vrije Universiteit *amsterdam*

Faculty of Economics and Business Administration

Exam: Business Mathematics / Quantitative Research Methods I

Code: E_BK1_BUSM / E_IBA1_BUSM / E_EBE1_QRM1

Examinator: dr. R. Heijungs
Co-reader: dr. G.J. Franx

Date: XX October, 2014

Time: XX:XX

Duration: 2 hours

Calculator allowed: Yes

Graphical calculator allowed: No

Number of questions: 3

Type of questions: Open / multiple choice

Answer in: Dutch or English (BK, EBE) / English (IBA)

Remarks:
(1) You will receive a special answer sheet for question 1
(2) You will receive normal empty paper for questions 2 and 3
(3) Please write your name and student number (7 digits) on paper (1) and (2)
(4) You may keep the questions

Credit score:

Start	Question 1	Question 2	Question 3
10	42	24	24

Grades: XX November, 2014.

Inspection: Will be announced on BlackBoard.

Number of pages: (7 (including front page and formula sheet))

Good luck!

Question 1 (42 points)

Question 1 consists of 14 short subquestions. Each subquestion counts for 3 points. You must give an answer only, on a separate sheet. Note the following in answering the subquestions:

- The indication "exact" means that you have to fill in an exact number, such as 12, $\frac{2}{3}$, $\sqrt{3}$, and e^{-2}.
- The indication "1 decimal" means you have to fill in a number at the specified accuracy, such as "–23.0". In addition, you may have to specify additional text, such as "euro".
- The indication "2 significant digits" means you have to fill in a number at the specified accuracy, such as "$12 \cdot 10^3$". In addition, you may have to specify additional text, such as "euro".
- The indication "text", means you have to supply a phrase, such as "There is no stationary point".
- The indication "formula" means that you have to fill in a mathematical expression, such as "$\sqrt{a^2+1}$".
- The indication "choose one" means that you have to choose one option, such as "B".
- The indication "choose one or more" means that you have to choose one or more options, such as "B and D and F".

(a) An American purchaser offers 5 \$/inch^2 for your woven fabric. Given that one inch is 2.54 cm, and one dollar is 0.77 euro, how much is the offer in €/cm^2? (1 decimal)

(b) Given is the matrix $\mathbf{X} = \begin{pmatrix} 1 & -4 & 0 \\ 9 & 3 & 4 \\ -5 & 2 & 8 \end{pmatrix}$. Compute $\sum_{i=2}^{3} x_{i,i-1}$. (exact or formula)

(c) Given is the function $f(x) = \frac{10^{\sqrt{x}}}{x-1}$. State the maximum domain of f. (formula)

(d) Given is the relation between x and y of the form $yx + y^2x^2 = 12$. Calculate $\frac{dy}{dx}$. (formula)

(e) Evaluate the following integral: $\int_1^5 \left(\frac{1}{2}x^2 + 5\right) dx$. (1 decimal)

(f) Solve for z: $\sqrt{3^{z-2}} = 81$. (exact)

(g) Solve for $\mathbf{Q}$: $\mathbf{Q}'\mathbf{A} - \mathbf{B} = \mathbf{C}$, where $\mathbf{A}$, $\mathbf{B}$, and $\mathbf{C}$ are invertible matrices of order 4×4. (formula)

(h) Given is $f(x,y) = \frac{x^3}{y-1}$. Calculate $\frac{\partial f}{\partial y}$. (formula)

(i) Given is a data vector $\mathbf{x}$ with length 23, mean 5.2 and coefficient of variation 12.2. Calculate the variance. (2 significant digits)

(j) Given is a matrix $\mathbf{A}$ of order 3×4 and a matrix $\mathbf{B}$ of order 4×4. What is the order of $\mathbf{AB}'\mathbf{A}'$? (formula)

(k) The correlation coefficient of two data vectors $\mathbf{x}$ and $\mathbf{y}$ is -0.9. Which statements are correct? (choose one or more)

(A) A positive value of $\mathbf{x}$ is in most cases associated with a negative value of $\mathbf{y}$.

(B) The data points fall more or less on a straight line with slope -0.9.

(C) The mean of $\mathbf{x}$ differs from the mean of $\mathbf{y}$ by -0.9.

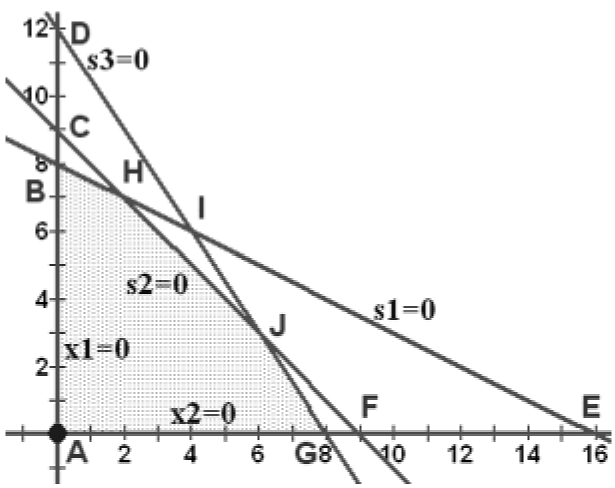
(D) The covariance of $\mathbf{x}$ and $\mathbf{y}$ is negative.

(E) None of the answers above is correct.

(l) Given is a function $f(x) = e^{2x} + x^6 + 5$. Find $\frac{d^{99}f(x)}{dx^{99}}$. (formula)

(m) Given is a function $g(t) = e^{2t} - \sqrt{4t}$. Find $\int g(t)dt$. (formula)

(n) A mathematical programming problem is graphically illustrated as below, with objective function $y = e^{x_1+x_2}$ to be maximized. All constraints are inequalities.



Formulate the problem in mathematical terms. (formula)

Question 2 (24 points)

Question 2 must be answered on the empty exam sheets. Please start at the top of page. You must specify all steps you take and use good notation principles.

(a) A company produces canned meat. Its cost function is given by $C = aL^{0.2}M^{0.8}$, where C is the cost (in euro) of producing canned meat, L is the cost (in euro) of its labour input, and M is the cost (in euro) of its input of raw meat. Investigate if C has stationary points, and if it has so, give the value of these points. (6 points)

(b) The market for canned meat is described by the demand function $Q = \alpha - \beta P^{\gamma}$, where $\alpha, \beta.$ and γ are constants, Q is the quantity, and P is the price, and by the supply function $Q = \delta + \epsilon P^{\gamma}$, where δ and ϵ are constants. In equilibrium, price and quantity are determined by these two equations simultaneously. Find a formula for the price P^* in equilibrium. (6 points)

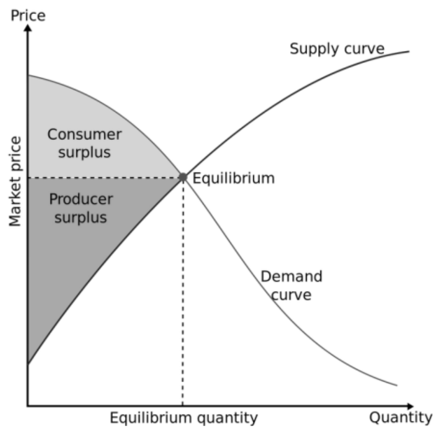
(c) The parameters of the demand function can be affected by marketing. Find a formula for the elasticity of the demanded quantity with respect to the parameter β. (4 points)

(d) The company has a profit function $\pi = bM\sqrt{L}$ (with b a positive constant) and wants to maximize profit. The budget available to the company for running its business is given by a constraint of 500 monetary units. This means that it is subject to $L + M = 600$. Formulate the constrained problem of maximizing profit, and find the optimum labour input. (8 points)

Question 3 (24 points)

Question 3 must be answered on the empty exam sheets. Please start at the top of page. You must specify all steps you take and use good notation principles.

(a) Given is a market for a product with a non-linear supply function $P = S(Q)$, and a non-linear demand function $P = D(Q)$. At market equilibrium, the price (P) and quantity (Q) of the product are indicated by symbols with an asterisk (P^* and Q^*). The diagram below defines in a conceptual way the consumer surplus.



Observe that it is an area, bounded by positive functions. Therefore, integrals provide a way to calculate the consumer surplus. Give a formula for the consumer surplus S_C, and solve this integral as far as possible. (4 points)

(b) If the quantity Q is tomatoes in kg, and the price P is in €/kg, what is the unit of the producer surplus S_P? (3 points)

(c) By lowering the price, the producer can increase the number of products sold. Give an expression for the rate of change of quantity per unit of price change. Introduce variables (x, f, etc.) whenever you need them, but define them clearly if you do so. (3 points)

(d) We next consider a market for two products. The market is expressed by the following set of equations:

$$\begin{cases} Q_1 = 5 - 8P_1 + 2P_2 \\ Q_2 = 12 - 4P_1 + 3P_2 \\ Q_1 = 7 + 4P_1 \\ Q_2 = 4 + 9P_2 \end{cases}$$

Write these equations into matrix form $\mathbf{Ax} = \mathbf{b}$, where $\mathbf{A}$ and $\mathbf{b}$ consist of numbers only, and where $\mathbf{x}$ consists of P_1, Q_1, P_2, and Q_2. (4 points)

(e) It turns out that the profit of the company is given by $\pi = 6P_1^2 - 3P_1P_2 + 3P_2^2 - 7P_1 + 112$. Find the stationary points and investigate their nature (10 points).

Business Mathematics/Quantitative Research Methods 1
Formula sheet (September 2014)

Descriptive statistics

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

$$s_x^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$$

$$s_{x,y} = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})$$

$$CV_x = \frac{s_x}{\bar{x}} \quad r_{x,y} = \frac{s_{x,y}}{s_x s_y}$$

Derivatives

$$\frac{df(x)}{dx} = f'(x) = \lim_{h\to 0}\frac{f(x+h) - f(x)}{h}$$

$$\frac{d^2 f(x)}{dx^2} = f''(x) = \frac{d}{dx}\left(\frac{df(x)}{dx}\right)$$

$$\frac{\partial f(x,y)}{\partial x} = \lim_{h\to 0}\frac{f(x+h,y) - f(x,y)}{h}$$

Function	**Derivative**	**Remark**
A	0	constant function
$Af(x)$	$Af'(x)$	$A \in \mathbb{R}$
x^a	ax^{a-1}	$a \neq 0$
$f(x) + g(x)$	$f'(x) + g'(x)$	sum rule
$f(x) \times g(x)$	$f'(x)g(x) + f(x)g'(x)$	product rule
$\frac{f(x)}{g(x)}$	$\frac{f'(x)g(x) - f(x)g'(x)}{\left(g(x)\right)^2}$	quotient rule
$f\left(g(x)\right)$	$f'\left(g(x)\right) \times g'(x)$	chain rule
e^x	e^x	exponential function
$\ln\|x\|$	$\frac{1}{x}$	$x \neq 0$, logarithmic function

Summation

$$\sum_{i=1}^{n} x_i = x_1 + x_2 + \cdots + x_n$$

$$\sum_{i=1}^{n}\sum_{j=1}^{m} x_{i,j} = \sum_{i=1}^{n}\left(\sum_{j=1}^{m} x_{i,j}\right) = \sum_{j=1}^{m}\left(\sum_{i=1}^{n} x_{i,j}\right)$$

Equations

$$ax^2 + bx + c = 0 \Rightarrow x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Integrals

$$F'(x) = f(x) \Leftrightarrow \int f(x)dx = F(x) + C$$

$$\int_a^b f(x)dx = F(b) - F(a)$$

Matrices

$$(\mathbf{AB})' = \mathbf{B}'\mathbf{A}' \quad (\mathbf{A}')^{-1} = (\mathbf{A}^{-1})' \quad (\mathbf{AB})^{-1} = \mathbf{B}^{-1}\mathbf{A}^{-1}$$

Functions

$$a^x = e^{x \ln a}$$

Approximations and elasticities

$$f(x) \approx f(a) + f'(a)(x-a) + \frac{1}{2}f''(a)(x-a)^2$$

$$\mathrm{El}_x f(x) = \frac{x}{f(x)} f'(x)$$

Extreme values

$$\left(\frac{\partial^2 f}{\partial x^2}\right)\left(\frac{\partial^2 f}{\partial y^2}\right) - \left(\frac{\partial^2 f}{\partial x \partial y}\right)^2 > 0$$

Constrained optimization

$$\begin{cases} \max f(\mathbf{x}) \\ \text{subject to } \mathbf{g}(\mathbf{x}) = \mathbf{c} \end{cases}$$

$$\mathcal{L}(\mathbf{x}, \boldsymbol{\lambda}) = f(\mathbf{x}) - \boldsymbol{\lambda} \cdot (\mathbf{g}(\mathbf{x}) - \mathbf{c}) \quad \frac{df^*}{dc} = \lambda$$

Curve fitting

$$y = ax + b$$

$$b = \frac{n\sum xy - \sum x \sum y}{n\sum x^2 - (\sum x)^2} \quad a = \frac{\sum y - b\sum x}{n}$$

Linear programming

$$\begin{cases} \max f(\mathbf{x}) = \mathbf{c} \cdot \mathbf{x} \\ \text{subject to } \mathbf{Ax} \le \mathbf{b} \\ \text{and } \mathbf{x} \ge \mathbf{0} \end{cases}$$