^2 + bx + c = 0 \Rightarrow x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Integrals

$$F'(x) = f(x) \Leftrightarrow \int f(x)dx = F(x) + C$$

$$\int_a^b f(x)dx = F(b) - F(a)$$

Matrices

$$(\mathbf{AB})' = \mathbf{B}'\mathbf{A}' \quad (\mathbf{A}')^{-1} = (\mathbf{A}^{-1})' \quad (\mathbf{AB})^{-1} = \mathbf{B}^{-1}\mathbf{A}^{-1}$$

Functions

$$a^x = e^{x \ln a}$$

Approximations and elasticities

$$f(x) \approx f(a) + f'(a)(x-a) + \frac{1}{2}f''(a)(x-a)^2$$

$$\mathrm{El}_x f(x) = \frac{x}{f(x)}f'(x)$$

Extreme values

$$\left(\frac{\partial^2 f}{\partial x^2}\right)\left(\frac{\partial^2 f}{\partial y^2}\right) - \left(\frac{\partial^2 f}{\partial x \partial y}\right)^2 > 0$$

Constrained optimization

$$\begin{cases} \max f(\mathbf{x}) \\ \text{subject to } \mathbf{g}(\mathbf{x}) = \mathbf{c} \end{cases}$$

$$\mathcal{L}(\mathbf{x}, \boldsymbol{\lambda}) = f(\mathbf{x}) - \boldsymbol{\lambda} \cdot (\mathbf{g}(\mathbf{x}) - \mathbf{c}) \quad \frac{df^*}{dc} = \lambda$$

Curve fitting

$$y = ax + b$$

$$a = \frac{n\sum xy - \sum x \sum y}{n\sum x^2 - (\sum x)^2} \quad b = \frac{\sum y - a\sum x}{n}$$

Linear programming

$$\begin{cases} \max f(\mathbf{x}) = \mathbf{c} \cdot \mathbf{x} \\ \text{subject to } \mathbf{Ax} \le \mathbf{b} \\ \text{and } \mathbf{x} \ge \mathbf{0} \end{cases}$$