Exam *Advanced Programming in Quantitative Economics*

20-24 August 2012

MSc/PhD Summercourse
University of Aarhus

The exam below can be solved over the course of the next weeks, in groups of a maximum of two students (one-person groups are permitted, even preferred). Only one set of answers per group of students is necessary. The final results have to be handed in by email before Saturday, Oct. 20 2012, 18.00h to `c.s.bos@vu.nl`. Marking will take place in the two weeks after this date.

1 Setting and model

Your exercise for this exam is to estimate a so-called GARCH model Engle (1982); Bollerslev (1986) on daily financial returns r_t. The basic model reads

$$r_t = \mu + \epsilon_t, \qquad \epsilon_t \sim \mathcal{N}(0, \sigma_t^2), \tag{1}$$

$$\sigma_{t+1}^2 = \omega + \alpha\epsilon_t^2 + \delta\sigma_t^2. \tag{2}$$

Equation (1) is like a standard regression equation with only a mean; the loglikelihood for observation r_t is like the loglikelihood of the normal model with mean μ and variance σ_t^2.

The added value of the GARCH model is the variance equation (2). The variance slowly evolves over time, where a large shock ϵ_t^2 leads to a sudden increase in the next-period variance σ_{t+1}^2.

The variance σ_t^2 of course needs to stay positive. A sufficient condition for this is to ensure that the parameters ω, α, δ are non-negative. Furthermore, e.g. if $\delta > 1$ or $\alpha > 1$ you can see that the variance sequence becomes explosive. Hence, a further restriction is that $\alpha + \delta < 1$.

The first observation would be distributed as

$$r_0 \sim \mathcal{N}(\mu, \sigma_0^2)$$

where until now the initial variance, σ_0^2, is not defined yet. In general the choice is made to either use the unconditional expectation of the variance,

$$\sigma_0^2 \equiv \frac{\omega}{1 - \alpha - \delta},$$

or to use the empirical variance of the returns,

$$\sigma_0^2 \equiv \operatorname{var} r_t,$$

as initial value.

2 Extension: GAS-t or Beta-t-GARCH model

The GARCH model fits many aspects of the financial data series, but usually the returns of financial assets have heavier tails than the standard GARCH model. Creal et al. (2012) and also Harvey and Chakravarty (2008) set up an extension with a Student-t density, where the observations are supposed to come from

$$r_t = \mu + \epsilon_t, \qquad \epsilon_t \sim t(0, \sigma_t^2, \nu), \tag{3}$$

$$\sigma_{t+1}^2 = \omega + A\sigma_t^2 \left(\frac{(\nu+1)y_t^2}{(\nu-2)\sigma_t^2 + y_t^2} - 1 \right) + D\sigma_t^2. \tag{4}$$

Note that here the disturbances are distributed as the Student-$t(0, \sigma_t^2, \nu)$ density, with degrees-of-freedom parameter ν and variance σ_t^2. This density is related to the standard Student-t density as

$$\begin{aligned} a &\sim t_\nu & p(a) &= p_t(a) \\ \epsilon &= \sigma \left(\frac{\nu-2}{\nu} \right)^{\frac{1}{2}} a & p(\epsilon) &= \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\sigma\sqrt{(\nu-2)\pi}\Gamma\left(\frac{\nu}{2}\right)} \left(1 + \frac{\epsilon^2}{\sigma^2(\nu-2)} \right) \\ & & &\equiv \frac{1}{\sigma} \left(\frac{\nu}{\nu-2} \right)^{\frac{1}{2}} p_t\left(\left(\frac{\nu}{\nu-2} \right)^{\frac{1}{2}} \frac{\epsilon}{\sigma} \right) \end{aligned}$$

The parameters in this model are almost comparable to the parameters in the GARCH model, with $A \approx \alpha$, and $D \approx \alpha + \beta$. Hence the restriction here should be that $A > 0$, $0 < D < 1$.

You could, as an voluntary extra, try to fit this GAS-t model, and compare the resulting estimates of the variance process σ_t^2 to the variance process you get for the standard GARCH model.

3 Data

Find your own data for this exercise. Your dataset should be on one stock, stock index, or possibly exchange rate. The data should be at the daily frequency, so e.g. each day's closing price, and be at least 1000 observations (roughly 4 years) long. You do not have to take weekends or missing observations into account.

Download these data from a website of choice. You could use Yahoo Finance, or Google Finance. Prepare the data for yourself, transforming the prices P_t to *percentage* returns:

$$\begin{aligned} p_t &= \log P_t \\ r_t &= 100(p_t - p_{t-1}) \end{aligned}$$

For nicer graphing, you might want to keep track of the dates, together with the returns.

4 On the restrictions

When estimating the GARCH model, one has to ensure that the restrictions on the parameters are satisfied. You can do this either using the SQP approach, or using the transformation approach. With the latter option, it is relatively hard to implement the restriction that $\alpha + \delta < 1$. In general it appears to be sufficient to impose $0 < \omega, 0 < \alpha < 1, 0 < \delta < 1$, and only indicate failure if the sum of α and δ happens to be too large.

5 Questions

Choose a data-set, prepare it, give some statistics on prices and returns. Then estimate the parameters of the GARCH model, taking the restrictions into account. Make sure that the estimations start at the same initial parameters. Report, for both estimators, what your estimates and standard errors are, with the optimal value for the loglikelihood and the number of function evaluations you needed for obtaining this optimum.

An incomplete list of other things you might consider in your report is

- indication of the problem (no need to extensively repeat the exam)
- data description including peculiarities, where you got it, how you transformed, what average returns, prices etc you found
- parameter estimates of the model, including final log-likelihood and standard errors, number of function evaluations, for MaxSQP(F) and/or MaxBFGS.
- the final estimate of the variance process: What does σ_t^2 look like?

Such a list is not exhaustive; own imagination is appreciated.

Write a report, maximum 5 pages excluding graphs/tables, on the analysis of the problem and the questions you answered. Hand in by email a zip- or 7z-file with the pdf-report and the programs 'ready-to-run', including data files, and if necessary a `readme.txt` if I would need to run the programs in some specific order.

6 Help

This project is relatively large, especially if you don't start it in a structured fashion. Check clearly, before you start, how to split it up in more manageable subtasks.

Make sure that, while working on the project, you do not get stuck. If you do not know how to continue, also after sleeping for a good night, get in touch either with Nima (`nimanonejad@gmail.com`), Federico (`fcarlini@econ.au.dk`) or me (`c.s.bos@vu.nl`), explaining clearly what the problem seems to be.

7 Evaluation of results

The report and programs will be evaluated on the basis of

1. Structure of solution (relating to analysis of problem) [20%]
2. Readability of programs/comments [20%]
3. Correctness of programming [20%]
4. Robustness of programming [20%]
5. Choice of descriptive statistics/graphs [10%]
6. Report, relating to structure of solution [10%]

Between brackets the approximative weight of each part in the final mark.

Please indicate clearly on top of your report and in your email, whether you need a full mark or only a pass/fail grade.

References

Bollerslev, T. (1986). Generalized autoregressive conditional heteroskedasticity. *Journal of Econometrics 31*(3), 307–327.

Creal, D., S. J. Koopman, and A. Lucas (2012). Generalized autoregressive score models with applications. *Journal of Applied Econometrics*. Forthcoming.

Engle, R. F. (1982). Autoregressive conditional heteroscedasticity with estimates of the variance of United Kingdom inflation. *Econometrica 50*, 987–1008.

Harvey, A. C. and T. Chakravarty (2008). Beta-t-(e)garch. Cambridge Working Papers in Economics 0840, University of Cambridge.