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Shortfall allowed: Loss aversion and habit formation

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SHORTFALL ALLOWED: LOSS AVERSION AND HABIT FORMATION

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ABSTRACT

In this paper we analyze a model of consumption and investment when preferences are loss averse around a habit level and investment yields an uncertain return. Loss aversion is the most natural way of modeling the presence of a habit as it explicitly models aversion to below-habit consumption.

Existing approaches either neglect the possibility of below-habit consumption or model habit formation by dividing consumption through a habit. Confronting the traditional model using ratios with the outcome under loss aversion shows that the loss averse model yields a more natural interpretation thus appears more realistic. Starting with an analytical solution for a piecewise-linear utility function, the results are found to be robust for the more general Kahneman-Tversky formulation of the value function.

Keywords: habit formation, loss aversion, consumption and investment, uncertainty, behavioral value function

JEL Codes: D1, D8, D9, E2

Habit formation met verliesaversie

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SAMENVATTING

In dit artikel analyseren we een consumptie- en investeringsmodel waarin agenten verliesaverse zijn rond hun habit (gewoonte) niveau. De opbrengst van spaargeld is onzeker. Verliesaversie is de meest natuurlijke manier om habit formation in een model te introduceren, omdat het habit niveau een natuurlijke rol heeft als referentiepunt. Daarmee kan ook consumptie beneden het habit niveau gemodelleerd worden.

Bestaande literatuur richt zich slechts op consumptio boven de habit, of gebruikt een methode waarbij als variabele consumptie gedeeld door de habit wordt gebruikt. We vergelijken de uitkomsten van het model met verliesaversie met een traditionele aanpak en vinden dat verliesaversie wat meer realistische uitkomsten geeft. De uitkomsten zijn robuust voor een andere specificatie van verliesaversie.

Trefwoorden: habit formation, verliesaversie, consumptie en investeringen, onzekerheid, behavioral value funtie

JEL Codes: D1, D8, D9, E2

1 INTRODUCTION

In much of the habit formation literature: only above-habit consumption is considered. Acknowledging that consumption below an aspiration level is very likely in real life, this paper introduces a better way to deal with habit formation, and compares the outcomes with an existing surrogate way of dealing with the issue.

The economics of household consumption and savings is a well-established field of research in economics. Despite the large body of literature, however, the explanatory power of the standard life-cycle framework is still disappointing. That is, confronting the predictions of the traditional consumption/savings model with real-world phenomena does not give satisfying results. A popular extension of the standard model is therefore to include the effect of habit formation. In the last decades more and more attention is paid to the notion that people form *habits*, i.e., consumption is evaluated and decisions are taken considering a level of consumption to which one has got used to.

Habit formation is relevant in many areas of economics where consumption preferences are modeled. (Constantinides 1990) models habit formation in an attempt to resolve the equity premium puzzle. Recent papers by Seckin(2000a,b) explore the effect of habit formation on precautionary savings in a classical model of consumption and savings. Habit formation is also in the loss averse model of (Bowman, Minehart, and Rabin 1999). One of the earliest references is (Duesenberry 1949), who argues that consumption decisions are not independent through time. Consumption creates a habit that influences future valuation, so the per-period utility function is not time-separable. This complicates the analysis of standard consumption models, see the discussion in (Browning and Lusardi 1996).

Although there is consensus on habit formation being a real world phenomenon, there is no consensus on how to include it in models of economic behavior. There is no outspoken debate on the issue, but from examining the existing literature two competing models are found. The first models the difference between consumption and a habit. The second models the ratio of consumption to a habit level. Experimental evidence favors the first type, but the classical CRRA utility functions that are used do not permit consumption below the habit. In this paper we want to explicitly allow consumption below a habit level. The ratio-approach using a CRRA utility function (which is the most used flavor) is compared with an approach where an agent is *loss averse* with respect to a habit level of consumption. Using loss aversion as a utility function is the most straightforward way to cope with below-habit consumption and is rooted in psychological evidence on behavior, see (Kahneman and Tversky 1979), (Tversky and Kahneman 1992). Our results can be compared with the work of (Bowman, Minehart, and Rabin 1999), who have a more abstract representation of loss aversion.

This paper shows that including habit formation gives a tractable solution, while increasing insight in the effects of loss aversion on consumption and savings. By finding an analytical characterization of the optimal solution, loss aversion can be added to the standard toolkit for economists in modeling consumption and investment under uncertainty. This is encouraging as habit formation with CRRA utility does not lend itself to simple tractable solutions. The analysis in this paper further differs from existing approaches in that I will concentrate on the level of individual decision making, i.e., the relation between consumption and wealth.

The rest of this paper proceeds as follows. Section 3 introduces the existing approaches to modeling habit formation and discusses their shortcomings. Section 3.1 presents an alternative model based on loss aversion for which an analytical solution is found. As a robustness check, Section 4 numerically compares the results with those when the value function from Kahneman-Tversky is taken as utility function directly. Section 5 concludes.

2 EXISTING APPROACHES TO HABIT FORMATION

In the treatment of habit formation in the literature, almost all studies use a traditional specification utility function (although the argument is affected by habits). That is, a function $u(C)$ maps an amount C to a positive measure of utility. It is such that more consumption is preferred over less, and the marginal utility of consumption is decreasing, so $C' > 0$, and $C'' < 0$. However, there is ample evidence that this formulation of utility found its way into mainstream economics not for its realism, but more for its favorable mathematical properties: it is differentiable everywhere, and a specific notion of risk aversion is easily derived from the ratio of the first and second-order derivatives.

Two approaches exist to incorporating habit formation in traditional (i.e., using a concave utility function in the levels) models of consumption. The first approach considers the difference between consumption and the habit, and is often used in an econometric or empirical context. The second approach considers the ratio of consumption and the habit, and is found in equilibrium/theoretical contexts.

Difference approach

When one comes to think about habit formation and possible specifications for $U(c_t, h_t)$, an obvious first choice is to have a linear specification. This boils down to utility being derived from the difference between consumption and the habit level. A general representation can be given as

$$U(c_t, h_t) = u(c_t - h_t), \tag{1}$$

where $u(\cdot)$ is a classical (smooth) function on $(0, \infty)$. A specific example, and also the most commonly used one, is the power utility function given by

$$u(x) = \frac{1}{1-\gamma} x^{1-\gamma}, \tag{2}$$

where x is the argument. An important property of the power utility is that it has Constant Relative Risk Aversion (CRRA). Introduced by (Pratt 1964), the rate of relative risk aversion for a function in consumption c is defined as

$$RRA = \frac{c \cdot u'}{u''}. \tag{3}$$

As the relative risk aversion of the function in (2) is constant and equal to γ, it is said to be a CRRA utility function.

The habit h_t is either exogenous, as in (Campbell and Cochrane 1999), or given by past consumption, i.e., $h_t = \alpha \cdot c_{t-1}$, as in (Dynan 2000), (Alessie and Lusardi 1997). Many studies also follow (Ryder and Heal 1973) in that the habit or subsistence level of consumption is an exponentially weighted sum of past consumption, e.g. (Constantinides 1990), (Detemple and Zapatero 1991).

A major problem with the power utility as it is predominantly used, is that it cannot have negative arguments. So if the argument of the function in (2) is $c_t - h_t$, consumption may never be below the habit, and the models using such a utility function can only consider above-habit consumption. Most studies deal with the imposed constraint by specifying an evolution of the natural log of $(c_t - h_t)$, see for example (Campbell and Cochrane 1999). It is clear that this results in c_t always being strictly greater than h_t.

The difference approach is used in many well-known and recent work. (Constantinides 1990) uses it to explain the equity premium puzzle (the result of which is challenged in (Chapman 2002)). (Matsen 2003) argues that that welfare gains of international trade are lower with habit persistence. (Dynan 2000) tests for the presence of habit formation in household data. In a dynamic model, explicit solutions for optimal consumption are derived in (Detemple and Zapatero 1992). (Campbell and Cochrane 1999) explain aggregate stock market behavior with the difference approach to habit formation. (Detemple and Zapatero 1991) derives axiomatic results for consumption policies. (Chapman 1998) expresses his concerns over the possibility of negative marginal utility with habit formation, which he deems unrealistic.

Ratio approach

Recognizing the problem of nonnegativeness, Abel (1990, 1999) proposes to take the *ratio* of c_t and h_t, so that the resulting term can be used in a traditional utility function. Abel explicitly motivates this choice by pointing out the problems with the alternative specification of taking the difference. Based on the power utility (that has constant relative risk aversion, CRRA), the formulation becomes

$$u(c,h) = \frac{1}{1-\gamma}\left(\frac{c}{h^{\eta}}\right)^{1-\gamma}. \qquad (4)$$

According to (Campbell and Cochrane 1999) the ratio approach eliminates changing risk aversion, which can be understood as relative risk aversion not depending on the value of h in (4). Using the ratio approach (Gomes and Michaelides 2003) perform a numerical analysis of consumption and investment in a model with labor income risk. Also, (Fuhrer 2000) proposes (4) to model habit formation as an explanation for the hump-shaped response of consumption to income shocks. Such an effect is often found in impulse-response analyses of monetary policy models.

Because (4) allows for below-habit consumption, the next section will compare the outcome of the loss averse model with the outcome of a model having (4) as the utility function. We can already observe that a potential problem with the ratio approach lies with h appearing in the numerator of (4), so decreasing h has a multiplicative effect on utility.

3 A MODEL BASED ON LOSS AVERSION

Consider an agent with current wealth w who lives two periods and has to decide on the level of his current savings s. His first and second period budget constraints are

$$c_1 = w - s \tag{5}$$

$$c_2 = s \cdot r, \tag{6}$$

where c_1, c_2 denote consumption over period i, $i = 1,2$, and r is the *uncertain* return on investment. We assume $c_1 > 0$ and that the return r has an arbitrary absolute-continuous probability distribution function $G(\cdot)$ with support $(0,\infty)$. Having an arbitrary probability distribution $G(\cdot)$ contrasts with (Aizenman 1998) and (Bowman, Minehart, and Rabin 1999), who also examine loss aversion but restrict uncertainty in income to a binomial type of probability distribution. If in the following the terms negative and positive return are used, they indicate that we are talking about r in terms of the *net* return, i.e., $r-1$, which can take on values in $(-1,\infty)$.

One interpretation is that of a worker who chooses how much to invest in pension savings (or human capital), receiving the fruits of that investment after retirement. The return r can represent the gross return on investment, which can be produced by a simple savings account, a portfolio of stocks, etc. A macroeconomic interpretation is one where aggregate investment in one period pays off in the second period, i.e., economic growth.

To study habit formation, we allow the benchmark in the second period to be determined by consumption in the first period as

$$h_2 = (1-\alpha)\cdot h_1 + \alpha \cdot c_1, \tag{7}$$

where α denotes the degree of habit persistence. $\alpha = 0$ implies a fixed customary level for both periods. $\alpha = 1$ implies that the period 2 habit level is equal to period 1 consumption.

The two-period model analyzed in this paper is

$$\max_{c_1} U(c_1, h_1) + \rho \cdot \mathbb{E}\left[U(s \cdot r, h_2)\right], \tag{8}$$

where $U(\cdot)$ is the instantaneous utility function. In the presence of a habit level of consumption, the choice of utility function is motivated by two core principles:

1 consumption is evaluated relative to a habit level of consumption,

2 the sensitivity for consumption below the habit is larger than above the habit.

Item 1 is core to any treatment of habit formation, while principle 2 is new. The finding that people do not evaluate outcomes in terms of the levels, but rather in terms of changes was pointed out most pronounced in the article of (Kahneman and Tversky 1979). In an experimental setting they find that students evaluate risky prospects relative to a reference point. Moreover, people are more sensitive to losses than to gains. The phenomenon is called *loss aversion*.

Principle 2 can be made operational by specifying the following per-period loss averse utility function

$$u(c_t, h_t) = c_t - \lambda \cdot (h_t - c_t)^+, \tag{9}$$

where $\lambda > 0$ is a loss aversion parameter, $(y)^+$ is used to denote the maximum of 0 and y. The parameter h_t is the benchmark level of consumption, representing the level of consumption below which a loss is suffered, be it physical, financial or mental. Obviously, the sensitivity for consumption below h_t is equal to $1+\lambda$, and for above-habit consumption it is 1.

The utility function in (9) incorporates the main features of Kahneman and Tversky's value function, as it measures utility relative to a reference point, and treats positive and negative deviations from the reference point asymmetrically. The penalty parameter λ represents a measure for the costs of not achieving a desired consumption level. The utility function in (9) has been used before by (Benartzi and Thaler 1995) in an attempt to explain the equity premium puzzle. It is also the piecewise-linear equivalent of the one used by (Aizenman 1998), where λ is the disappointment aversion rate and $h_t \equiv h$ the certainty equivalent consumption. (Bowman, Minehart, and Rabin 1999) derive axiomatic results for loss averse utility functions of which (9) is a special case. The precise Kahneman-Tversky value function is used in Section 4 as a robustness check of the results derived for the piecewise-linear model.

Note that the two-period model can still be considered to be within the standard life-cycle framework. Having two periods can be seen as modeling life-time consumption in which smoothing happens at low frequencies, i.e., across two specific stages of the life-cycle. (Browning and Crossley 2001) notes that many psychological or behavioral explanations rule out the life-cycle framework, referring to work as (Thaler 1994), in which no formal optimization of consumption and savings is considered. Clearly, that is not the case here.

The traditional approaches to habit formation imply than an increase in current consumption increases the marginal utility of consumption in future periods. If current consumption increases the habit level, future consumption will be closer to the habit and thus have larger marginal utility. Our utility function in (9) induces another, more pronounced effect: an increase in consumption in one period affects marginal utility in the future only if it is below the habit level. The explicit modeling of the feeling of the consumer about the habit gives a stronger (above or below) effect of current consumption on future utility. The issue of sustaining a once started consumption pattern becomes explicit here.

With (9) as the instantaneous utility function, the agent determines savings through the following maximization problem

$$\max_{c_1}\ c_1 - \lambda \cdot (h_1 - c_1)^+ + \mathbb{E}(c_2) - \lambda \cdot \mathbb{E}(h_2 - c_2)^+, \tag{10}$$

subject to budget constraints (5) and (6), and habit formation evolution given by (7). Note that we abstract from time-discounting. In the above model, it would be appropriate to include a parameter ρ for the last two terms, representing a discount with respect to future consumption. However, this only increases the number of parameters in the model, without changing the results. If appropriate, in the following we will add a note to show how the results would change if time-discounting was included.

As the only decision in the model is c_1, in the following we will use c to denote first-period consumption.

3.1 A comparison of optimal consumption rules

In this section, the two-period model with loss aversion around the habit level is found to have analytical solutions that satisfy common intuition. Moreover, comparing the results with the outcomes of a traditional CRRA utility function with habit formation (which is analytically intractable) shows the present model to yield more realistic outcomes.

Theorem 3.1. *Consider the cases of a positive(I) and negative(II) expected return separately.*

I. If $\mathbb{E}[r] > 1$*, define*

$$\overline{w}_p = h_1 \cdot \left(1 + 1/r_p^*\right), \tag{11}$$

where r_p^* *is the* $\overline{r}$ *that solves*

$$\lambda - \mathbb{E}[r] - \lambda \int_0^{\overline{r}} (\alpha + r) dG = 0. \tag{12}$$

The optimal consumption rule is given by

$$c_1^* = \begin{cases} w - \frac{\alpha w + (1-\alpha) h_1}{r_p^* + \alpha} & \text{if } w \leq \overline{w}_p, \\ h_1 & \text{if } w > \overline{w}_p. \end{cases} \tag{13}$$

II. If $\mathbb{E}[r] < 1$*, define*

$$\overline{w}_L = h_1 \cdot (1 + 1/\overline{r}_n^L), \tag{14}$$

$$\overline{w}_H = h_1 \cdot (1 + 1/\overline{r}_n^H), \tag{15}$$

where $\overline{r}_n^L$ is the $\overline{r}$ that solves

$$\lambda - \mathbb{E}[r] - \lambda \int_0^{\overline{r}} (\alpha + r) dG = 0, \tag{16}$$

and $\overline{r}_n^H$ solves

$$-\mathbb{E}[r] - \lambda \int_0^{\overline{r}} (\alpha + r) dG = 0. \tag{17}$$

From equations (16) *and* (17) *it follows that $\overline{r}_n^L > \overline{r}_n^H$, so $\overline{w}_L < \overline{w}_H$. Optimal consumption is given by*

$$c^* = \begin{cases} w - \frac{\alpha \cdot w + (1-\alpha) h_1}{\overline{r}_n^L + \alpha} & \text{if } w \leq \overline{w}_L, \\ h_1 & \text{if } \overline{w}_L < w < \overline{w}_H, \\ w - \frac{\alpha \cdot w + (1-\alpha) h_1}{\overline{r}_n^H + \alpha} & \text{if } w \geq \overline{w}_H. \end{cases} \tag{18}$$

Proof: See appendix.

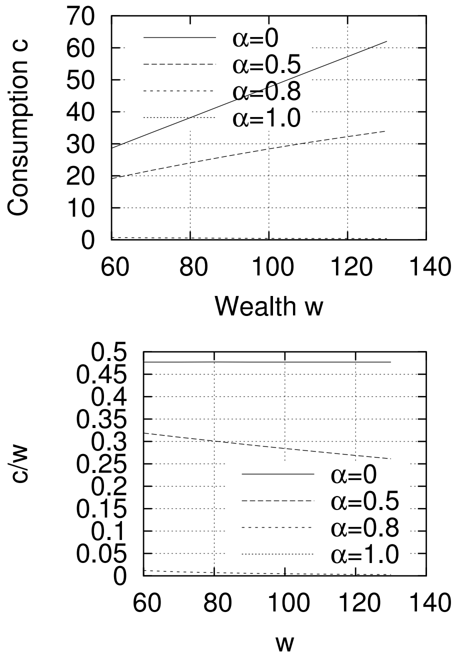
As the outcomes differ for the cases of positive and negative expected return, the consequences of Theorem 3.1 are analyzed separately for these two cases. We start with the case of positive expected return, case I in the theorem.

3.2 **Positive expected return**

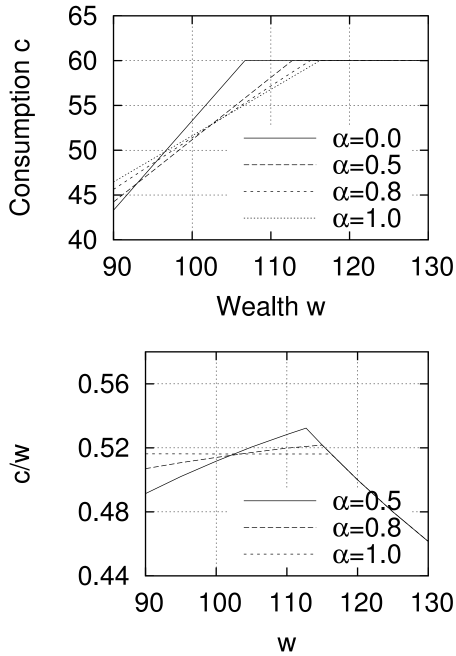
Figure 1 compares the optimal consumption rules for CRRA utility with loss averse utility with different degrees of habit persistence. For ease of exposition the top row plots the levels of consumption. The bottom figures show consumption as a fraction of wealth, c/w.

The left-hand side of Figure 1 shows fairly standard results for the CRRA utility. Without habit persistence ($\alpha = 0$), consumption is a constant fraction of wealth. With habit persistence, the value of consumption in the next period is negatively affected by current consumption, as current consumption is in the numerator of the second-period utility function. Hence, for $\alpha > 0$ consumption becomes a decreasing fraction of wealth.

A problem that can be spotted easily from the formulas as well as Figure 1(a) is that for a large degree of habit persistence, it becomes attractive to consume extremely little. Little first-period consumption influences the habit h_2 in the numerator of the second period CRRA utility function, so the second period utility becomes huge. This does not seem realistic consumer behavior: consume very little in one period so that habits adjust downward strongly, and enjoy the very high above-habit utility in the next period.



(a) CRRA



(b) Loss aversion

Figure 1 c^* **and** c^*/w **as a function of wealth under positive expected return**

For a positive expected return, $h_1 = 60$, this figure shows the optimal consumption as a function of initial wealth for varying degrees of habit persistence. $\alpha = 0$ corresponds to the absence of habit formation, i.e., $h_2 \equiv h_1$. $\alpha = 1$ implies $h_2 = c_1$. The parameters for the CRRA utility are $\gamma = 0.5$, and $\sigma = 0.5$. For the loss averse utility $\lambda = 1$. The return distribution is subnormal(0.085, 0.16).

For loss aversion we will see that consuming very little in the first period to decrease the habit is not a fruitful strategy, as consumption shortfall is punished larger than consumption surplus is valued.

The top graph in Figure 1(b) illustrates the results of Theorem 3.1 most clearly. The general shape of the absolute consumption rule is one where first-period consumption increases with wealth, up to a point $\overline{w}_p$ where consumption stays equal to the level of the initial habit h_1. It follows from the theorem that both the level $\overline{w}_p$ and the slope are a function of α. The consequence for relative consumption (bottom graph) is that it is increasing in wealth below $\overline{w}_p$, and decreasing in wealth above $\overline{w}_p$. When α increases, we see that the threshold $\overline{w}_p$ increases and the slope of the absolute consumption rule decreases. Comparing habit-formers($\alpha = 1$) to agents without habit formation($\alpha = 0$) shows that for high wealth levels a lower consumption is accepted for the benefit of a lower second period reference point. For low wealth, consumption may be higher, as the second period reference point is already lower because of habit formation.

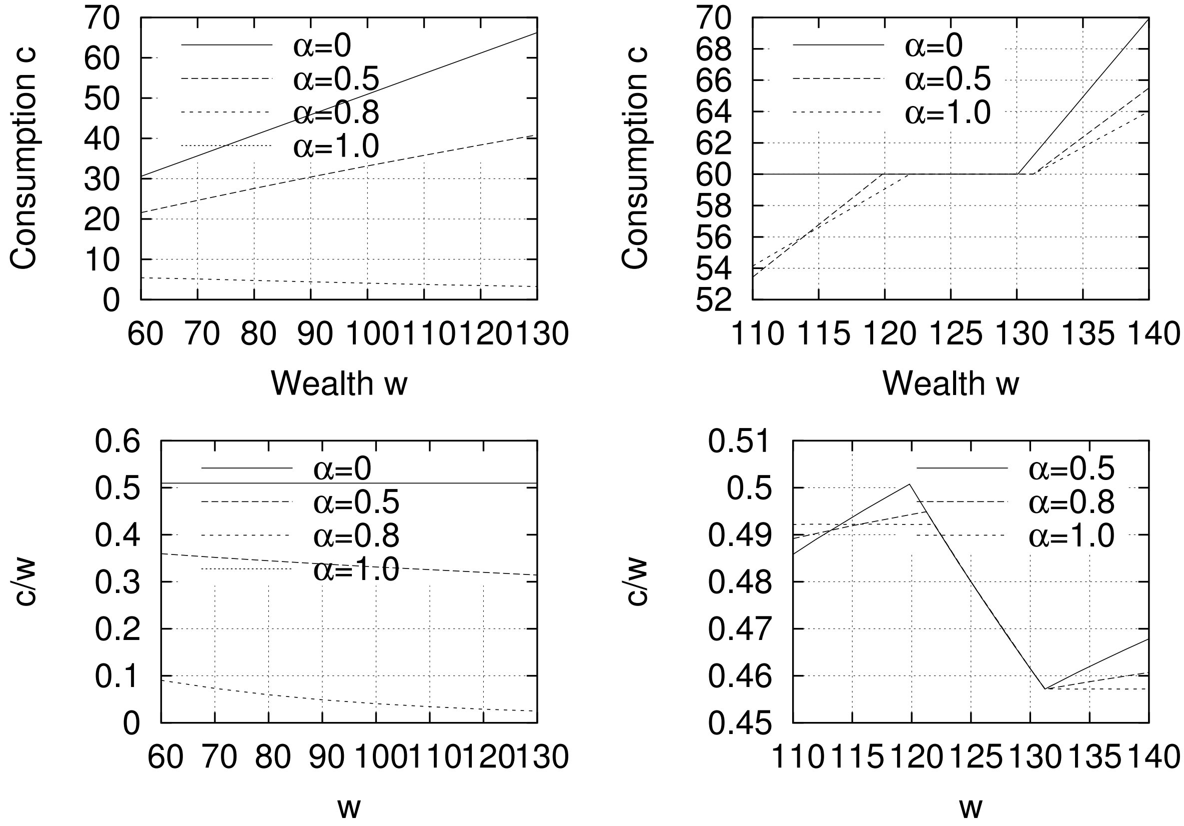
Let us elaborate on the implications of the results. The traditional way of modeling habit formation leads to a monotone nonincreasing consumption rule. But, intuitively speaking, if habits are important to a consumer, one would expect different behavior for "poor" and "rich" consumers (that is, defined relative to the habit). This is clearly visible for the loss averse consumer in Figure 1(b), but absent for the CRRA consumer in Figure 1(a).

A possible interpretation of the consumption rules in Figure 1(b) is the consumption choice after a recent period of economic growth. After a high(low) recent growth rate and sluggish adjustment of habits, wealth would be located to the right(left) end on the horizontal axis. An implication for policy makers is then that lowering taxes does not influence absolute consumption if it is above habits. Any higher wealth is allocated to future periods.

Note that the slope of c^* as a function of w decreases in the level of habit persistence. This is in accordance with the argument put forward by (Fuhrer 2000), who analyses a possible hump-shaped response in monetary transmission. The stronger the habit persistence, the more careful people adjust consumption to income growth, not to commit themselves to high future habit levels.

3.3 Negative expected return

From the outset, a negative expected return on investment might seem unrealistic, but (i) the model finds different outcomes so we should deal with it, and (ii) it happens in practice. An example of the latter is a situation with very high inflation, e.g. Argentina in 2002. For that case r represents a real return.



(a) CRRA

(b) Loss aversion

Figure 2 Consumption with habit formation and negative expected return

For a negative expected return, $\lambda = 1$, $h_1 = 60$, this figure shows the optimal consumption as a function of initial wealth for varying degrees of habit formation. $\alpha = 0$ corresponds to no habit formation, $\alpha = 1$ implies $h_2 = c_1$. The return distribution is lognormal(-0.04, 0.07).

For the case of a negative expected return, the optimal consumption rule is visualized in Figure 2. With the negative expected return the CRRA consumer does not have a drastic different consumption rule. The loss averse consumer do change their consumption rule, giving a more realistic picture. The consumption behavior in Figure 2(b) is markedly different from the results in Figure 1(b). The appeal to realism is relevant considering that, ceteris paribus, the consumption behavior of an individual in a country with a negative return on savings *will* be markedly different from the behavior under a normal situation, i.e., a positive savings return.

For $\alpha = 0$, behavior is the exact opposite of before. Below a threshold level $\overline{w}_H$, consumption is constant and equals the first-period benchmark. Above the threshold, any surplus wealth is consumed. Only for low wealth it is necessary to invest with a focus on the second period benchmark. Expected return might be negative, but the punishment on below-habit consumption in the second period drives savings.

When there is habit persistence($\alpha > 0$), we see that below the threshold $\overline{w}_L$, first period consumption is not kept at the benchmark level. Consuming less increases savings *and* decreases the second period benchmark. The combined effect increases total utility. For $\alpha = 0$, the increase of savings alone does not justify a consumption below the benchmark, as the return on savings is negative. The change in slope of the consumption decision is comparable with the positive return case.

For larger wealth levels, above $\overline{w}_H$, habit formation leads to a decrease in consumption relative to the case $\alpha = 0$. That is because an increase in consumption does not only decrease period 2 expected consumption, but also increases the period 2 benchmark and thus decreases utility further.

3.4 **Rational habit formation**

In the above analysis, we have assumed that the degree of habit persistence is given by a parameter α, an exogenous constant. Now suppose that a consumer can choose the level of habit persistence, a phenomenon which could be called *rational* habit formation. To do so, α needs to become an endogenous parameter that can be optimized. Theorem 3.2 gives the optimal degree of habit persistence.

Theorem 3.2. *If $w > h_1(1+\overline{r})/\overline{r}$, having a fixed habit ($\alpha = 0$) is optimal. Otherwise, full habit formation based on past consumption ($\alpha = 1$) is optimal.*

The intuition behind the result of Theorem (3.2) is that a not-so-wealthy household will benefit from adjusting the reference point downwards, as this increases the likelihood of a surplus in the second period. A wealthy household will want to keep the reference point low, as to experience a surplus in both periods.

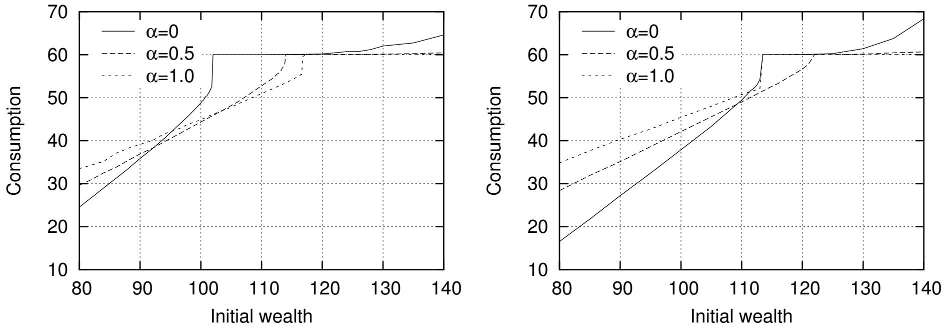


Figure 3 Results for the Kahneman-Tversky value function

The left panel shows the results for a positive expected return, r lognormal(0.085, 0.16), the right panel for a negative expected return r lognormal(-0.04, 0.16). $h_1 = 60$, while $\alpha = 0$ corresponds to no habit persistence, $\alpha = 1$ to full habit persistence.

4 ROBUSTNESS: KAHNEMAN-TVERSKY VALUE FUNCTION

As a robustness-check we consider the consequences of taking the value function estimated in (Tversky and Kahneman 1992) directly as utility function. It is given by

$$v(c) = \begin{cases} x^{0.89} & \text{if } x \geq 0, \\ -2.25 \cdot (-x)^{0.89} & \text{if } x < 0, \end{cases} \tag{19}$$

where the variable x represents the deviation from the reference point. In the context of consumption and savings, we simply take $x = c - h$ and solve the optimization problem with (19) as instantaneous utility function. The results are in Figure 3 for both positive and negative expected return.

Comparing Figure 3 with Figures 1 and 2, we see a large degree of similarity. There are two differences, however. First, for $\alpha = 0$ in the positive-return case, for Kahneman-Tversky utility the first-period consumption increases beyond the benchmark, while for bilinear utility the benchmark is never exceeded. This is a consequence of the convexity of the behavioral value function in the domain of losses. The convexity is not large enough to increase consumption significantly above the benchmark when there is habit persistence though. A second difference is in the shape of the lines in Figure 3. They resemble the shape of the value function itself: a small area below the reference point has a very high slope, the marginal utility of consumption goes to infinity when consumption approaches the benchmark level. In the figure, we can observe that when consumption is close to the benchmark level of 60, the optimal decision becomes extremely sensitive to the initial wealth level. This is very different from the bilinear objective, where consumption was a bilinear function of wealth.

For the negative expected return-case, the consumption patterns for Kahneman-Tversky utility stay roughly the same. A difference with Figure 2 is that consumption above the benchmark does not take off, while for the bilinear objective, consumption rises linearly with wealth. The difference is caused by the concavity in gains for the Kahneman-Tversky objective: the marginal value of first period consumption above the benchmark is decreasing in the level. Together with an increased second-period reference point and high marginal utility around the reference point, increasing first period consumption is not attractive in the case of Kahneman-Tversky utility.

5 CONCLUSION

This paper has challenged the basic tenets of traditional habit formation modeling. As long as a classical smooth utility as CRRA is used, only a shallow image of habit formation can be taken into account. In such a case, both the utility function and the resulting consumption rules could have more realism. For example, we show that the ratio approach can lead to a degenerate solution for very high levels of habit persistence. Therefore, the model presented here has introduced a more appropriate and direct modeling of habit formation, namely in defining habit formation as being averse to shortfall below the habit. This appeals directly to the theory of loss aversion, in which the reference point needs to be defined. Thus, the contribution of this paper can also be found in connecting the theory of habit formation to that of loss aversion, and being the first to derive analytical consumption rules under uncertainty and habit formation.

We have shown that habit formation has a significant impact on consumption and savings behavior. For positive as well as negative expected return, it increases the wealth level at which the first-period benchmark is consumed. Consumption below the benchmark decreases first-period utility, but increases second-period utility through the combined effect of increased savings and a lower reference point. The figures of optimal consumption have shown that the average slope of the consumption function as a function of wealth becomes smaller, i.e., consumption changes take place more gradually.

In addition, we find that the results found do not change significantly when the Kahneman-Tversky value function is taken as instantaneous utility function. This implies that the results for the model analyzed in this paper can be relevant for empirical work on consumption, investment, and savings.

APPENDIX

Proof of Theorem 3.1:

The optimization is given by

$$\max_{c_1} \; c_1 - \lambda \cdot (h_1 - c_1)^+ + \mathbb{E}[s \cdot r] - \lambda \cdot \mathbb{E}[h_2 - s \cdot r]^+, \tag{A1}$$

$$\text{s.t.} \quad s = w - c_1, \tag{A2}$$

$$\text{and} \quad h_2 = (1-\alpha) \cdot h_1 + \alpha \cdot c_1. \tag{A3}$$

Denote the value of the objective function (A1) for a fixed value of c_1 by $v(c_1)$. The first order condition for an interior optimum is given by

$$\mathbb{E}\left[\frac{\partial v}{\partial c_1}\right] = 0, \tag{A4}$$

where

$$\mathbb{E}\left[\frac{\partial v}{\partial c_1}\right] = \begin{cases} 1 + \lambda - \mathbb{E}[r] - \lambda \cdot \int_0^{\overline{r}} (\alpha + r) dG & \text{if } c_1 \leq b, \\ 1 - \mathbb{E}[r] - \lambda \cdot \int_0^{\overline{r}} (\alpha + r) dG & \text{if } c_1 > b, \end{cases} \tag{A5}$$

and $\overline{r}$ is defined as

$$\overline{r} = \frac{(1-\alpha) \cdot h_1 + \alpha c_1}{w - c_1} - 1. \tag{A6}$$

The second order condition to problem (A1) is given by

$$\mathbb{E}\left[\frac{\partial^2 v}{\partial c_1^2}\right] = -\lambda \frac{(w - c_1) \cdot \alpha + (1-\alpha) \cdot h_1 + \alpha \cdot c_1}{(w - c_1)^2} \cdot (\alpha + \overline{r}) \cdot g(\overline{r}), \tag{A7}$$

where $g(\cdot) > 0$ is the density function of the return r. For any $c_1 < w$, expression (A7) is negative, ensuring that any $c_1 < w$ that satisfies (A5) is an interior optimal solution.

For $\mathbb{E}[r] \geq 1$ it follows from (A5) that $c_1^* \leq h_1$, as for $c_1 > h_1$ the derivative with respect to c_1 is negative for all $\overline{r}$. Hence, the appropriate derivative when there is an interior solution is the first line in (A5). If there is no interior solution, either $c_1^* = 0$ or $c_1^* = h_1$. If for a given α, $\overline{r}^\alpha$ solves (A5), then the wealth level $\overline{w}_\alpha^p$ at which $c_1^* = h_1$ can be derived from the definition of $\overline{r}$ in (A6). It is given by

$$\overline{w}_\alpha^p = h_1 \cdot (1 + 1/\overline{r}^\alpha), \tag{A8}$$

which is decreasing in $\overline{r}^\alpha$. As $\alpha > 0$, it follows from (A5) that $\overline{r}^\alpha$ is decreasing in α, so $\overline{w}_\alpha^p$ is increasing in α. This completes the proof of part I.

Unlike the case without habit formation, if $\mathbb{E}[r] < 1$ the derivative for $c_1 < h_1$ is not positive for all $\bar{r}$. This can only be true for all α when we would demand $(\lambda+1)(\mathbb{E}[r]) < 1$, which is the exact opposite of the (reasonable) assumption we made in Theorem 1. We have to conclude that both lines in (A5) can constitute a local optimum, which need to be analyzed both.

Having the two areas for a possible optimum, we analyze the wealth levels for both areas at which c_1 is exactly equal to h_1. Define

$$\overline{w}_L = h_1 \cdot (1 + 1/\bar{r}_n^L), \tag{A9}$$

$$\overline{w}_H = h_1 \cdot (1 + 1/\bar{r}_n^H), \tag{A10}$$

where $\bar{r}_n^L$ and $\bar{r}_n^H$ are the $\bar{r}$ that solve (A5) for consumption lower and higher than the benchmark b, respectively. It follows directly from the first order conditions that $\bar{r}_n^L > \bar{r}_n^H$, so $\overline{w}_H > \overline{w}_L$. The latter implies that for wealth below $\overline{w}_L$, the optimal consumption follows from the definition of $\bar{r}$ and the value of $\bar{r}^*$ that solves (A5) for $c_1 \le h_1$. For wealth above $\overline{w}_H$, the $\bar{r}^*$ for $c_1 > h_1$ is the relevant one. If wealth lies between the two thresholds, optimal consumption is equal to h_1. □

Proof of Theorem 3.2

We can write the value of the objective function in the optimum with habit persistence as

$$\begin{aligned} v^*(w) = {} & c_1^* \left(1 + \lambda - \mathbb{E}[r] - \lambda \int_0^{\bar{r}^*} (\alpha + r) dG\right) \\ & - \lambda \cdot h_1 + w \cdot \mathbb{E}[r] - \lambda(1-\alpha) h_1 G(\bar{r}^*) + \lambda w \int_0^{\bar{r}^*} r dG, \end{aligned} \tag{A11}$$

where c_1^* is the value of the optimal consumption level. The result follows by observing that the coefficient for c_1^* between brackets is the expression for the derivative, that is assumed to be zero in the optimum. Thus, the value of the objective function at the optimum is given by

$$v^*(w) = -\lambda \cdot h_1 + w \cdot \mathbb{E}[r] - \lambda(1-\alpha) h_1 G(\bar{r}^*) + \lambda w \int_0^{\bar{r}^*} r dG. \tag{A12}$$

To find how the optimal objective value varies with α, we determine the derivative of v^* with respect to α. It is given by

$$\frac{\partial v^*}{\partial \alpha} = \lambda h_1 G(\bar{r}) - \lambda(1-\alpha) h_1 g(\bar{r}^*) \cdot \frac{\partial \bar{r}^*}{\partial \alpha} + \lambda w \bar{r}^* g(\bar{r}) \frac{\partial \bar{r}^*}{\partial \alpha}, \tag{A13}$$

where $\bar{r}^*$ is the $\bar{r}$ that solves the first order condition. Given an interior solution, the first order condition (A4) implies the following expression for $\partial \bar{r}^* / \partial \alpha$

$$\frac{\partial \bar{r}^*}{\partial \alpha} = -\frac{G(\bar{r}^*)}{(\alpha + \bar{r}^*) g(\bar{r}^*)}, \tag{A14}$$

which is negative. Substitution in (A13) gives

$$\frac{\partial v^*}{\partial \alpha} = \lambda h_1 G(\bar{r}^*) + \lambda(1-\alpha) h_1 \frac{G(\bar{r}^*)}{\alpha + \bar{r}^*} - \lambda w \bar{r}^* \frac{G(\bar{r}^*)}{\alpha + \bar{r}^*}. \tag{A15}$$

Now, the first term on the right-hand side of (A15) is positive, as well as the second term. Only the third term is negative, involving the initial wealth level w. As the derivative is monotone in w, there is a threshold value w^*, above which the derivative of the objective with respect to α is negative. In that case, the optimal α is equal to 0. Otherwise, it is equal to 1. The level w^* is given by

$$w^* = h_1 \frac{1 + \bar{r}^*}{\bar{r}^*}, \tag{A16}$$

which completes the proof. □

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