Minimum Funding Ratios for Defined-Benefit Pension Funds[*]

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Abstract

We compute minimum nominal funding ratios for Defined Benefit (DB) plans based on the expected utility that can be achieved in a Defined Contribution (DC) pension scheme. Using Monte Carlo simulation, expected utility is computed for three different specifications of utility: power utility, mean-shortfall and mean-downside deviation. Depending on risk aversion and the level of sophistication assumed for the DC-scheme, minimum acceptable funding ratios are between 0.87 and 1.20 in nominal terms. For relative risk aversion of 5 and a DC-scheme with a fixed-contribution setup, the minimum nominal funding ratio is between 0.87 and 0.98. The attractiveness of the DB plan increases with the expected equity premium and the fraction invested in stocks. We conclude that the expected value of intergenerational solidarity, providing time-diversification to its participants, can be large. Minimum funding ratios in real (inflation-adjusted) terms lie between 0.56 and 0.79. Given a DB pension fund with a funding ratio of 1.30, a participant in a DC plan has to pay a 2.7 to 6.1%-point higher contribution on average to achieve equal expected utility.

Keywords: defined-benefit pension fund, minimum funding ratio, individual efficiency, defined-contribution

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1 Introduction

This paper addresses the issue of comparing a defined-benefit pension scheme with a defined-contribution pension scheme. Most countries have one, or both of these schemes in place that provide employer-sponsored retirement benefits. The key difference between DB and DC is that DB embeds intergeneration solidarity whereas DC does not. Under DB, retirement benefits in terms of final pension are accumulated while contribution is a fraction of wage, regardless of age. However, since the present value of the final pension increases with age, the younger workers are indirectly subsidizing the old. Negative shocks to the financial position of the fund are mostly borne by the younger generation, who have a prospect of receiving the same solidarity when they are older. This mechanism is best described as intergenerational solidarity. Under DC, where workers contribute to an individual retirement fund, this mechanism is absent. Given the existence of both systems, in some countries occurring side by side, the subject of this paper is the value of intergenerational solidarity, as measured by the minimum funding ratio under which an individual would still voluntarily join the defined benefit fund.

From existing research we know that intergenerational solidarity is welfare enhancing for participants, given an initially sound financial position of the DB pension fund. (The financial position of a DB fund is usually expressed as the funding ratio, assets divided by liabilities.) [Bodie et al., 1985], [Cui et al., 2006] and [Gollier, 2008] analyze the value of intergenerational transfers and find that the embedded solidarity in funded pension schemes is welfare enhancing. Risk sharing between generations creates a diversification opportunity that is not available elsewhere. Each generation pays relatively more when young, but receive insurance to downside shocks in later life. In this paper we take the existing insights one step further by computing the minimum nominal funding ratio of a DB pension fund at which a 25-year old would only just join, if given the choice between DB and DC. The relevance for policymakers and regulators is that the results provide a baseline result on the funding ratios below which intergenerational solidarity might come under pressure. A defined-contribution (DC) pension scheme can be seen as a fall back option for societies that do not want to keep the DB funds up and running. As such, the resulting funding ratios in this paper provide a relevant benchmark to evaluate DB pension fund attractiveness.

The method of analysis in this paper is Monte Carlo simulation. This has the disadvantage of not getting analytical expressions, but it can incorporate the typical ingredients of a DB pension fund, and allows for evaluation with different utility functions. The simulation approach also mimics the ALM modeling approach that is used by most pension funds in the Netherlands, see [Boender, 1997]. Based on a real life ALM model [Boender et al., 2000] compare DB and DC in terms of riskiness of retirement benefits, given a specified pension ambition. Our analysis differs in that we compute indifference funding ratios for several utility specifications.

The rest of this paper proceeds as follows. Section 2 presents the model used for simulating the DB pension fund as well as the method used for comparing DB and DC. Section 4 presents the nominal DB funding ratios for which DC and DB have equal utility and also the average contribution level that makes DB and DC participants equally well-off. Section 5 studies a reverse question by computing the fixed level of contributions necessary in a DC scheme to achieve equal utility to a DB scheme. Section 6 concludes.

2 The DB pension fund

The defined-benefit (DB) pension fund that we consider in this paper is a private, employer-funded pension scheme where participants pay a fraction of wages as contribution, and receive a yearly increase in future pension benefits of 1.75% of their average wage. The accrual of 1.75% is such that, at retirement, the benefits accrue to 70% of final wage.[1] A level of 70% represents the expectation of current pension plan participants, seen as a target level of pension benefits that allows a continuation of lifestyle beyond retirement. The embedded solidarity in the DB pension contract modeled here is seen in the fact that the contribution level is not age-dependent: older participants are closer to retirement than younger ones, but still pay the same contribution as a fraction of wages. Thus, the younger generation ‘sponsors’ the older. The benefits of this time-diversification effect are the subject of study in this paper. We should note, however, that our comparison of DB and DC does not take into account the added benefits of portability in DC, and default risks in DB. The portability of a DC-pension compares favorably to defined-benefit, which implictly taxes an employee who leaves the pension plan before the age of 40 (approximately). That has a negative effect on labor mobility. Also, we disregard the effect of DB plans being vulnerable to the default of a sponsoring company. If such a risk materializes, active participants suffer from the lack of new employees that can provide the time-diversifcation that, as we show in this paper, is quite valuable.

To avoid aging effects, we assume that the age of pension fund participants exactly matches that of the latest 1995-2000 survivor table from the Dutch Society of Actuaries. The number of participants in the pension fund is equal to the sum of those aged 25 and over in the survivorship table, divided by 1000, which gives 544,000 participants, of which 160,000 are retired and 384,000 are active. The wage for an active participant (aged under 65) is equal to the average income per active worker in the Netherlands, as supplied by the Netherlands Bureau of Statistics (CBS). The average wage amounts to € 29,300 in 2007, the latest year for

[1] In fact, this is more than provided for by the pension fund in the Netherlands, where the first pay-as-you-go pillar of pension provisions, the AOW, provides a minimum level of retirement benefits. Changing accrual to 1.25%, a target of 50%, and assuming 20% is provided by AOW, does not impact the outcomes. Computations are available upon request.

which data is available[2]. The wage increases with the yearly inflation rate. The initial sum of wages is €10.96 bln.

Our economy has a constant inflation rate of 2%, a nominal risk-free interest rate of 4% and normally distributed stock return with a mean return of 7% and standard deviation of 18%. The inflation rate is based on the ceiling of the ECB's inflation target of 2 percent. The risk-free rate is somewhat lower than for example the US T-bill long-term return of 5% (see [Siegel, 2002] and [Hoevenaars et al., 2008]), but such a return is not riskless in the sense of a fixed rate of return. The mean stock return of 7% implies an equity premium of 3%, which is the same as in [Van Rooij et al., 2004] and [Lucas and Zeldes, 2006]. [Cui et al., 2006] use an equity premium of 4% and volatility of 20%. Note that the current setup does not assume mean reversion in stock returns. The baseline investment policy for the DB fund is assumed to be 50% stocks and 50% bonds, which is a representative portfolio composition for pension funds. We analyze the sensitivity of the results to alternative asset mixes. The funding ratio of the DB pension fund is defined as assets divided by the liabilities.

The present value of liabilities is computed using a fixed 4% discount rate. Thus, the resulting funding ratio (assets divided by liabilities) is a nominal funding ratio. This conforms to Dutch and US regulatory practice, where pension funds have to report nominal funding ratios, and are called under- or overfunded in terms of the nominal funding ratio. Sensitivity to market valuation of liabilities and real funding ratios are separately examined in Section 4.3. The initial value for the liabilities in the simulation is €52.7 bln, computed as the actuarial value of total pension benefits for all members of the pension fund. Given our stationary setup (population equal to mortality table) this is by definition equal to the difference between pension payments and the value of new pension rights, divided by 0.04. Table 1 summarizes the parameter values for our economy and the properties of the defined benefit pension fund.

<<INSERT TABLE 1 HERE >>

The funding policy prescribes how contribution is set and indexation-cuts are applied, given the financial situation of the pension fund. The policies are summarized in Table 2. The DB plan includes a form of conditional indexation, i.e., the inflation correction (here: 2%) is only applied to the pension rights when the funding ratio is high enough.

<< INSERT TABLE 2 HERE >>

Note that DB pension plans do not necessarily have constant contribution levels across age cohorts. For example, one can imagine DB plans with cohort-specific accounts, while

[2] I have also done the simulation with an age-specific wage profile, with no significant impact on the results. The intuition is that the final pension of DC and DB participants is affected in the same way by a rising wage profile.

retaining some form of intergenerational solidarity, see [Teulings and De Vries, 2006]. This is beyond the scope of the current paper, however.

2.1 Simulation setup

The simulation of economic variables is done in the following way. First, we define two asset returns in a vector x_t as

$$x_t = \begin{pmatrix} x_{1,t} \\ x_{2,t} \end{pmatrix} \tag{1}$$

where $x_{1,t}$ is the risk free rate and $x_{2,t}$ is the return on stocks. In the simulation, both are drawn independently from a normal distribution, with parameter values as outlined above. These two variables are the only source of uncertainty in the model, since we have stationary demographic dynamics and the inflation rate is assumed fixed. Moreover, in the baseline setup the interest rate is fixed and only the stock return is simulated. Section 4.3 introduces uncertainty in the interest rate, to evaluate the impact of market valuation with stochastic interest rates on the outcomes. The details are discussed in Section 4.3.

With a realization of x_t for t=1 to 40, the assets of the pension fund are updated by

$$A_{t+1} = A_t \cdot (\alpha \cdot x_{2,1} + (1-\alpha) \cdot x_{1,t}) - PP_t + C_t, \tag{2}$$

where A_t are the assets, α is the fraction in stocks, PP_t is the pension payments and C_t the contributions in period t. Pension payments are influenced by the indexation level I_t, which, together with contributions C_t comprises the funding policy of the DB pension fund. Appendix B contains a complete description of the funding policy rules used in the simulation. The policy rules for indexation and contribution are piecewise linear functions of the nominal funding ratio (total assets, A_t, divided by liabilities L_t), where specific levels of the funding ratio form the 'kinks' where the marginal impact of the funding ratio on policy changes. See also Table 2.

The investment policy of the DB-pension fund is simply a fixed fraction, α, of assets. For the individual saver, we consider a linear investment rule in terms of the time to retirement, see Section 3.

Given the initial level of liabilities, described above, liabilities evolve as

$$L_{t+1} = L_t \cdot (1+r) \cdot I_t + NR_t - PP_t, \tag{3}$$

where r is the interest rate, I_t the indexation cut in year t, NR_t the new pension rights and PP_t the pension payments. Equation (3) is almost a definition: new pension rights add to liabilities, while pension payments are paid out of them. We take indexation to impact all participants (active and retired) equally, proportional to their share of pension liabilities, so it multiplies with L_t. The same holds for interest income, so that L_t grows with the interest rate.

Given the evolution of assets and liabilities, the simulation is easily performed by generating for each period a new stock return $x_{2,t}$, computing the new funding ratio and the new values for the policy decisions on contribution and indexation. This is a typical simulation setup for Asset/Liability Management and creates a representative set of scenarios for contribution, acquired pension rights at retirement and funding ratios.

3 Comparison of DB and DC

To make an even comparison of DB with DC, our primary method of comparison is that of matching contributions. The first method is a 1-on-1 match of every contribution payment by the DB-participant at each period in each scenario. This gives the most direct comparison of how the pay-off between DC and DB compares, since the contribution paths are exactly similar. The second method assumes DC-contributions are a fixed percentage of wages, equal to the average fraction paid by the DB-participant over all periods and scenarios. Thus, the second method is more typical of how participants in a DC-scheme would contribute, namely, through a fixed fraction of their wage. We assume that the DB and DC-participant live from 25 to at least 65, and that they cannot switch pension plans or change employer. This fits with our aim of giving a clean comparison of what utility the participants can derive from a pension plan. In practice, a participant to a DB-scheme will incur a loss when switching employers. At the same time, a typical participant of a DC-scheme might lack the discipline to contribute significantly to his pension plan when young, which is automatic in a DB-scheme.

With respect to the investment policy, we assume a fixed mix for the pension fund, i.e., a fixed fraction of wealth is held in stocks. For the DC scheme we want take lifecycle considerations into account. For a given utility specification it is possible to optimize over the investment strategy, see [Gollier, 2005] and [Siegmann, 2005]. However, individual savers do not seem to be good optimizers, e.g., [Benartzi and Thaler, 2001]. Therefore, in the DC scheme we just allow for a simple parameter that specifies a time-dependent decision rule on the fraction α invested in stocks. Specifically, consider the following linear decision rule for the asset mix with parameter t_0 as

$$\alpha(t) = \frac{t - 65}{t_0 - 65}, \tag{4}$$

where t is the age of the DC-participant and t_0 is the age at which the fraction in stocks starts decreasing. In addition, $\alpha(t)$ is limited to lie between 0 and 1. Equation (4) describes an age-dependent asset mix with an initial fraction of 1 and t_0 the number of years before retirement that the stock investment is decreased with $1/t_0$ per year. At retirement (65) the fraction is 0. The decision rule in (4) reflects an investment policy that has a high allocation to stocks when the participant is young and the wage income is a relatively high part of lifetime income. When the participant is older, the allocation to stocks is low, as the pension wealth is high relative to the remaining lifetime wages. This is the most basic representation of life-cycle investing. In the following, we consider values for t_0 of both 10 and 20, representing a more (10) and less (20) aggressive investment strategy towards retirement. Note that the linear investment rule in (4) is the same as used in [Boender et al., 2000] and [Gollier, 2008] for an individual investor.

A final challenge is to make pension outcomes comparable, as the outcome of a DC plan is typically a sum of money, while a DB-participant obtains a pension in terms of the fraction of final wage. We choose to convert the amount built-up by the DC-participant to a pension in terms of final wage, by dividing by the annuity cost of an indexed pension and then by final salary. For correctly comparing DB and DC outcomes, the annuity costs should reflect (i) the cost of purchasing an indexed pension, and (ii) the expected level of indexation cuts in the DB pension. The first requirement is easily met by computing an annuity cost at the 2% real interest rate. The second requirement is met by looking backwards at the average indexation cuts experienced by the DB-participant. If we assume that future indexation cuts will match the historical average, we can multiply the annuity cost by the ratio of the average final DB-pension over 0.7, the fully indexed pension level. Thus, an average DB pension of 63%, a 10% shortfall, is matched by a 10% cheaper annuity for the DC-participant.[3] Note that the ultimate annuity cost is not the price of an available pension annuity, but the proxy for the implicit costs of the defined benefit pension promise.

Differences between DB and DC plans that we do not consider are for example the administration costs (lower for DB) and portability (straightforward under DC, complicated under DB). We also leave out the issue of a sponsoring company of a DB plan going bankrupt, which complicates our analysis and is dependent on the legal treatment of a company pension plan. We assume a strict setup of entry and exit: an individuals enter the DB and DC plan aged 25 and retire at 65. The way of computing the ’final pension’ for DC members at 65 is not completely in accordance with empirically observed behavior, but it does make the pension outcomes comparable numerically, which is the main goal of this paper.

3.1 Measuring utility of pension outcomes

We use a utility function to map the range of pension outcomes from the Monte Carlo simulation to a risk-adjusted outcome. Three types of utility functions that are often used in

[3] An alternative method is to simulate retirement-paths for the DB and DC-participant. However, this requires us to compare streams of pension payments, adding to the complexity of the model.

the financial literature are considered: power utility, mean-shortfall, and mean-downside deviation.

The first utility function is power utility. Given a pay-off *x*, power utility is defined as

$$u(x) = \frac{1}{1-\gamma} x^{1-\gamma} \tag{5}$$

where γ is the coefficient of relative risk aversion. Relative risk aversion is constant since power utility is of the Constant Relative Risk Aversion (CRRA) family of utility functions. The power utility function is used abundantly in both theoretical and empirical research because of its attractive properties. Also, as pointed out by [Campbell and Viceira, 2002], the long-run behavior of the economy suggests that risk aversion cannot depend strongly on wealth. With respect to the level of relative risk aversion, [Dalal and Arshanapalli, 1993] find a value of 1.3 based on the holdings of risky assets by US-households, while [Chiappori and Paiella, 2006] find a median value of 3 and average of 4.2 based on financial holdings of Italian households. In the results we present outcomes for values of γ of 1, 3 and 5.

The second type, mean-shortfall is given by

$$u(x) = \begin{cases} x & \text{if } x \geq r, \\ -\gamma \cdot (r - x) & \text{if } x < r, \end{cases} \tag{6}$$

where *r* is the reference value of, in this case, the level of final pension. Loss aversion boils down to penalizing realizations of *x* below *r* with a penalty of γ. This specification is a linearized version of that originally proposed by [Kahneman and Tversky, 1979] and also used as such by [Benartzi and Thaler, 1995]. [Kahneman and Tversky, 1979] estimate γ to be 2.25. In the results we present outcomes for a value of 2.25 as well as 5.

The third type, mean-downside deviation is comparable to mean-shortfall, but gives more weight to larger shortfalls below the reference point by using a quadratic penalty specification:

$$u(x) = \begin{cases} x & \text{if } x \geq r, \\ -\gamma \cdot (r - x)^2 & \text{if } x < r. \end{cases} \tag{7}$$

Downside deviation is proposed by [Boender, 1997] to use in a pension fund context and actively used in most ALM practices at pension funds in the Netherlands. Its widespread use by DB pension funds suggest that outcomes with downside deviation could be seen as most representative from the perspective of the pension fund sector itself. Also [Sortino and Van der Meer, 1991] gives an economic rationale for using downside deviation as a risk measure in an investment context.

4 Results

This section presents the results of the simulation model, in terms of the funding ratios that makes a DB-participant just as well of as a DC-participant, in terms of achieving equal utility. Hence, the higher the reported funding ratios, the less attractive a DB-scheme is for an individual. The lower the reported funding ratio, the more attractive a DB-scheme is. For example, a funding ratio below 100% indicates that the intergenerational benefits of the DB-scheme are large, i.e., an individual would voluntary join an (nominally) underfunded DB-scheme.

Based on 1000 simulations, Table 3 shows the results for both exact and average matching of contributions, and four different stock investment choices for the DC saver. The indifference funding ratio (IFR) is computed using an iterative goal seek procedure.

<<INSERT TABLE 3 HERE >>

Panel A gives the results when the average contribution rate under DB is matched by the DC individual, panel B gives the results for exact matching of DB-contributions by the DC-participant.

Consider first the results in Panel A of Table 3. The indifference funding ratios vary between 0.87 and 1.16 for power utility, between 1.02 and 1.08 for expected shortfall and between 1.00 and 1.06 for downside deviation. As the range of outcomes is biggest for power utility maximizers, the rest of the discussion will focus on those outcomes.

For all four investment variants, the funding ratios are decreasing in the level of risk aversion. This is as expected, as the guaranteed DB pension (minus indexation cuts) becomes more attractive at higher levels of risk aversion. In our setup, this translates to a lower funding ratios that is necessary to arrive at equal utility compared to DC.

In panel A, the best DC investment strategy is the one that gives the highest funding ratio. E.g., a high funding ratio of, say, 1.2, means that the DC scheme is preferred for all DB-funding ratios below 120%. Hence, we see that there is no clear dominating investment strategy for the DC scheme at all risk aversion levels. For the highest risk averse individual (CRRA 5), a fixed 50% mix is optimal, while for a low risk averse individual (CRRA 1), the 10-year life-cycle strategy gives the highest indifference funding ratio (1.16).

In panel B, the IFRs are all strictly higher than the corresponding values in panel A. This shows that the contribution policy followed by the DB fund, increasing contributions in bad times and decreasing them in good times, is also beneficial for a DC pension scheme. As in panel A, indifference funding ratios vary between 0.96 and 1.20, funding ratios are decreasing in the risk aversion parameters, the lowest funding ratio occurs for CRRA 5 and the highest for CRRA 1. For power utility, the 20-year investment policy dominates all others, a result also found by [Boender et al., 2000]. The most risk averse agent with power utility, will demand a premium of between 5 to 17% funding ratio to participate in the DB fund. Otherwise, he can get a better deal in expected utility terms by choosing the DC plan.

As noted before, the inherent problem with exact matching of contributions is that it is very difficult, not to say impossible, for an individual worker to commit to a contribution policy comparable with that of a DB plan. In the DB plan, contributions are the highest when funding ratios, and past returns, are low. These are also the states in the economy where paying a high pension contribution is not attractive. Given the lack of self-discipline in pension savers, and the nature of most DC plans in practice, i.e., funded by a constant contribution percentage, panel B is difficult to regard as attainable given the demonstrated lack of discipline regarding the way people save for their pension, see [van Rooij et al., 2007]. That said, the funding ratios in panel B can be considered an upper bound to funding ratios that make a DC participant just as well of as a DB participant.

Finally, an important finding in Table 3 is that the average indifference funding ratio is close to 1.0. This strengthens the motivation of current regulatory practice in the Netherlands, where the new policy framework (the FTK) aims to prevent nominal funding ratios dropping below 100%. In the wording of this paper, we can interpret these objectives as ensuring that the DB fund remains attractive for new entrants, relative to a DC alternative. Although participation is mandatory, maintaining individual efficiency for each participant prevents intergenerational tensions within the pension fund. Such tensions might eventually lead to declining support for DB. Although a transition from DB to DC is not necessarily utility-decreasing from the perspective of an individual who is at the start of his career, it does threaten the financial soundness of existing DB funds. Given the existence of DB funds, it is natural to assume that regulators and policy makers are keen to protect the existing pension funds. Our analysis suggests such an interest is well-served by ensuring nominal funding ratios that are at least between 12% and 20% (nominally) overfunded, assuming the least risk averse participants should still prefer DB to DC.

4.1 Sensitivity to the asset mix

The comparison between DC and DB pension outcomes might be skewed towards DC since we can select 'the best' investment strategy for DC. I.e., we selected the 10 and 20-year periods based on the relative performance of DC. So, for a fair comparison we should also consider whether there is a better performing asset mix for the DB pension fund. This is done in Table 4, by using a 60% and 80% stock fraction for the asset mix of the DB fund.

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Table 4 shows that with higher stock investment for DB, indifference funding ratios remain above the 100% nominal level for only the least risk averse agents. For the most risk averse participant (a relative risk aversion coefficient of 5), indifference funding ratios are between 0.85 and 0.98. For the least risk averse participants with log utility (CRRA 1), indifference funding ratios are between 1.06 and 1.15. Observing that the indifference funding ratios are increasingly smaller for the 60% and 80% asset mix, our analysis suggests that having a risky investment strategy has a large value for participants in a DB fund. This is in concordance with the findings of [Cui et al., 2006] and [Boender et al., 2000], who also find that intergenerational transfers are most valuable when the investment mix of the pension funds is the most risky. The diversification benefits provided by the intergenerational solidarity are used to limit the downside risk, while gaining from the upside potential of risky investments. Individual saving as in DC, lacks this diversification opportunity, an effect that is more pronounced at riskier DB investment policies.

A second explanation is the fact that a DB pension fund has an existing pool of assets, while an individual saver starts with an empty fund. Thus, a young participant profits to a greater extent from the higher expected return from the riskier asset mix.

4.2 Sensitivity to the equity premium

The results might be sensitive to the assumed size of the equity premium. Table 5 presents indifference funding ratios for an equity premium of 2, 3, and 4%, respectively.

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Across the board, the indifference funding ratios in Table 5 are decreasing in the equity premium, suggesting a higher equity premium increases the relative performance of DB, and vice versa. Looking at the 20-year DC investment policy, we see that for the equity premium of 4%, indifference funding ratios are between 0.99 and 1.12. For the 2% equity premium, they are between 1.12 and 1.24.

As in the previous subsection, the sensitivity of the results to the equity premium confirms previous research, i.e., intergenerational diversification is most valuable in the context of risky investing. The intuition is that the participant of a DB pension fund profits the equity premium right from the start of his working life, through the existing asset value of the fund. In contrast, the individual saver does not earn the equity premium when young, when his retirement fund has only little asset value. Thus, the value of the intergenerational transfers in a DB pension fund are increasing in the size of the equity premium, as the excess return on existing assets (and buffer) is shared with all generations within the fund.

4.3 Market valuation of liabilities and real funding ratios

Until now we have assumed a fixed risk-free rate, that represents both the bond return as well as the discount rate for the pension liabilities. However, many countries have adopted a fair value approach to value pension liabilities. Market valuation uses market interest rates with matching maturities for discounting future pension payments and provide a ’market value’ of the current liabilities. To incorporate market valuation in our model, we make the risk-free interest stochastic, having a normal distribution around the mean of 4%, with a standard deviation of 1%. Assuming a duration of the liabilities of *D*, we model a bond return, y_t, as being 4% plus *D* times the change in the riskfree rate. I.e.,

$$y_t = x_{t,1} - D \cdot \Delta x_{t,1}, \tag{8}$$

where $x_{t,1}$ is the short-term interest rate, as in Equation (1).

The resulting long-term bond return is then used as the return on the bond portfolio and as the discount rate for the liabilities. This approach is similar to the construction of the liability return series in [Hoevenaars et al., 2008]. The resulting minimum funding ratios are in Table 7, for durations of 5, 10 and 15 years.

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The table shows that the DB plan becomes more attractive under stochastic interest rates and bond returns. The effect is stronger for longer durations. The intuition is that, although DB funding ratios and contributions become more volatile, it has mostly a large negative impact on the annuity costs for a DC participant. A stochastic interest rate leads to extra risk at retirement, when the monetary outcome of the plan is converted to an annuity.

Panel B in Table 7 lists the minimum funding ratios in real terms, i.e., adjusted for 2% inflation. These are obtained by dividing by 1.02^D, where *D* is the duration of the liabilities. Real funding ratios are used in some countries, such as in the UK, and represent the fraction of total liabilities that is covered in real terms. The results show that minimum real funding ratios for a liability duration of 15 years, which is the most common for pension funds, lie between 0.56 and 0.79.

5 Indifference contribution levels

Given an initial funding ratio of a DB pension fund, we now solve for the contribution rate under DC that gives equal utility. This, we measure the value of the intergenerational solidarity inherent in DB by the difference in average contributions. Table 6 lists the results, where again an iterative goal seek procedure was used to find the contribution level that results in equal utility for DC and DB.

<<INSERT TABLE 6 HERE >>

As expected, the average contribution rate for DB is decreasing in the initial funding ratio of the fund, from 13.6% at a funding ratio of 0.9 to 7.8% at a funding ratio of 1.4. The average contribution in the DB plan roughly decreases with one percentage point for every 10%-point increase in the initial funding ratio. In contrast, the average contribution under DC increases markedly with every increase in the initial DB funding ratio. The most risk averse individual, with relative risk aversion of 5, achieves equal utility by paying a 12.4% contribution rate at the lowest DB funding ratio, to 15.4% at a DB funding ratio of 1.4.

Another way of interpreting the results in Table 6 is that for a well funded DB pension plan with a funding ratio of 1.3, a DC plan with equal expected utility needs to have contribution rate that is 2.7 to 6.1%-points higher, depending on the risk aversion of the participant. For a DB fund that is only barely solvent (funding ratio of 1), the comparable DC contribution rate is between 2% lower and 0.8% higher. Hence, a practical conclusion of this analysis is that for employers who consider switching from DB to DC, retaining equal expected utility for employees, this is quite expensive at high funding ratios of the DB fund. At low funding ratios, however, the least risk averse participant could lower their average pension contribution by switching to DC while still increasing the expected utility their pension.

6 Conclusions and discussion

In this paper we have simulated a defined benefit (DB) pension fund where the only uncertainty is given by the returns on the equity investment of the fund. Set up in real terms with fixed population dynamics (no aging), the modeled stationary pension fund is the most simple representation of the essence of defined benefit pension funding. The simulated contribution paths for the individual DB-participant are matched by the individual contributions to a DC-scheme in which contributions are paid and invested in stocks and bonds. Given both matches in the average contribution rate as well as an exact match, the resulting utility of the pension outcome are evaluated using three different utility specifications. The results are cast in the initial DB-funding ratios that make the DB and DC participant equally well-off.

Overall, our results are in line with previous research, e.g. [Boender et al., 2000], [Cui et al., 2006] and [Gollier, 2008], who find sizable welfare gains stemming from the intergenerational transfer implicit in the DB pension fund. The value of intergenerational risk sharing is increasing in the fraction invested in equities and in the equity premium. Both effects are due to the fact that a participant of the DB fund profits from the equity premium throughout his active working life, while an individual saver starts with an empty retirement fund.

In the base case, we find minimum acceptable funding ratios that lie between 0.96 and 1.20. Above a funding ratio of 1.20, the DB pension fund is attractive for all prospective participants. Below 0.96, even the most risk averse individual would rather not join, if given the choice. Between 0.96 and 1.20 the benefits for an individual to join the DB pension fund depends on his risk aversion, as well as the contribution and investment policy he can commit to in managing a private retirement fund. Given the documented difficulties that laypersons have in managing financial wealth, the present analysis suggests that a minimum acceptable funding ratio for a representative worker would be closer to 0.96 than to 1.20.

An analysis of the required contribution to achieve equal utility reveals that for a funding ratio of 1.3, which is the approximate regulatory target funding ratio for Dutch pension funds, the contribution rate for DC needs to be 2.7 to 6.1%-point higher to achieve equal expected utility.

For the US, empirical research by [Samwick and Skinner, 2004] finds that 401(k) plans, which are of the DC-type, have actually outperformed the DB plans in the period 1989-2001, for all but the most risk averse participants. Rather than disprove the present analysis, it clearly shows that the relative performance of DB and DC will ultimately depend on the realizations of wages, inflation and asset returns. Our analysis suggests a more nuanced picture of how the two systems compare, with a crucial role for the initial funding ratio of the DB pension fund.

Finally, note that our comparison of DB with DC pension does not take into account the portability of a DC-pension versus the negative effect on labor market mobility of a DB-plan. Also, participants to a DB-plan can suffer badly from the default of a sponsoring company.

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Appendix A: Tables and figures

Table 1: Pension fund setup

This table shows the values of the financial variables used for the modeled pension fund, using a nominal discount rate of 4%. The survivor table used for the composition of participants per age is the Dutch survivorship table 1995-2000, as published by the Dutch Actuarial Society.

Variable	Value
Number of active participants (aged < 65)	383,022
Wage of individual participant	29,300
Number of pensioners (age ≥ 65)	151,565
Yearly pension accrual (of current wage)	1.75%
Maximum pension at retirement	20,510
Liabilities	52.762 bln.
LW-ratio (liabilities/wages)	4.8

Table 2: Funding policy rules

This table shows contribution and indexation rules as a function of the nominal funding ratio.

Funding ratio	Contribution
below 1.05	max. 6% contribution increase
above 1.05	contribution to reach 1.32 in 15 years, with max. 3%-point increase per year max. 6%-point decrease per year absolute max. of 25%

Funding ratio	Indexation
below 0.95	2% indexation cut
0.95 - 1.10	proportional cut
1.10 - 1.32	full indexation
over 1.32	full indexation + compensation of past cuts

Table 3: Nominal indifference funding ratios

This table shows the initial nominal DB funding ratios for which DB and DC have equal expected utility. The funding ratios displayed in the table are the initial funding ratios for the DB pension fund for which the corresponding DC-scheme has equal expected utility. Panel A gives the outcomes when the average DB contribution is used as a fixed contribution under DC. Panel B gives the outcomes given an exact match in the DC scheme of DB contributions, at each time-period and scenario. The first column gives the utility function used, CRRA for a power utility function, LA for mean-expected shortfall and DD for downside deviation. The second column gives the risk aversion parameter. The reference point for LA and DD utility is at the maximum level of pension benefits under the DB plan. Note that CRRA with a risk aversion of 1 is log utility. The third and fourth column give the outcomes for a static mix for DC of 50% and 75% stocks. The stock fraction for DB is 50% stocks everywhere. The last two columns give the outcomes for a stock fraction for DC that is 100% at the start, decreasing to 0 from 10 years and 20 years before retirement, respectively.

Panel A: Average match

Utility	Risk aversion	50%	75%	10 yrs.	20 yrs.
CRRA	1	1.09	1.15	1.16	1.12
	3	1.02	0.98	0.98	1.02
	5	0.98	0.87	0.88	0.96
LA	2.25	1.05	1.11	1.11	1.08
	5	1.02	1.05	1.05	1.04
DD	10	1.04	1.06	1.06	1.05
	20	1.01	1.00	1.00	1.02

Panel B: Exact match

Utility	Risk aversion	50%	75%	10 yrs.	20 yrs.
CRRA	1	1.12	1.18	1.20	1.17
	3	1.03	1.06	1.09	1.10
	5	0.96	0.98	1.03	1.05
LA	2.25	1.09	1.15	1.17	1.14
	5	1.07	1.11	1.13	1.12
DD	10	1.06	1.11	1.14	1.12
	20	1.04	1.07	1.10	1.10

Table 4: Effects of a higher stock fraction in DB

This table shows the initial nominal DB funding ratios for which DB and DC have equal expected utility. The funding ratios displayed in the table are the initial funding ratios for the DB pension fund for which the corresponding DC-scheme has equal expected utility. Panel A gives the outcomes when the average DB contribution is used as a fixed contribution under DC. Panel B gives the outcomes given an exact match in the DC scheme of DB contributions, at each time-period and scenario. The first column gives the utility function used, CRRA for a power utility function, LA for mean-expected shortfall and DD for downside deviation. The second column gives the risk aversion parameter. The reference point for LA and DD utility is at the maximum level of pension benefits under the DB plan. Note that CRRA with a risk aversion of 1 is log utility. The third and fourth column give the outcomes for a static mix for DB of 60% stocks and for DC a policy of getting out of stocks 10 years and 20 years before retirement, respectively. Idem for the the last two columns but there with 80% stocks in the DB pension fund.

Panel A: Average match

		60%		80%	
	Risk aversion	10	20	10	20
Utility					
CRRA	1	1.12	1.09	1.09	1.06
	3	0.96	0.99	0.92	0.96
	5	0.86	0.94	0.84	0.92
LA	2.25	1.08	1.05	1.05	1.01
	5	1.02	1.01	0.99	0.98
DD	10	1.03	1.02	1.00	0.99
	20	0.98	0.99	0.95	0.96

Panel B: Exact match

		60%		80%	
	Risk aversion	10	20	10	20
Utility					
CRRA	1	1.15	1.12	1.09	1.06
	3	1.03	1.03	0.93	0.93
	5	0.96	0.98	0.85	0.86
LA	2.25	1.14	1.11	1.09	1.06
	5	1.10	1.08	1.04	1.03
DD	10	1.09	1.08	1.03	1.00
	20	1.06	1.05	0.99	0.98

Table 5: Sensitivity to the equity premium

For three different levels of the equity premium, this table shows the initial DB funding ratios for which DB and DC have equal expected utility. The funding ratios displayed in the table are the initial funding ratios for the DB pension fund for which the corresponding DC-scheme has equal expected utility. The default equity premium of 0.03 corresponds to a 7% yearly stock return. Investment policy for the DC scheme is 100% equity, linearly decreasing to 0% at retirement in the last 10 (columns 3-5) or 20 years (columns 6-8) before retirement. Exact matching of DB contribution is used for the contribution policy under DC.

		10 yr.			20 yr.		
Utility		0.02	0.03	0.04	0.02	0.03	0.04
CRRA	1	1.25	1.20	1.16	1.24	1.17	1.12
	3	1.14	1.09	1.05	1.17	1.10	1.04
	5	1.07	1.03	0.99	1.12	1.05	0.99
LA	2.25	1.21	1.17	1.14	1.20	1.14	1.10
	5	1.17	1.13	1.10	1.18	1.12	1.07
DD	10	1.18	1.14	1.11	1.19	1.12	1.07
	20	1.14	1.10	1.07	1.16	1.10	1.05

Table 6: Iso-utility levels of contribution

This table shows the levels of average pension contribution, as a percentage of wages, that give equal utility for DC and DB. The first column gives the initial funding ratio in nominal terms of the defined benefit (DB) pension plan. The second column gives contribution in the DB plan for the initial funding ratios in the first column. For 3 different levels of risk aversion the third to fifth columns give the average contributions for the DC participants that are necessary to achieve equal utility as in the DB plan. The investment policy in the DC-scheme is the 20-year policy.

		DC contribution						
Funding ratio	DB contribution	CRRA(1)	CRRA(3)	CRRA(5)	LA(2.25)	LA(5)	DD(10)	DD(20)
0.9	0.136	0.099	0.114	0.124	0.104	0.109	0.109	0.114
1.0	0.126	0.106	0.122	0.134	0.112	0.118	0.117	0.123
1.1	0.114	0.110	0.128	0.141	0.118	0.126	0.122	0.130
1.2	0.103	0.114	0.132	0.147	0.122	0.130	0.126	0.134
1.3	0.090	0.117	0.136	0.151	0.126	0.134	0.130	0.138
1.4	0.078	0.119	0.138	0.154	0.128	0.138	0.132	0.141

Table 7: Market valuation of liabilities

This table shows the minimum funding ratios for a DB plan that gives participants equal utility as a DC plan, when the pension liabilities in the DB plan are discounted at a stochastic interest rate. The discount rate is equal to 5, 10 or 15 times the change in the long-term interest rate, itself a normal distribution with mean 4% and standard deviation 1%. It is also equal to the return on the bond portfolio, so that a pension fund with 100% in bonds is fully matched and hedged against interest rate changes. The third column lists the outcomes for the baseline scenario as shown in Table 3, for the 20-year rundown period for DC a 50% stock fraction for DB and exact matching of contributions. Columns 4 to 6 list the outcomes for different interest rate durations of the liabilities and the bonds. Panel B lists the results in terms of minimum *real* funding ratios, obtained by dividing the nominal ratios in panel A by 1.02 ^*D*, where *D* is the duration of the liabilities (zero for the riskfree setup).

A. Nominal

			Duration		
Utility		Riskfree	5.00	10.00	15.00
CRRA	1	1.17	1.12	1.05	1.06
	3	1.10	1.05	0.96	0.88
	5	1.05	1.01	0.89	0.75
LA	2.25	1.14	1.09	0.98	1.02
	5	1.12	1.06	0.93	0.97
DD	10	1.12	1.07	0.98	0.95
	20	1.10	1.05	0.94	0.89

B. Real

			Duration		
Utility		Riskfree	5.00	10.00	15.00
CRRA	1	1.17	1.01	0.86	0.79
	3	1.10	0.95	0.79	0.65
	5	1.05	0.91	0.73	0.56
LA	2.25	1.14	0.99	0.80	0.76
	5	1.12	0.96	0.76	0.72
DD	10	1.12	0.97	0.80	0.71
	20	1.10	0.95	0.77	0.66

Appendix B: Dynamics and policy rules of the defined-benefit pension fund

Wages, pension rights and payments

Throughout the paper we assume a fixed level p of inflation, i.e., 2%. For a given level of inflation, the sum of wages, Wt, evolves as

$$W_{t+1} = W_t \cdot \pi_t \quad (9)$$

where W_0 is simply the sum of wages for each employee in each age group.

In a DB pension fund, active employees acquire new pension rights as a fraction of their current wage. For a stable demographic pyramid as we have, the new pension rights in the first year of the simulation, NR_0, are given by

$$NR_0 = 0.0175 \cdot \sum_{i=25}^{64} W_{i0} \cdot {}_{65-i|}a_i^r, \quad (10)$$

where 0.0175 reflects the build-up of pension rights over 40 years (70% / 40), W_{i0} is the wage for the group of age *i* in year 0, and the final term is the actuarial cost of one unit of pension at 65 (retirement), for an individual aged *i* and given an interest rate *r*.

Pension payments are initially (in year 0 of the simulation) equal to the number of retirees times 70% of the wage at 65. In any year of the simulation, pension payments increase with indexation, which is a funding policy decision, together with the contribution level.

The funding policy

Given the level of pension payments PP_t, assets A_t, an interest rate of r, inflation of π and total wages of W_t, the level of contributions (as a fraction of wages) that keeps assets constant is given by

$$C_t^{eq} = \frac{PP_t - A_t \cdot (r - \pi)}{W_t} \tag{11}$$

Given a target funding ratio of 132%, and a recovery period of 15 years, the unconstrained level of contributions needed to reach the target is given by

$$C_t^{unc} = \begin{cases} C_t^{eq} & \text{if } \delta_t \geq 1.32 \\ C_t^{eq} + 1/15 \cdot (1.32 - \delta_t) \cdot L_t/W_t & \text{if } \delta_t < 1.32, \end{cases} \tag{12}$$

where δ_t denotes the current (nominal) funding ratio, i.e., assets divided by liabilities. To prevent overly large jumps in the contribution level (as a fraction of wages W_t), contribution changes are constrained to 3%-points upwards and 6%-points downwards. Below the minimum funding ratio of 105%, upward jumps of 6%-points are allowed. So, formally, we have

$$C_t^* = \begin{cases} \min\{C_t^{unc} + (\delta_t - 1.05) * L_t/W_t, C_{t-1} + 0.06\} & \text{if } \delta_t < 1.05 \\ \min\{C_t^{unc}, C_{t-1} + 0.03\} & \text{if } C_t^{unc} > C_{t-1} \\ \max\{C_t^{unc}, C_{t-1} - 0.06\} & \text{otherwise} \end{cases} \tag{13}$$

A final restriction is that contributions cannot be negative, and cannot exceed 25% of wages, i.e.,

$$C_t = \max(0, \min(0.25, C_t^*)) \tag{14}$$

which is the level of contributions used in the evolution of pension fund assets, and subsequently matched by the individual in the DC-pension plan.

Indexation cuts are a piecewise linear function of the funding ratio: below a threshold level of 95% funding, no indexation according to the inflation rate of π is granted. Between 95% and 110% indexation is proportional to the funding ratio. Between 110% and 132%, full indexation is granted, but no prior cuts are compensated. Above 132%, prior indexation cuts are compensated for. Formally, this boils down to the following policy rule of indexation I_t:

$$I_t = \begin{cases} 0 & \text{if } \delta_t < 0.95 \\ \pi \cdot (\delta_t - 0.95)/0.15 & \text{if } 0.95 < \delta_t < 1.10 \\ \pi & \text{if } 1.10 < \delta_t < 1.32 \\ \pi + \min\{\delta_t - 1.32, cumCut_t\} & \text{if } \delta_t > 1.32 \end{cases}, \tag{15}$$

where p is the level of inflation and $cumCut_t$ is the cumulative indexation cut at time t, given by

$$cumCut_t = \sum_{t=0}^{T-1} (\pi - I_t). \tag{16}$$