Systemic Risk & Diversification across European Banks and Insurers*

Jan Frederik Slijkerman† Dirk Schoenmaker‡
Casper G. de Vries§

October 25, 2011

Abstract

The mutual and cross company exposures to fat tail distributed risks determine the potential impact of a financial crisis on banks and insurers. We examine the systemic interdependencies within and across the European banking and insurance sectors during times of stress by means of extreme value analysis. While insurers exhibit a slightly higher interdependency in comparison with banks, the interdependency across the two sectors turns out to be considerably lower. This suggests that downside risk can be lowered through financial conglomeration.

1 Introduction

This paper studies the systemic interdependencies within and across the banking and insurance sectors in times of stress. Banks and insurers are

---

*We are grateful to a referee for the excellent comments on a previous version and Linda van Goor for an insightful discussion. Furthermore we like to thank conference and seminar participants at the DNB, the BIS-CEPR-JFI meeting, the Free University Amsterdam and the Bundesbank Dresden 2010 conference.

†AEGON Asset Management and Erasmus University Rotterdam. The opinions expressed in this paper are those of the author and do not necessarily represent the views of AEGON Asset Management.

‡Duisenberg School of Finance

§Erasmus University Rotterdam, Tinbergen Institute & Duisenberg School of Finance
both exposed to fat tailed distributed shocks through their assets and liabilities that create linkages and common exposures. Financial innovation improved the ways in which risks can be spread and transferred from the banking sector to the insurance sector and vice versa. Diversification lowers the risk of isolated shocks for a financial entity, but may simultaneously increase the systemic risk. The credit crisis shows how problems in one part of the banking sector can easily spread to other parts of the banking sector due to these risk transfers. At the start of the credit crisis EU banks had exposure to US sub-prime mortgages of about equal size as US banks, a perfect example of international risk diversification and contagion. Other parts of the financial sector can also be easily affected. During the burst of the internet hype, banks came off lightly while insurers carried substantial losses as a result of their equity and bond exposures. The credit crisis shows that risks have been shifted between the banking sector and the insurance industry by means of credit risk transfers, warranting the bailout of large insurers as well as those of banks.

As we will show, risk transfers between the banking and insurance books are nevertheless a useful diversification device in times of stress. This is so because risks of banks and insurers differ, due to the differences in their business models. Banks transform liquid liabilities of depositors into illiquid assets (loans) and the foremost risk drivers are the business cycle and the interest rate. A life insurance company per contrast has a better match between its asset and liability maturity structure, but a major risk is the longevity risk. It can often hold assets until maturity when the time to pay has come; covering a period that extends over business cycles. Non-life insurance risk is again different. Claim risk is largely unrelated to the business cycle, while the investment risk on the premium income is. As of today these differences and their interrelation in times of financial hardship have received little attention.

Our main research question therefore concerns how the downside risk in the banking sector differs from the downside risk in the insurance sector and how these are related in times of crisis. To investigate these issues we estimate the downside dependence between combinations of financials, both within a sector and across sectors. As the risk profile of both sectors is different, we find that there is scope for diversification of worst outcomes. To understand the possible differences in cross-sector risk, we develop an analytical factor model to interpret the sources of systemic risk.

Given the importance of the payment and clearing functions for the real
economy, academic research into systemic risk has traditionally focussed on the banking sector; see De Bandt and Hartmann (2002) for a survey. The stability of the insurance sector is therefore of a somewhat lesser public concern than the fragility of the banking sector. The systemic importance of the insurance industry is therefore more indirect by its influence on the banking sector. The recent rescue of AIG, for example, was motivated by the large scale insurance of credit instruments that AIG had sold to the banking industry. It was feared that too many banks would be downgraded if these policies could no longer be honored. The presence of financial conglomerates, the linkages and the differences in risk profiles make that an assessment of the downside risk derived from banking and insurance is of great interest.

Traditionally, research in the area has concentrated more on the possible benefits of mergers across sectors. Early work discussed the potential benefits from the abolishment of the Glass-Steagall act in the US for individual firms (which forbid bank holding companies to perform insurance activities), see Laderman (1999), Berger (2000), Estrella (2001) and Carow (2001). These earlier studies conclude there are gains from diversification. But a more recent US study by Stiroh and Rumble (2006) finds that the diversification benefits are more than offset by the costs of the increased exposure to new volatile activities. Moreover, as was shown theoretically by Shaffer (1985), while diversification may benefit individual institutions, diversification often increases the systemic risk.

Moving to regulatory requirements, Kuritzkes et al. (2003) argue that there is scope for a reduction of 5-10% in capital requirements for a combined bank and insurance company. The regulatory framework during our sample period, though, did not allow for cross hedging between business lines (BCBS, 2004). The different entities of a conglomerate are supervised separately according to sector specific regulation. Due to the two pillar system, the Basel II and Solvency II regulations fall short in recognizing the potential benefits of cross sector mergers for containing the risks in the financial system, but also do not recognize explicitly the potential negative effects of

\[\text{One of the first studies considering systemic risk of insurers was written by the Group of Thirty (1997). More recently Swiss Re (2003b) concluded that there is ample systemic risk in the reinsurance sector. Plantin and Rochet (2007, ch.8) argue why there is less concern for systemic risk in the insurance industry than in the banking sector, since there are fewer feedback mechanisms. Nevertheless, they also reckon the systemic dangers of fire sales by life insurance companies to satisfy capital requirements after a stock market plunge.}\]
To analyze this issue we focus on the downside risk exclusively, rather than using global risk measures, like the variance, that are employed in other studies. Using global risk measures such as the variance-covariance matrix are appropriate if other aspects such as upside potential also play a role (as in asset allocation questions). In the banking sector the focus on downside risk for risk management issues has become mainstream through the Value at Risk (VaR) methodology. In insurance, the study of ruin has traditionally put an emphasis on downside risk issues. On the industry level and the financial sector as a whole, the emphasis is on the systemic stability. Systemic risk by its very nature is concerned with the downside risk of the system.

The downside risk focus has another advantage, as it enables more easily to capture the stylized fact that the return series of financial assets are fat tailed distributed, see e.g. Jansen and De Vries (1991). The more common assumption that returns are normally distributed considerably underestimates the downside risk. Hence, given the focus on downside risk, we will not start from this premise and allow for fat tails to capture the univariate risk properties. For the multivariate question of downside risk diversification benefits and systemic risk issues, the normal distribution based correlation concept may also dramatically fail to capture the degree of dependence. For example, one can have multivariate Student-t distributed random variables that exhibit fat tails and are dependent, but which are nevertheless uncorrelated; this is impossible for normally distributed random variables. The downside risk measures that we consider are derived from Extreme Value Theory (EVT) and easily allow for the observed non-normality.

Except for Gully et al. (2001), Bikker and Lelyveld (2002) and Lelyveld and Knot (2009), most studies focus on U.S. data, as in De Nicolo and Kwast (2002), and assume that the returns are normally distributed. Our empirical research is focused on European data and applies extreme value theory, allowing for fat tail risk and asymptotic dependence. In the empirical section we measure the downside risk and systemic dependence between combinations of financials, both within a sector and across sectors. The extreme value based techniques avoid correlation based techniques that focus primarily on the central order statistics, but rather uses the extreme order statistics as in Hartmann et al. (2004).

In the remainder of this paper we first explain the use of the downside risk measure instead of the correlation measure. Next we provide an economic rationale for the dependence between different financial institutions to exist,
even in the limit. Thereafter, we explain the methodology, give a description of the data and present the results. Finally, we summarize our findings and draw some policy conclusions.

2 Dependence

To understand the dependence between two random variables that follow a normal distribution, it suffices to have the mean, variance and correlation coefficient as these completely characterize their joint behavior. The correlation measure itself, however, is often not a very useful statistic for financial risk analysis for a number of reasons.

As a first reason recall that the correlation measure can be zero, while there is nonetheless dependence in the data. Consider e.g. the two portfolios $X + Y$ and $X - Y$, where $X$ and $Y$ are two asset returns. If the two asset are independently and identically distributed\(^2\), then the two portfolios are uncorrelated. If $X$, $Y$ are normally distributed, the two portfolios are also independent. But the two portfolios are dependent if the $X$, $Y$ are fat tailed distributed, like in the case of a Student-t distribution (with degrees of freedom above 2), as the two portfolios have their largest realizations along the two diagonals. In fact, one shows that in the Student-t case for large $s$, the conditional probability $P(X + Y > s | X - Y > s)$ tends to $1/2$, whereas under independence the conditional probability equals the unconditional probability $P(X + Y > s)$, which tends to zero as $s$ increases.

A second reason is the empirical observation that the return series are not normally distributed. In Figure 1a we have depicted the daily stock returns of ABN AMRO Bank and AXA since 1992 until 2003. In the Figure 1b we randomly generated returns from a bivariate normal distribution using the estimated means, variances and correlation from the actual data. Comparing the two plots, one sees that the outliers more or less align along the diagonal as in the above portfolio example; which is a clear sign of systemic risk. Looking univariately along the axes, moreover, note that the actual returns exhibit many more outliers than the normal remakes. This is the well known fat tail phenomenon. If the tails are so fat that the second moment is unbounded, the correlation measure is not appropriate. For the non-life insurance industry second moment failure is considered an important issue and one reason for capping insurance contracts.

\(^2\)This is for simplicity; the argument can also be made in a CAPM setting.
A third reason is that for our purposes we are only interested in downside dependence, while the correlation concept is a global measure. Empirically, the correlation coefficient is mainly driven by the observations from the center and not by the infrequent tail observations. Thus while one would like to overweight the tail events, the correlation measure rather does the reverse. For all these reasons we turn to a measure that focuses directly on the downside dependence and does justice to the fat tail phenomenon.

![Scatter plots showing downside dependence](image)

Figure 1: The normal distributed remake underestimates the systemic risk

### 2.1 Downside Dependence

The above discussion of the correlation coefficient identified a number of reasons for turning to an alternative measure for identifying systemic risk. This measure preferably focuses on the interdependence between downside losses and should be robust towards fat tails. Our preferred systemic risk indicator is the expected number of failures, given that at least one firm is failing, see Huang (1992). Let \( \kappa \) be the number of firms that crash and let \( A \) and \( B \) be the stochastic loss returns of two financials. Let \( t \) be the loss level that triggers a failure. One can easily allow for different thresholds per firm, say \( s \) and \( z \), but this reduces the clarity of presentation.\(^3\) Thus we focus on the diagonal.

\[^3\text{In practice one scales the different thresholds towards a common failure factor } t \text{ by defining } s = \theta t \text{ and } z = \gamma t. \text{ For the theoretical analysis this means that one can redefine the loss returns } A \text{ and } B, \text{ by dividing these with the desired scales } \theta \text{ and } \gamma \text{ respectively.}\]
With two firms, the conditional expected number of failures is

\[ E[\kappa | \kappa \geq 1] \]

\[ = \frac{1 \cdot P(A > t, B \leq t) + 1 \cdot P(A \leq t, B > t) + 2 \cdot P(A > t, B > t)}{1 - P(A \leq t, B \leq t)} \]

\[ = \frac{P(A > t) + P(B > t)}{1 - P(A \leq t, B \leq t)} , \]

at the common high loss level \( t \). Note that this conditional expectation can be readily extended to more than two firms. We will use the conditional expected number of failures

\[ E[\kappa | \kappa \geq 1] = \frac{P(A > t) + P(B > t)}{1 - P(A \leq t, B \leq t)} \quad (1) \]

as our measure of downside dependence.

In a bivariate setting, the conditional failure expectation minus one equals the conditional probability on a systemic crisis. Since, given that at least one firm crashes, the joint failure probability is

\[ \frac{P(A > t, B > t)}{1 - P(A \leq t, B \leq t)} = E[\kappa | \kappa \geq 1] - 1 . \quad (2) \]

Hence, alternatively we refer to (1) as the measure of systemic risk. Hartmann et al. (2004) provide further motivation for this measure.

Unless one is willing to make further assumptions, as in the options based distance to default literature, it is impossible to pin down the exact level of \( t \) at which there will be a failure, or at which supervisors declare the institution financially unsound. For this reason we do take limits in the theoretical analysis and consider

\[ SR(\kappa) \equiv \lim_{t \to \infty} E[\kappa | \kappa \geq 1] . \quad (3) \]

Extreme value theory then shows that even though the measure is evaluated in the limit, it nevertheless provides a reliable benchmark for the dependency at high but finite levels of \( t \).\(^4\) Note that for the introductory example where

\(^{4}\)Recall that it is straightforward to allow for different failure levels across firms.

In the empirical analysis we look along the diagonal, i.e. take \( \theta = \gamma = 1 \).
\[ A = X + Y \] and \[ B = X - Y \] and if the two factors \( X \) and \( Y \) are iid Student-t distributed, then \( SR(\kappa) = 4/3 > 1 \). But in the case that \( X \) and \( Y \) are iid Normally distributed, \( SR(\kappa) = 1 \). Moreover, note that the measure does not require bounded second moments and zooms in on the downside risk.

Recently the parametric approach to dependency by means of copulas has gained some popularity. In the interest of robustness we prefer not to choose a particular copula and follow the non-parametric multivariate EVT approach. Note that the connection between the two concepts in the limit is as follows

\[
SR(\kappa) = \lim_{t \to \infty} \frac{P(A > t) + P(B > t)}{1 - P(A \leq t, B \leq t)} = \lim_{p \to 1} \frac{2(1 - p)}{1 - C(p, p)}, \tag{4}
\]

where \( P(A > t) = P(B > t) = 1 - p \) and \( C(p, p) \) is the limit copula. Thus if the copula is known, the failure measure can be calculated. Kole (2006) discusses the use of different copulas in this context. Other alternative measures comprise the conditional probability of a specific failing institution, given the demise of another: \( P(A > t|B > t) \). With the methods developed below, the limiting value of this measure are rather straightforward to compute as well. But the point is that as the number of financials increases, the number of different partial measures one has to report rapidly increases. The \( SR(\kappa) \), however is good in any dimension as it summarizes the systemic risk in a single measure.

In case that the risk would be thin tailed distributed, the so called Ledford-Tawn measure would provide more detailed information (by a logarithmic transformation of the probabilities). In this case the \( SR(\kappa) \) would just indicate asymptotic independence. But for the application to financials the Ledford-Tawn measure is less informative than the \( SR(\kappa) \) due to the fat tailed nature of the risks.

### 2.2 Economic Rationale for Downside Dependence

To provide a rationale for the downside dependence between banks and insurers, we start from an elementary factor model. The factors are assumed to follow a distribution with non-normal heavy tails. Firms and sectors partly differ with respect to their risk factors and this determines the differences in downside dependence within and between the sectors. The model is in the vein of De Vries’ (2005) portfolio approach to downside risk for banks.
The investments of banks and insurers are to a certain degree similar. Both invest in syndicated loans, have proprietary investments in equity and both hold mortgage portfolios. Moreover, the costs arising out of liabilities for banks and insurers are to some degree similar. Both, for example, sell products with a guaranteed interest rate. Financial instruments can transform insurance risk to financial investments (e.g. catastrophe bonds), or can transform default risk to insurance risk via credit default swaps. The securitization of bank loan portfolios has widened the scope of investments for insurers. There are also differences. Banks live from the interest rate spreads (intermediation margins), while life insurers receive premiums and have to pay the long interest rate. The deposit contract exposes the banks to the risk of immediate callability, while insurers do not have such a liquidity risk. The semi-reduced form approach of a factor model captures these similarities and differences through the factors (and their coefficients).

Suppose the risk of all firms in the financial sector can be decomposed into three elements. Financials face a common risk component (macro risk), $F$; an insurance or bank sector specific risk (sector risk), labelled $A$ and $B$ respectively; and firm specific idiosyncratic risk, denoted $Y_i$ and $Z_j$ for banks and insurers respectively. All these factors are assumed to be fat tailed distributed. We take this to mean that the tails of the distributions exhibit power like behavior, as in the case of the Pareto distribution.\footnote{These distributions are such that for a sample of \( n \) i.i.d. draws \[ \lim_{s \to \infty} \frac{\mathbb{P} \{ \max(X_1, \ldots, X_n) > s \}}{\mathbb{P} \{ \sum_{i=1}^{n} X_i > s \}} = 1. \] Thus the sum is almost entirely driven by the maximum of the observations.} For ease of presentation we assume that the entire loss distribution is Pareto distributed. But we emphasize that the results carry over to all distributions that exhibit regular varying tails, such as the Student-t distribution, see below.

Assume that the downside risk of the factors ($A, B, F, Y_i, Z_j$) are (unit scale) Pareto distributed on $[1, \infty)$

\[
P(A > t) = P(B > t) = P(F > t) = P(Y_i > t) = P(Z_j > t) = t^{-\alpha}, \tag{5}
\]

and where $\alpha > 0$. The shape parameter $\alpha$ in the power determines the number of moments that are bounded; a finite variance for example requires that $\alpha > 2$. This is in contrast to distributions like the normal distribution that has all moments bounded due to the tails which are of exponential order.
There is considerable evidence that the shape parameters are equal across different firms, see e.g. Jansen and de Vries (1991) and Hyung and de Vries (2002). Nevertheless, in the analysis below we briefly explain how the results have to be adjusted in case that the shape parameters differ.

The above framework applies generally to the entire class of heavy tailed distributions that are characterized by the following tail property of the symmetric distribution\footnote{For simplicity of presentation we focus on symmetric distributions.}

\[
\lim_{t \to \infty} \frac{R(-tx)}{R(-t)} = \lim_{t \to \infty} \frac{1 - R(tx)}{1 - R(t)} = x^{-\alpha}
\]

for some \( \alpha > 0 \).

For example consider the case of the Student-t distribution with \( v \) degrees of freedom. Invoking L’Hôpital’s rule, one can use the density \( r(t) \) to show that

\[
\lim_{t \to \infty} \frac{r(tx)}{r(t)} = x \lim_{t \to \infty} \left( \frac{1 + t^2/v}{1 + t^2x^2/v} \right)^{(v+1)/2} = x^{-v}
\]

and hence \( \alpha = v \). Furthermore,

\[
a = \lim_{t \to \infty} \frac{r(t)}{vt^{-v-1}} = \frac{1}{v} \lim_{t \to \infty} t^{v+1} \left\{ \frac{\Gamma((v+1)/2)}{\Gamma(v/2)} \frac{1}{\sqrt{v\pi} t^{v+1} (1/t^2 + 1/v)^{(v+1)/2}} \right\}
\]

\[
= \frac{1}{v} \frac{\Gamma((v+1)/2)}{\Gamma(v/2)} \frac{1}{\sqrt{v\pi}} v^{(v+1)/2} = \frac{\Gamma((v+1)/2)}{\Gamma(v/2)} \frac{1}{\sqrt{\pi}} v^{v/2-1}
\]

Thus for large \( t \)

\[
P(X > t) \simeq a t^{-\alpha}.
\]

Define the rescaled random variable \( Y = a^{1/\alpha} X \), hence

\[
P(Y > t) = P(a^{1/\alpha} X > t) = P(X > a^{-1/\alpha} t) \simeq t^{-\alpha}.
\]

This gives an expression that is analogues to the pure Pareto case in (5), except for the fact that the Pareto expression is only good in the tails, as it only holds exactly in the limit

\[
\lim_{t \to \infty} \frac{P(Y > t)}{t^{-\alpha}} = 1.
\]
In the following we investigate the downside dependence between two financials within a sector and across sectors. To this end define the equity loss returns of a bank $G_i$ and an insurer $H_j$ as a portfolio of risk factors consisting of the following elements:

$$G_i = F + B + Y_i \text{ and } H_j = F + A + Z_j.$$  \hfill (6)

For brevity we assume unitary scale coefficients for each factor, but this can be easily relaxed as we briefly discuss in the analysis that follows. In practice the evidence is that the scales are considerably different across different companies, in contrast to the shape parameter $\alpha$, see Hyung and De Vries (2002).

2.2.1 Within sector dependence

Consider first the dependency between two banks. The probability of a large loss $t$ can be calculated with the help of Feller’s convolution theorem (1971, VIII.8). A brief exposition of this theorem is given in section 6.1 of the Appendix. The Feller theorem holds that the probability of the sum in (6) converges to the sum of the marginal probabilities as $t$ gets large. In other words, the convolution probability is determined by the probability mass along the axes above the portfolio line $F + B + Y_i = t$. Since the bank portfolio consists of three independent risk factors, the probability of a crash of a bank is

$$P(F + B + Y_i > t) = 3t^{-\alpha} + o(t^{-\alpha}).$$  \hfill (7)

Note that in case the scale factors differ from unity and be equal to $f$, $b$ and $y_i$ say, the result would just be

$$P(F + B + Y_i > t) = (f + b + y_i) t^{-\alpha} + o(t^{-\alpha}).$$

Thus differences in scale only imply a quantitative difference.

But in case that the shape parameter $\alpha$ differs across the factors, this induces a qualitative difference. For example suppose that the idiosyncratic factor $Y_i$ has a higher shape parameter $\alpha_i > \alpha$ than the other two factors. Then

$$P(F + B + Y_i > t) = 2t^{-\alpha} + o(t^{-\alpha}),$$

since the idiosyncratic factor contributes terms which are of smaller order (tend to zero faster). The rule of thumb is therefore that one can ignore factors that have thinner tails (higher shape parameters). The interested reader
can easily adjust the analysis below for such cases. The empirical analysis automatically takes care of this possibility (since the extremes contributed by the other factors will dominate in the data).

Next, we determine the probability that two banks crash simultaneously

\[ P(G_1 > t, G_2 > t) = P(F + B + Y_1 > t, F + B + Y_2 > t). \]

To determine this systemic failure probability, we can again make use of Feller’s convolution theorem. Recall that the theorem holds that only the probability mass along the axes counts. The intersection of the two sets determined by the portfolio inequalities \( F + B + Y_1 > t \) and \( F + B + Y_2 > t \), i.e. when both inequalities hold simultaneously, only has points above \( t \) along the \( F + B \) axis in common\(^7\). Points along the \( Y_1 \) or the \( Y_2 \) axes larger than \( t \) cannot simultaneously satisfy both inequalities (e.g. in the three dimensional space \((Y_1, Y_2, Z)\), the point \((0, 2t, 0)\) satisfies \(Z + Y_2 > t\), but not \(Z + Y_1 > t\)). This implies that

\[ \lim_{t \to \infty} \frac{P(F + B + Y_1 > t, F + B + Y_2 > t)}{P(F + B > t)} = 1, \]

It follows that

\[ P(G_1 > t, G_2 > t) = P(F + B > t) + o(t^{-\alpha}) = 2t^{-\alpha} + o(t^{-\alpha}). \quad (8) \]

The probability of a joint crash among two insurers is similar, thus \( P(H_1 > t, H_2 > t) = 2t^{-\alpha} + o(t^{-\alpha}). \)

**2.2.2 Cross-sector dependence**

Since the sector risk for the two companies is different, there are less common components in the portfolio of the two firms. The probability of a joint crash of an insurer and a bank is entirely determined by the single common factor \( F \). If the portfolio inequalities \( F + B + Y_1 > t \) and \( F + A + Z_1 > t \) hold simultaneously, there is only probability mass of order \( t^{-\alpha} \) above \( t \) along the \( F \) axis in common, and no mass of this order along the \( B + Y_1 \) and \( A + Z_1 \) axes. This implies that

\[ P(G_1 > t, H_1 > t) = P(F + B + Y_1 > t, F + A + Z_1 > t) = t^{-\alpha} + o(t^{-\alpha}). \quad (9) \]

\(^7\)Note that the sum \( F + B \) can be treated as a single random variable \( Z \), say.
2.2.3 Systemic risk

On the basis of (7), (8) and (9) we can evaluate our measure for systemic risk or downside dependence (1). To this end, recall

\[ 1 - P(A \leq t, B \leq t) = P(A > t) + P(B > t) - P(A > t, B > t). \]  

(10)

Combining (7), (8) and (10) one obtains the within sector systemic risk (4) as

\[ SR(\kappa) = \lim_{t \to \infty} \frac{P(G_i > t) + P(G_j > t)}{1 - P(G_i \leq t, G_j \leq t)} \]

\[ = \frac{3t^{-\alpha} + 3t^{-\alpha}}{3t^{-\alpha} + 3t^{-\alpha} - 2t^{-\alpha}} = \frac{6}{4}. \]

In words, the \( SR(\kappa) \) value says that in one out of the two cases when there is a bank failure, the other bank fails as well. Note that differences in scale would only qualitatively change this result, i.e. \( SR(\kappa) \) would still be higher than one. Different shape parameters, however, may also have a qualitative effect (for example, if both the idiosyncratic factors have larger shape parameters than \( F \) and \( B \), then \( SR(\kappa) = 1 \)).

The cross sector systemic risk is found analogously from (7), (9) and (10)

\[ SR(\kappa) = \lim_{t \to \infty} \frac{P(G_i > t) + P(H_j > t)}{1 - P(G_i \leq t, H_j \leq t)} = \frac{6}{5}. \]

(12)

The conditional expectation is higher in the case of within sector dependence than in the case of cross sector dependence, since the sectorial risks differ. In the empirical section we estimate the systemic risk measure and test for the predicted difference. If the within sector and cross sector dependencies turn out to be equal, that would indicate that the sector risks are similar or unimportant; if these differ, that would vindicate the above sectorial factor structure.

2.2.4 Dependence and the normal distribution

It is interesting to note that the dependence in the tail disappears if we assume that the independent factors, \( A, B, F, Y_i \) and \( Z_j \) are standard normally distributed. Note that normality immediately implies that \( G_i, G_j, H_i \) and \( H_j \) are all correlated. If we assume that the returns on the individual
projects exhibit heavy tails as before, there is dependence in the tails and the expected number of failures (1) converges to a number larger than one as the failure level $t$ increases. Even though there is positive correlation if the returns of both $G_i$ and $H_i$ follow a bivariate normal distribution, however, all dependence between the firms disappears as $t$ increases. Thus under normality

$$SR(\kappa) = \lim_{t \to \infty} \frac{P(G_i > t) + P(G_j > t)}{1 - P(G_i \leq t, G_j \leq t)} = \lim_{t \to \infty} \frac{P(G_i > t) + P(H_j > t)}{1 - P(G_i \leq t, H_j \leq t)} = 1.$$  

The proof for this result is similar to the proof of proposition 2 in De Vries (2005) and follows directly from the general result by Sibuya (1960). This explains why Figure 1a differs so much from Figure 1b, especially in the North-East and South-West corners. The disappearance of the dependency in the tail area is not unique for the normal distribution. The same holds for exponentially distributed factors, see De Vries (2005). In the empirical section we compare the semi-parametric estimate of (1), which allows for heavy tails, with the parametric estimates based on the bivariate normal model for the returns.

2.3 Effects of Mergers

Lastly, we investigate how the systemic failure probability is affected by mergers. We consider a bilateral merger within a sector and across sectors. First we consider an economy made up of just two financial firms. Then we consider an economy made up of two banks and two insurers respectively.

In an economy with just two banks or two insurers, we have from (8) the systemic failure probability of order $2t^{-\alpha}$. Suppose that the threshold failure level for a merged firm increases commensurable with its portfolio size. In that case the systemic failure probability increases to

$$P(G_1 + G_2 > 2t) = P(2F + 2B + Y_1 + Y_1 > 2t)$$
$$= P(F + B + \frac{1}{2}Y_1 + \frac{1}{2}Y_1 > t)$$
$$= (2 + 2^{1-\alpha}) t^{-\alpha} + o(t^{-\alpha}).$$  

(13)

In a two firm economy, the systemic risk due to a merger is equal to the risk

$^8$Use that $P(\frac{1}{2}Y_1 > t) = P(Y_1 > 2t) = 2^{-\alpha}t^{-\alpha}$ and apply Feller’s convolution theorem.
of failure of the new firm. Thus (13) exceeds (8), specifically
\[ \lim_{t \to \infty} \frac{P(G_1 + G_2 > 2t)}{P(G_1 > t, G_2 > t)} = 1 + 2^{-\alpha} > 1, \quad (14) \]
as noted by Shaffer (1985) in a non-parametric setting. But (13) is also lower
than (7) due to the diversification effect if \( \alpha > 1 \). Thus a merger increases
systemic risk, while at the same time it lowers the risk that an individual
bank goes bust.

In case the financial sector consists of just one bank and one insurer, the
systemic failure probability is of order \( t^{-\alpha} \) by (9). A merger increases the
systemic risk to
\[ P(G_1 + H_1 > 2t) = (1 + 2^{2-\alpha}) t^{-\alpha} + o(t^{-\alpha}), \]
so that
\[ \lim_{t \to \infty} \frac{P(G_1 + H_1 > 2t)}{P(G_1 > t, H_1 > t)} = 1 + 2^{1-\alpha} > 1. \quad (15) \]
Note that if \( \alpha > 1 \) then \( 1 + 2^{1-\alpha} > 1 + 2^{-\alpha} \), so that a within sector merger
increases systemic risk by less than a cross sector merger, due to the fact that
the systemic risk among a separate bank and insurer is lower than among
two separate banks (or insurers). However, also note that
\[ \lim_{t \to \infty} \frac{P(G_1 + G_2 > 2t)}{P(G_1 + H_1 > 2t)} = 2 + 2^{1-\alpha} \frac{1}{1+2^{2-\alpha}} > 1 \]
if \( \alpha > 1 \). Thus a cross sector merger caries less risk than a within sector
merger.

A two firm financial sector is a bit of an awkward framework for the
evaluation of the effect of mergers on the systemic risk. Therefore, we also
consider a larger two banks and two insurers economy, where two out of
the four firms may merge. We compare the systemic risk in the four firm
economy with that of a three firm economy in which two of the four firms
have merged. In the four firm economy as well as in the three firm economy
the only common factor is the market factor. Moreover, the double weight on
the market factor for the merged firm cancels against the expanded threshold
\( 2t \). Hence,
\[ \lim_{t \to \infty} \frac{P(G_1 + G_2 > 2t, H_1 > t, H_2 > t)}{P(G_1 > t, G_2 > t, H_1 > t, H_2 > t)} = 1 \quad (16) \]
and
\[ \lim_{t \to \infty} \frac{P(G_1 > t, G_2 + H_1 > 2t, H_2 > t)}{P(G_1 > t, G_2 > t, H_1 > t, H_2 > t)} = 1. \]

To a first order the probabilities in the numerators and denominators are of the same size \( t^{-\alpha} \). Thus in the enlarged economy a merger between a subset of the firms has no first order effects on the systemic risk, in contrast with the two firm cases (14) and (15). But there are first order individual firm benefits from a merger through the diversification effect.

### 3 Estimators

The basis for estimation of the systemic risk measure (1) is a simple non-parametric count measure. From elementary probability theory we have that

\[ P(A \leq t, B \leq t) = 1 - P(\max[A, B] > t) \]

and by using (10)

\[ P(A > t) + P(B > t) = 1 - P(A \leq t, B \leq t) + P(A > t, B > t) = P(\max[A, B] > t) + P(\min[A, B] > t). \]

One can therefore rewrite the conditional expectation (1) as follows

\[ E[\kappa|\kappa \geq 1] = 1 + \frac{P(\min[A, B] > t)}{P(\max[A, B] > t)}. \]

The estimation of (1) can thus be reduced to the estimation of two univariate probabilities. The probabilities in the numerator and denominator can be easily estimated by counting the number of minima and maxima that exceed the threshold \( t \). Our count estimator thus reads

\[ E[\kappa|\kappa \geq 1] = 1 + \frac{\#(\min[A, B] > t)}{\#(\max[A, B] > t)}. \]

In the applications we take \( t = 0.075 \) (i.e. a 7.5% loss return on a single day) in (18), close to the boundary of the sample, and we count the number of realizations of the min and max series that are above this threshold.\(^9\) The

\(^9\) Note that the loss returns are positive numbers, after multiplying the returns with \(-1\), appearing in the first quadrant.
7.5% loss return is close to the fifth highest loss return, see the Table 5 with summary statistics in the Appendix.

Next, we discuss the asymptotic properties of the appropriately scaled estimator. Divide both the numerator and denominator in (18) by the sample size $n$. This turns the numerator and denominator into correlated U-statistics, see e.g. Serfling (1980, ch.5), since in this way one averages the excesses of the maxima and minima series. A Cramér delta argument applied to the ratio then yields the asymptotic normality as $n \to \infty$ for fixed thresholds $t$.

Subsequently, from statistical EVT it follows that one may let $t \to \infty$, provided that this happens not too fast, i.e. such that $M/n \to 0$ and where $M = \#(\max[A, B] > t)$, cf. de Haan and Ferreira (2006, p.260) or Huang (1992). Alternatively, one can apply the asymptotically normally distributed tail probability estimator from de Haan and Ferreira (2006, th. 4.4.7) for the numerator and the denominator in (18) and again invoke the Cramér delta method to establish asymptotic normality for the ratio as $t \to \infty$.

To gain insight into the estimator (18), we conducted a small simulation experiment. To this end we generated two series with 3120 draws (the number of observations in the real data) of pseudo random variables from the standard normal and Student-t distribution with 3 degrees of freedom. Both series were rescaled to give these the same means, variances and correlation pattern as in the ABN AMRO and AXA series from Figure 1a.

In Figure 2 we plot the ratio of the number $\min[A, B] > t$ to $\max[A, B] > t$ by varying the threshold $t$. The thresholds are the order statistics from the two series; the x-axis gives the indices from the descending order statistics. The y-axis gives $E[\kappa|\kappa \geq 1] - 1$ from (2), plotted against the increasing rank order of the descending ordered statistics. As the rank along the x-axis increases, we move into the center of the sample and obtain more pairs with maxima and minima that exceed the threshold order statistic. For any finite sample from whatever distribution, eventually $E[\kappa|\kappa \geq 1] - 1$ equals 1 at the lowest threshold $t$, when $t$ equals the smallest order statistic. But this is not the relevant area, since $SR(\kappa) = \lim_{t \to \infty} E[\kappa|\kappa \geq 1]$ should be judged from using a low number of order statistics only. Hence, the plots are based only on the first 750 descending order statistics from the combined series (where the choice for 750 is somewhat arbitrary).

---

10 The standard deviations are 0.019 and 0.023 for ABN AMRO and AXA respectively. The correlation coefficient is 0.575. The means were less than $10^{-3}$ in absolute value.
The right hand panel in Figure 2 shows the result for the normal distribution. The plot first lingers at zero and then gradually moves upward. Since the normal distribution implies that all dependency vanishes asymptotically, the plot first remains close to zero and only then gradually increases.

The left hand panel in Figure 2 displays the result for the correlated Student series. This plot differs markedly from the correlated normal based plot. Almost immediately the series jumps to a level around 0.2 at which it stabilizes after some gyrations. Since far out in the tail areas there are just a few observations for which \( \min[A, B] > t \), the estimator (18) is initially unstable. But it rapidly settles around 0.2. This is indeed the level that one would expect on the basis of calculations analogous to the calculations used to derive (11) and (12). In particular, for the Student-t series with 3 degrees of freedom (ignoring the means), one derives\(^{11}\)

\[
SR(\kappa) - 1 = \frac{\rho^3}{(1 - \rho^2)^{3/2} + (\sigma_1/\sigma_2)^3} = 0.17106.
\]

Next we turn to actual data. Using the same data as were used to cross-plot the stock returns of AXA and ABN-AMRO in Figure 1, Figure 3 gives the results for the estimate of (2) systemic risk. The results are very similar.

\(^{11}\)See footnote 10 for the numerical values used.
to the results of the Student-t simulation in Figure 2. On the left side of the graph the plot is initially quite variable due to a lack of observations in the tail area, but it quickly stabilizes around a level of 0.28. Since in our simulation we choose the degrees of freedom equal to the power $\alpha$ that is observed in the real data and set the correlation equal to the correlation that is observed between the AXA and ABN-AMRO returns, it is not so surprising that the estimator (18) stabilizes at a similar level as in the simulation experiment for the Student-t. The more important feature, though, is the fact that the plot immediately jumps to this level and does not gradually increase from zero, as it would if the data are asymptotically independent, like in the case of the normal distribution.

Below, we also investigate the robustness of our procedure by varying the threshold $t$. It is shown that the estimates do not change much, which is the force of statistical EVT. Moreover, in the Appendix we construct confidence bands by the Jackknife resampling procedure.
4 Empirical results

We present the estimates of the systemic risk within and across the banking and insurance sectors.

4.1 Data

Our sample consists of the ten largest European banks and the ten largest European insurers. These firms were selected on the basis of balance sheet criteria such as the amount of customer deposits and life and non-life premium income. Insurers can provide both life insurance and non-life insurance (e.g. property and casualty insurance). We use daily data from January 1992 until December 2003. A precise description of the dataset is given in the Appendix. From the daily price quotes we construct the daily loss returns that are the empirical counterparts of $G_i$ and $H_j$ from the theory section. Table 4 summarizes our classification of a financial intermediary as a bank or insurer. Summary statistics of the loss returns are provided in Table 5.

4.2 Systemic risk estimates

We estimate the within sector and cross sector downside dependencies by means of (18), using $t = 0.075$, i.e. a 7.5% loss return on a single day is taken as the threshold for the systemic risk. Since we have 10 banks and 10 insurers in our dataset, we have results for 45 possible combinations of banks, 45 possible combinations of insurers and 100 possible combinations between banks and insurers. Table 1 summarizes the estimation results for the 190 different combinations in total. The results for all 190 pairwise combinations are reported in the Tables 6, 7 and 8 in the Appendix, as well as confidence bands based on the Jacknife resampling procedure.

The summary results in Table 1 give the average and the median of all the $SR(\kappa) - 1$ estimates. Recall that $SR(\kappa) - 1$ is an estimate of the conditional joint failure probability (2). These results clearly indicate that the cross-sector dependence between banks and insurers is lower than the dependence between two firms from within the same sector. The average probability that two banks crash, given that one crashes is 10.4%. For insurers this probability is similar and equals 11.7%. The probability that an insurer crashes given that a bank crashes or that a bank crashes, given that an insurer crashes is only 7.4%, however. In other words, while two banks on average jointly fail
Table 1: Summary non-parametric estimation results for $t = 0.075$

<table>
<thead>
<tr>
<th></th>
<th>Bank</th>
<th>Insurer</th>
<th>Bank</th>
<th>Insurer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bank</td>
<td>0.1038</td>
<td>0.0744</td>
<td>0.095</td>
<td>0.069</td>
</tr>
<tr>
<td>Insurer</td>
<td>0.0744</td>
<td>0.1170</td>
<td>0.069</td>
<td>0.107</td>
</tr>
</tbody>
</table>

$SR(\kappa) - 1$

Table 2: Summary non-parametric estimation results for $t = 0.07$

<table>
<thead>
<tr>
<th></th>
<th>Bank</th>
<th>Insurer</th>
<th>Bank</th>
<th>Insurer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bank</td>
<td>0.1150</td>
<td>0.0884</td>
<td>0.0968</td>
<td>0.0842</td>
</tr>
<tr>
<td>Insurer</td>
<td>0.0884</td>
<td>0.1314</td>
<td>0.0842</td>
<td>0.1190</td>
</tr>
</tbody>
</table>

$SR(\kappa) - 1$

one out of every 9.5 times that there is a bank failing, a bank and an insurer fail jointly only one out of every 13.5 times that an insurer or a bank fails. It appears that the dependence is lower across the sectors.

To investigate the robustness of these conclusions and to show that this is not the result of sampling inaccuracy, we re-estimated (18) at the lower threshold $t = 0.07$. The results are collected in Table 2. While the averages and medians are somewhat different, the qualitative ranking is the same. Again, the cross sectorial dependence is lower than the within sector dependence. Moving into the sample by lowering the threshold $t$ to 0.07, of course produces somewhat higher estimates. But, as is shown below, under the presumption of normality, the estimates are much lower.

We formally tested whether the cross-sector dependence is dissimilar from the dependence within the same sector by applying the Wilcoxon-Mann-Whitney signed ranks test to the Tables 6-8. The null hypothesis is that estimates from two of the three tables are coming from the same distribution. The alternative hypothesis is that the values differ.

We find that the probability that the banking sector dependencies are similar to the cross sectorial dependencies is only 0.004%. We conclude that the risk profile of the two groups differs significantly. Using the same test

\[12\] We opted for this comprehensive test statistic given the limited amount of data. Alternatively, one could base oneself on the asymptotic normality of the individual pairwise estimates and use a Bonferroni bound. This approach is, however, overly conservative.
procedure, we also found that the probability that the downside risk for combinations of insurers is equal to combinations of banks and insurers is only 0.003%. Thus the dependence between banks and insurers is also significantly lower than the dependence among insurers. Moreover, the same test indicates that equality between the sectorial medians of banks and insurers is also not supported, but the rejection is less strong as the difference between the sectorial medians is smaller than the difference with the cross sectorial median.

On the firm level, there are sizable deviations from the average risk within the sector. Results for specific combinations of firms are given in the Tables 6, 7 and 8 in the Appendix. The largest conditional probability of a crash of two firms is 37.5% and involves two Spanish banks (Table 6). The 37.5% is way above the sector average of 10.3%. A possible explanation for this high probability are the common exposures of the two Spanish banks (housing exposures, government bond investments) to risks in Spain and Latin America.

We have also estimated (17), assuming a bivariate normal distribution for the returns. The results are given in the Tables 10, 12 and 11. A summary is given in Table 3. The results indicate again that the dependence between banks and insurers is also lower than the dependence among other combinations, lending robustness to our main conclusion. The order of magnitudes, though, are quite different. The assumption of normally distributed returns considerably underestimates the downside risk, both for the marginal and the multivariate risks. The average cross sector systemic risk on basis of the normality presumption is so low that only in one out of every 158 times that there is a failure, both the insurer and the bank are expected to fail jointly. While the count measure says that this joint failure happens approximately once per 13 instances of a failure. A comparable huge difference is found for the within insurance sector joint failure probability. The normal based estimate shows that this happens once per 75 times that there is an insurer
that crashes, which is way below the count measure estimate of once per 8.5 times. These huge differences are caused by the fact that under the assumption of joint normality, all dependence in the tail area eventually disappears. The non-parametric based estimator (18), though, shows that this in fact is not the case.

For specific combinations of firms we find similar considerable differences between the normal based results and the frequency based count measure. The conditional probability of a double crash for the combination of HSBC and RBS for example is 0.083, while the normal distribution based estimate is only 0.0044. Our measure therefore predicts that the conditional probability of a double crash is approximately 20 times higher for this combination than the normal based measure would make believe. For the pair AVIVA and AEGON, the estimate based on normality gives 0.0134. This is a factor 8 lower than the non-parametric based estimate of 0.111. Thus the normal based measure gives a completely different view of the tail dependence. It essentially rules out the possibility of a joint crash. Estimates that are not predisposed and can allow for the fat tail feature of the data are of an entirely different order. The estimates based on (18) appear to be more in line with the facts, since we do observe joint failures repeatedly.

5 Conclusion

The paper investigates the downside risk interdependencies within and between European insurers and banks. Banks and insurers hold numerous cross exposures and are heavily exposed to the real economy. A simple analytical factor model, in combination with the assumption that these factors are heavy tailed distributed, provides an explanation for the downside dependence structures between banks and insurers. It gives an expression for the likelihood of joint crashes, whereas the normal distribution based model would indicate that this is essentially a zero probability event.

The empirical section investigates the dependence between combinations of financials, both within a sector and across sectors. We find that downside dependence between a bank and an insurer is significantly different from the dependence structure between two banks or between two insurers. The average probability that two banks crash, given that one crashes is 10.3%. For insurers this probability is 11.7%. The probability that an insurer crashes given that a bank crashes or that a bank crashes, given that an insurer
crashes, is only 7.4%. The latter figure is in line with the empirical evidence reported in Lelyveld and Knot (2009) for European financial conglomerates. Moreover, it indicates that in general downside dependence is lower for cross-sector combinations. The theoretical model explains this by the fact that there are fewer common factors. But while such cross sectorial mergers may reduce the risk of individual financial institutions if cross hedging at the holding level were allowed, mergers can at the same time increase the systemic risk. We showed, though, that a merger of a subset of the firms embedded in a larger economy may to a first order have no impact on the systemic risk, while there are first order individual firm benefits from the formation of a conglomerate.

The current drive to unwind bancassurance conglomerates must therefore be due to other motives than risk, such as managerial complexity and the separate pillar structure of the regulatory frameworks for bank and insurance activities. See e.g. Schmid and Walter (2009) who generally find that scope is not value enhancing, but also show that combinations of commercial banking and insurance are on balance positive. This time due to the economy wide character of the crisis, banks and insurance companies were both severely affected. But this was different for example at the time of the bursting of the internet bubble, when insurers were hard hit, while banks came off lightly. Dissolving conglomerates now may therefore be a myopic error. The next crisis will surely be different.

The recent financial crisis has shown once again that fat tails and strong dependency are for real. Thus higher capital requirements and full recognition of off-balance commitments in risk weighted capital calculations are a necessity. Moreover, proper systemic risk evaluation requires aggregation across institutions and sectors, rather than the micro based approach that only looks at individual institutions as in the Value at Risk methodology. Our conditional failure index is a measure that goes into the direction of making operational the much needed macro approach to the financial stability issue.

References


[19] Laderman, E.S., The potential diversification and failure reduction benefits of bank expansion into nonbanking activities, Federal Reserve Bank of San Francisco, 1999


[26] Swiss Re, Reinsurance - a systemic risk?, Sigma no.5, Zurich, 2003

6 Theoretical Appendix

In this appendix we review the Feller convolution result and the case of the normal distribution for the $SR(\kappa)$ measure.

6.1 Feller’s theorem

We briefly introduce Feller’s convolution theorem (1971, VIII.8). This is needed to calculate convolutions of heavy tailed random variables. The convolution result is also used to determine the downside interdependence (systemic risk). The Feller theorem holds that if two independent random variables $A$ and $B$ satisfy

$$P(A > t) = P(B > t) = t^{-\alpha},$$

then their convolution satisfies

$$\lim_{t \to \infty} \frac{P(A + B > t)}{2t^{-\alpha}L(t)} = 1,$$

and where $L(t)$ is slowly varying (i.e. $\lim_{t \to \infty} L(at)/L(t) = 1$, for any $a > 0$). In other words, the theorem implies that for large failure levels $t$, the convolution of $A$ and $B$ can be approximated by the sum of the marginal distributions of $A$ and $B$. All that counts for the probability of the sum is the (marginal) probability mass that is located along the two axes above the points where the line $A + B = t$ cuts the axes.

To show this, first note that since $A$ and $B$ are independently Pareto distributed

$$1 - P(A \leq t, B \leq t) = 1 - [1 - t^{-\alpha}]^2 = 2t^{-\alpha} - t^{-2\alpha} \sim 2t^{-\alpha},$$

as $\lim_{t \to \infty} (2t^{-\alpha} - t^{-2\alpha})/t^{-\alpha} = 2$. Since (for positive random variables)

$$P(A + B > t) \geq 1 - P(A \leq t, B \leq t),$$

we have the bound $P(A + B > t) \geq 2t^{-\alpha}$. The Feller theorem maintains that $P(A + B > t)$ is in fact approximately $2t^{-\alpha}$ as $t$ becomes large, i.e. is also the lower bound. To verify this, we demonstrate that

$$P(A + B > t) - [1 - P(A \leq t, B \leq t)],$$
which comprises the probability mass in the triangle above the line \( A + B = t \) (with vertices \((0, t), (t, 0)\) and \((t, t)\)), is of an order smaller than \( t^{-\alpha} \). Note that by independence, for \( \lambda \in (0, 1) \)

\[
P(A > \lambda t, B > (1 - \lambda) t) = \left( \frac{1}{\lambda(1 - \lambda)} \right)^{\alpha} t^{-2\alpha}.
\]

Thus for any slab above the line \( A + B = t \) and with vertex at \((\lambda, 1 - \lambda)\) on the line \( A + B = t \), the probability mass is of an order smaller than \( t^{-\alpha} \) (i.e. \( \lim_{t \to \infty} t^{-2\alpha}/t^{-\alpha} = 0 \)). Note that this slab partly covers the triangle. By varying \( \lambda \), this shows that the entire triangle must carry probability mass of an order smaller than \( t^{-\alpha} \).

### 6.2 Normal Case

Assume all factors \( A, B, F, Y_i \) are standard normally distributed. From the additivity the properties of the normal distribution and using Laplace’s asymptotic expansion, we have that

\[
P(G_i > t) = P(F + B + Y_i > t) = P(\sqrt{3}F > t) \sim \frac{1}{\sqrt{2\pi}} \frac{\sqrt{3}}{t} \exp \left( -\frac{1}{2} \frac{t^2}{3} \right)
\]
as \( t \to \infty \). Analogously,

\[
P(G_i + G_j > t) = P(2F + 2B + Y_i + Y_j > t) = P(\sqrt{10}F > t) \sim \frac{1}{\sqrt{2\pi}} \frac{\sqrt{10}}{t} \exp \left( -\frac{1}{2} \frac{t^2}{10} \right).
\]

Furthermore,

\[
\frac{1}{2} \frac{P(G_i + G_j > 2t)}{P(G_i > t)} \sim \frac{1}{2} \frac{\sqrt{10}}{\sqrt{2\pi}} \frac{\sqrt{10}}{t} \exp \left( -\frac{1}{2} \frac{t^2}{10} \right) = \frac{\sqrt{10}}{4\sqrt{3}} \exp \left( -\frac{1}{30} t^2 \right) \to 0
\]
as \( t \to \infty \). The following bound

\[
\frac{P(G_i > t) + P(G_j > t)}{1 - P(G_i \leq t, G_j \leq t)} = \frac{1}{1 - \frac{P(G_i > t, G_j > t)}{P(G_i > t) + P(G_j > t)}} \leq \frac{1}{1 - \frac{P(G_i + G_j > 2t)}{2P(G_i > t)}}.
\]
therefore implies that
\[
\lim_{t \to \infty} \frac{P(G_i > t) + P(G_j > t)}{1 - P(G_i \leq t, G_j \leq t)} = 1.
\]

7 Empirical Appendix

In this appendix we discuss the selection of the data, and we give the detailed results for the pairwise downside risk estimates for the count measure $SR(\kappa)$ and under the assumption of normality.

7.1 Data selection

Since it is common for financial companies in Europe to exploit a broad portfolio of activities in banking and insurance, it is difficult to construct a dataset of companies pursuing pure banking or insurance strategies. Moreover, some activities as for example the provision of mortgages, are common for all companies in both banking and insurance. In this section we will explain when we define a company being a bank or an insurer.

We distinguish three different categories: banks, insurers (combining property&casualty and life insurance business) and financial conglomerates. The dataset contains companies from Europe (the EU and Switzerland). First, we have taken the largest firms by market capitalization in the following sectors from Datastream: banking, life insurance, insurance and other financial services. We classified these companies based on their annual accounts over 2002.

To be able to make a distinction between insurers and banks, we collected the following balance sheet items: ‘customer deposits’, ‘technical provisions’ and ‘life-insurance risk born by the policy holder’. We assume that these broad items are unique for specific sectors. The item ‘customer deposits’ is typical for banks, since they borrow money from the public. The item ‘technical provisions’ is typical for insurers, since it represents the size of provisions for future insurance claims. Another item typical for life insurance is ‘life-insurance risk born by the policy holder’, which represents provisions for future claims of life insurance policies. The three items were added up and we represented the customer deposits as a percentage of this sum of balance sheet items. When the percentage of deposits is larger than 90% we define a financial firm as a bank. When the sum of ‘technical provisions’
<table>
<thead>
<tr>
<th>Bank</th>
<th>Bank (%)</th>
<th>Insurer (%)</th>
<th>Life (%)</th>
<th>Non-life (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>HSBC</td>
<td>0.98</td>
<td>0.02</td>
<td></td>
<td></td>
</tr>
<tr>
<td>RBS</td>
<td>0.96</td>
<td>0.04</td>
<td></td>
<td></td>
</tr>
<tr>
<td>UBS</td>
<td>1.00</td>
<td>0.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td>BARCLAYS</td>
<td>0.95</td>
<td>0.05</td>
<td></td>
<td></td>
</tr>
<tr>
<td>BBVA</td>
<td>1.00</td>
<td>0.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td>BSCH</td>
<td>1.00</td>
<td>0.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td>DEUTSCHE BANK</td>
<td>0.98</td>
<td>0.02</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ABN AMRO</td>
<td>0.97</td>
<td>0.03</td>
<td>0.78</td>
<td>0.22</td>
</tr>
<tr>
<td>UNICREDITO</td>
<td>1.00</td>
<td>0.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td>STD CHARTERED</td>
<td>1.00</td>
<td>0.00</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Insurer</th>
<th>Bank (%)</th>
<th>Insurer (%)</th>
<th>Life (%)</th>
<th>Non-life (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>GENERALI</td>
<td>0.00</td>
<td>1.00</td>
<td>0.65</td>
<td>0.35</td>
</tr>
<tr>
<td>AXA</td>
<td>0.00</td>
<td>1.00</td>
<td>0.70</td>
<td>0.30</td>
</tr>
<tr>
<td>AEGON</td>
<td>0.03</td>
<td>0.97</td>
<td>0.96</td>
<td>0.04</td>
</tr>
<tr>
<td>AVIVA</td>
<td>0.00</td>
<td>1.00</td>
<td>0.75</td>
<td>0.25</td>
</tr>
<tr>
<td>PRUDENTIAL</td>
<td>0.06</td>
<td>0.94</td>
<td>0.98</td>
<td>0.02</td>
</tr>
<tr>
<td>ZFS</td>
<td>0.00</td>
<td>1.00</td>
<td>0.30</td>
<td>0.70</td>
</tr>
<tr>
<td>LEGAL &amp; GENERAL</td>
<td>0.00</td>
<td>1.00</td>
<td>0.94</td>
<td>0.06</td>
</tr>
<tr>
<td>ALLEANZA</td>
<td>0.00</td>
<td>1.00</td>
<td>1.00</td>
<td>0.00</td>
</tr>
<tr>
<td>ROYAL &amp; SUN</td>
<td>0.00</td>
<td>1.00</td>
<td>0.82</td>
<td>0.18</td>
</tr>
<tr>
<td>SKANDIA</td>
<td>0.08</td>
<td>0.92</td>
<td>0.99</td>
<td>0.01</td>
</tr>
</tbody>
</table>

Table 4: Selected data
and ‘life-insurance risk born by the policy holder’ represented as a percentage of the sum of all three items is larger than 90%, we define the firm as an insurer. Table 4 summarizes our classification for the different financial intermediaries.

We made a distinction between property and casualty insurers and life insurers and collected data on the net premium income of insurers. The net premiums are the gross premiums written minus reinsurance cover. Since an insurer might choose to buy reinsurance cover for some lines of business, we argue that the net premium income gives the best information whether an insurer is active in life insurance or in property and casualty insurance. The life-insurance premium income was represented as a percentage of the total premium income.

We use data from 1992-2003, since in 1992 Basle I came into effect. Data is on a daily basis. Firms which are part of a larger conglomerate, like Winterthur which is a holding of Credit Suisse, are excluded. Some firms are omitted because the available data series is too short. Summary statistics of the loss returns are provided in Table 5. For HSBC we lack a few observations at the start of the sample. But we prefer to keep the starting date in 1992 when Basle I came into effect. The missing data for HSBC do not hamper...
The largest realized daily loss of 24.6% was for Skandia, close to the largest loss of 24.3% realized by Royal & Sun. The 5th largest losses are already quite a bit smaller, respectively 12.9% for Skandia and 12.6% for Royal & Sun; but the 5th largest loss of ZFS is larger. The mean returns are positive, except for Royal & Sun. Standard deviations of the returns are very similar. Under the assumption of normality, the 5th largest loss returns are still close to their theoretical values of respectively 9.3% and 7.7% for Skandia and Royal & Sun. But the normal model fails for the largest losses, where it predicts respectively 10.7% and 8.8% for Skandia and Royal & Sun, which is less than 50% of the actual largest losses. This is the fat tail effect.

For the other companies similar observations apply. The threshold of 7.5% loss in the empirical application is chosen on the basis of the fact that at this level the fat tail property starts to kick in; at this level there are also still sufficient observations in the joint loss area for estimation.

### 7.2 Pairwise Multivariate Results

In this subsection we present the results of the estimator (18) of $SR(\kappa)$ for all bank and insurance combinations. The first Table 6 contains the results for all bank combinations. The second Table 7 contains the results for the insurers and the third Table 8 gives the cross sectorial results. In the latter table, the banks are listed on the rows and the insurers per column.

At the end of this subsection, we do a robustness check and report confidence bands for our estimates. To start with the observation on robustness, note that in theory the $SR(\kappa)$ measure evaluates $E[\kappa|\kappa \geq 1]$ at the limiting failure levels. In practice, the estimate of $SR(\kappa)$ is evaluated at finite failure levels. Thus in practice one estimates $E[\kappa|\kappa \geq 1]$, but at high failure levels. One may wonder how much this matters. Under independence, for example, $SR(\kappa) = 1$, but at finite failure levels nevertheless $E[\kappa|\kappa \geq 1] > 1$.

The same observation holds for jointly normal distributed returns. To investigate

\[ SR(\kappa) = \frac{P(A > t) + P(B > t)}{1 - P(A \leq t, B \leq t)} = \frac{2(1 - p)}{1 - p^2} = \frac{2}{1 + p} > 1. \]

This tends to one as $p$ is lowered to zero.

---

\[ 13 \text{If } P(A > t) = P(B > t) = 1 - p \text{ and } A \text{ and } B \text{ are independent, then} \]

\[ SR(\kappa) = \frac{P(A > t) + P(B > t)}{1 - P(A \leq t, B \leq t)} = \frac{2(1 - p)}{1 - p^2} = \frac{2}{1 + p} > 1. \]

This tends to one as $p$ is lowered to zero.
### Table 6: Banks vs Banks, t=0.075, Real data

<table>
<thead>
<tr>
<th></th>
<th>HSBC</th>
<th>RBS</th>
<th>UBS</th>
<th>BARCLAYS</th>
<th>BSCH</th>
<th>BBVA</th>
<th>DEUTSCHE BANK</th>
<th>ABN AMRO</th>
<th>UNICREDITO</th>
<th>STD CHARTERED</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.000</td>
<td>1.083</td>
<td>1.083</td>
<td>1.077</td>
<td>1.000</td>
<td>1.000</td>
<td>1.083</td>
<td>1.071</td>
<td>1.000</td>
<td>1.056</td>
</tr>
<tr>
<td>2</td>
<td>1.083</td>
<td>2.000</td>
<td>1.125</td>
<td>1.188</td>
<td>1.056</td>
<td>1.050</td>
<td>1.125</td>
<td>1.053</td>
<td>1.059</td>
<td>1.091</td>
</tr>
<tr>
<td>3</td>
<td>1.083</td>
<td>1.125</td>
<td>2.000</td>
<td>1.118</td>
<td>1.118</td>
<td>1.167</td>
<td>1.125</td>
<td>1.111</td>
<td>1.059</td>
<td>1.091</td>
</tr>
<tr>
<td>4</td>
<td>1.077</td>
<td>1.188</td>
<td>1.118</td>
<td>2.000</td>
<td>1.111</td>
<td>1.100</td>
<td>1.056</td>
<td>1.167</td>
<td>1.118</td>
<td>1.042</td>
</tr>
<tr>
<td>5</td>
<td>1.000</td>
<td>1.056</td>
<td>1.118</td>
<td>1.111</td>
<td>2.000</td>
<td>1.375</td>
<td>1.056</td>
<td>1.167</td>
<td>1.267</td>
<td>1.136</td>
</tr>
<tr>
<td>6</td>
<td>1.000</td>
<td>1.050</td>
<td>1.167</td>
<td>1.100</td>
<td>1.375</td>
<td>2.000</td>
<td>1.050</td>
<td>1.095</td>
<td>1.235</td>
<td>1.125</td>
</tr>
<tr>
<td>7</td>
<td>1.083</td>
<td>1.125</td>
<td>1.125</td>
<td>1.056</td>
<td>1.056</td>
<td>1.050</td>
<td>2.000</td>
<td>1.111</td>
<td>1.000</td>
<td>1.091</td>
</tr>
<tr>
<td>8</td>
<td>1.071</td>
<td>1.053</td>
<td>1.111</td>
<td>1.167</td>
<td>1.167</td>
<td>1.095</td>
<td>1.111</td>
<td>2.000</td>
<td>1.111</td>
<td>1.130</td>
</tr>
<tr>
<td>9</td>
<td>1.000</td>
<td>1.059</td>
<td>1.059</td>
<td>1.118</td>
<td>1.267</td>
<td>1.235</td>
<td>1.000</td>
<td>1.111</td>
<td>2.000</td>
<td>1.143</td>
</tr>
<tr>
<td>10</td>
<td>1.056</td>
<td>1.091</td>
<td>1.091</td>
<td>1.042</td>
<td>1.136</td>
<td>1.125</td>
<td>1.091</td>
<td>1.130</td>
<td>1.143</td>
<td>2.000</td>
</tr>
</tbody>
</table>

### Table 7: Insurers vs Insurers, t=0.075, Real data

<table>
<thead>
<tr>
<th></th>
<th>ROYAL &amp; SUN</th>
<th>AEGON</th>
<th>AVIVA</th>
<th>PRUDENTIAL</th>
<th>LEGAL &amp; GENERAL</th>
<th>ALLEANZA</th>
<th>SKANDIA</th>
<th>GENERALI</th>
<th>AXA</th>
<th>ZFS</th>
</tr>
</thead>
<tbody>
<tr>
<td>11</td>
<td>2.000</td>
<td>1.225</td>
<td>1.182</td>
<td>1.143</td>
<td>1.107</td>
<td>1.074</td>
<td>1.106</td>
<td>1.037</td>
<td>1.229</td>
<td>1.125</td>
</tr>
<tr>
<td>12</td>
<td>1.225</td>
<td>2.000</td>
<td>1.111</td>
<td>1.242</td>
<td>1.032</td>
<td>1.034</td>
<td>1.138</td>
<td>1.036</td>
<td>1.333</td>
<td>1.196</td>
</tr>
<tr>
<td>13</td>
<td>1.182</td>
<td>1.111</td>
<td>2.000</td>
<td>1.192</td>
<td>1.100</td>
<td>1.111</td>
<td>1.143</td>
<td>1.056</td>
<td>1.097</td>
<td>1.154</td>
</tr>
<tr>
<td>14</td>
<td>1.143</td>
<td>1.242</td>
<td>1.192</td>
<td>2.000</td>
<td>1.150</td>
<td>1.105</td>
<td>1.140</td>
<td>1.053</td>
<td>1.207</td>
<td>1.122</td>
</tr>
<tr>
<td>15</td>
<td>1.107</td>
<td>1.032</td>
<td>1.100</td>
<td>1.150</td>
<td>2.000</td>
<td>1.000</td>
<td>1.018</td>
<td>1.000</td>
<td>1.040</td>
<td>1.057</td>
</tr>
<tr>
<td>16</td>
<td>1.074</td>
<td>1.034</td>
<td>1.111</td>
<td>1.105</td>
<td>1.000</td>
<td>2.000</td>
<td>1.038</td>
<td>1.286</td>
<td>1.091</td>
<td>1.061</td>
</tr>
<tr>
<td>17</td>
<td>1.106</td>
<td>1.138</td>
<td>1.143</td>
<td>1.140</td>
<td>1.018</td>
<td>1.038</td>
<td>2.000</td>
<td>1.019</td>
<td>1.172</td>
<td>1.179</td>
</tr>
<tr>
<td>18</td>
<td>1.037</td>
<td>1.036</td>
<td>1.056</td>
<td>1.053</td>
<td>1.000</td>
<td>1.286</td>
<td>1.019</td>
<td>2.000</td>
<td>1.095</td>
<td>1.030</td>
</tr>
<tr>
<td>19</td>
<td>1.229</td>
<td>1.333</td>
<td>1.097</td>
<td>1.207</td>
<td>1.040</td>
<td>1.091</td>
<td>1.172</td>
<td>1.095</td>
<td>2.000</td>
<td>1.195</td>
</tr>
<tr>
<td>20</td>
<td>1.125</td>
<td>1.196</td>
<td>1.154</td>
<td>1.122</td>
<td>1.057</td>
<td>1.061</td>
<td>1.179</td>
<td>1.030</td>
<td>1.195</td>
<td>2.000</td>
</tr>
</tbody>
</table>

Table 6: Banks vs Banks, t=0.075, Real data

Table 7: Insurers vs Insurers, t=0.075, Real data
this issue, we also estimated $E[\kappa | \kappa \geq 1]$ under the assumption of independence for one pair of banks. We used the two Spanish banks singled out for discussion in the main text. From Table 6 we have that BSCH and BBVA have a conditional joint failure probability of 37.5%. For these two banks the conditional expected number of failures $\kappa$ in (1) under the assumption of independence we get

$$E[\kappa | \kappa \geq 1] = \frac{P(F_1 > t) + P(F_2 > t)}{1 - P(F_1 \leq t) * P(F_2 \leq t)} = 1 + \frac{1}{\frac{1}{0.0038} + \frac{1}{0.0035} - 1} = 1.0018.$$ 

The number 1.0018 is very close to one and considerably smaller than 1.375 from (18). Under the assumption of normally distributed returns, we find from the Table 10 in the next subsection an estimate for $E[\kappa | \kappa \geq 1]$ of 1.0793, also considerably lower than the count based measure 1.375. Both results therefore indicate that the way in which we measure $SR(\kappa)$ gives answers that considerably differ from the cases of asymptotic independence.

### 7.3 Confidence Bands

In the applications we took $t = 0.075$ in (18) close to the boundary of the sample. The choice of the threshold is driven by the desire to take it rela-
Table 9: Multivariate results and 90% confidence bands

<table>
<thead>
<tr>
<th>combinations</th>
<th>lower bound</th>
<th>point estimate</th>
<th>upper bound</th>
</tr>
</thead>
<tbody>
<tr>
<td>Royal &amp; Sun - AEGON</td>
<td>1.194</td>
<td>1.225</td>
<td>1.257</td>
</tr>
<tr>
<td>AEGON- AVIVA</td>
<td>1.097</td>
<td>1.111</td>
<td>1.125</td>
</tr>
<tr>
<td>RBS - Std Chartered</td>
<td>1.056</td>
<td>1.091</td>
<td>1.100</td>
</tr>
<tr>
<td>BSCH - BBVA</td>
<td>1.357</td>
<td>1.375</td>
<td>1.400</td>
</tr>
<tr>
<td>BSCH - Legal &amp; General</td>
<td>1.063</td>
<td>1.063</td>
<td>1.071</td>
</tr>
</tbody>
</table>

tively large, since this is the failure and systemic risk area, but this desire is tempered by the need to have sufficient data for estimation purposes. To establish confidence bands, we opted for the robust non-parametric Jackknife method, rather than relying on asymptotic theory with $t \to \infty$, given that in our sample size there are relatively few realizations that exceed $t$. Above we pointed out that the estimator (18) at fixed $t$ may be viewed as the ratio of two U-statistics. By Arvesen (1969, Th. 8), relying again on Cremér’s delta argument, it immediately follows that the Jacknifed estimate is asymptotically mean zero normally distributed.

Thus we can obtain a confidence band by the Jackknife resampling procedure. To this end the data were divided in 20 blocks of 156 observations. As a by-product, it is shown that our results do not change much if we omit a sequence of observations. We then apply estimator (18) 20 times, each time leaving one block of 156 observations out of the time series. To obtain the confidence band, the highest and lowest estimation results were removed, the interval between the next highest and lowest statistics then provide the 90% confidence interval. The point estimator is estimated using the full sample.

Finally, Table 9 reports some of the Jackknife confidence bands for a number $SR(\kappa)$ estimates. A selection of the results is compiled in this table for considerations of space. The bounds of the confidence interval do not deviate considerably from the point estimates and are of the same order. The central column gives the point estimate from (18). In the left and right column one finds the 90% confidence interval. In the case of the combination of BSCH and Legal and General, the point estimate of (18) hits the lower bound. This is the result of the quite limited sample, of only 12 years of daily data, which is small if one studies bivariate dependence.
### 7.4 Multivariate Normal Results

The tables with the systemic risk estimates are recalculated under the assumption of joint normality to put the count based measure into perspective. We start with the results for the banking sector in Table 10. The next table, Table 11, is the normal based systemic risk table for the insurers. Lastly follow the cross sectorial results under the presumption of joint normality in Table 12.

![Table 10: Banks vs Banks, t=0.075, Bivariate normal](image)

Table 10: Banks vs Banks, $t=0.075$, Bivariate normal
### Table 11: Insurers vs Insurers, t=0.075, Bivariate normal

<table>
<thead>
<tr>
<th>Year</th>
<th>ROYAL &amp; SUN</th>
<th>AEGON</th>
<th>AVIVA</th>
<th>PRUDENTIAL</th>
<th>LEGAL &amp; GENERAL</th>
<th>ALLEANZA</th>
<th>SKANDIA</th>
<th>GENERALI</th>
<th>AXA</th>
<th>ZFS</th>
</tr>
</thead>
<tbody>
<tr>
<td>11</td>
<td>1.0175</td>
<td>1.0300</td>
<td>1.0175</td>
<td>1.0112</td>
<td>1.0047</td>
<td>1.0158</td>
<td>1.0007</td>
<td>1.0190</td>
<td>1.0249</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>1.0175</td>
<td>1.0134</td>
<td>1.0184</td>
<td>1.0095</td>
<td>1.0092</td>
<td>1.0117</td>
<td>1.0022</td>
<td>1.0465</td>
<td>1.0399</td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>1.0200</td>
<td>1.0134</td>
<td>2</td>
<td>1.0317</td>
<td>1.0221</td>
<td>1.0050</td>
<td>1.0041</td>
<td>1.0147</td>
<td>1.0128</td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>1.0175</td>
<td>1.0184</td>
<td>2</td>
<td>1.0317</td>
<td>1.0221</td>
<td>1.0063</td>
<td>1.0048</td>
<td>1.0214</td>
<td>1.0131</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>1.0112</td>
<td>1.0095</td>
<td>1.0221</td>
<td>1.0317</td>
<td>1.0221</td>
<td>1.0063</td>
<td>1.0048</td>
<td>1.0214</td>
<td>1.0131</td>
<td></td>
</tr>
<tr>
<td>16</td>
<td>1.0047</td>
<td>1.0092</td>
<td>1.0050</td>
<td>1.0063</td>
<td>1.0030</td>
<td>1.0012</td>
<td>1.0146</td>
<td>1.0119</td>
<td>1.0091</td>
<td></td>
</tr>
<tr>
<td>17</td>
<td>1.0158</td>
<td>1.0117</td>
<td>1.0041</td>
<td>1.0048</td>
<td>1.0024</td>
<td>1.0033</td>
<td>1.0004</td>
<td>1.0128</td>
<td>1.0146</td>
<td></td>
</tr>
<tr>
<td>18</td>
<td>1.0007</td>
<td>1.0022</td>
<td>1.0016</td>
<td>1.0019</td>
<td>1.0012</td>
<td>1.0166</td>
<td>1.0004</td>
<td>1.0027</td>
<td>1.0019</td>
<td></td>
</tr>
<tr>
<td>19</td>
<td>1.0190</td>
<td>1.0465</td>
<td>1.0147</td>
<td>1.0214</td>
<td>1.0119</td>
<td>1.0128</td>
<td>1.0027</td>
<td>1.0442</td>
<td>1.0442</td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>1.0249</td>
<td>1.0399</td>
<td>1.0128</td>
<td>1.0131</td>
<td>1.0091</td>
<td>1.0088</td>
<td>1.0146</td>
<td>1.0019</td>
<td>1.0442</td>
<td></td>
</tr>
</tbody>
</table>

### Table 12: Banks vs Insurers, t=0.075, Bivariate normal

<table>
<thead>
<tr>
<th>Year</th>
<th>ROYAL &amp; SUN</th>
<th>AEGON</th>
<th>AVIVA</th>
<th>PRUDENTIAL</th>
<th>LEGAL &amp; GENERAL</th>
<th>ALLEANZA</th>
<th>SKANDIA</th>
<th>GENERALI</th>
<th>AXA</th>
<th>ZFS</th>
</tr>
</thead>
<tbody>
<tr>
<td>11</td>
<td>1.0013</td>
<td>1.0023</td>
<td>1.0032</td>
<td>1.0033</td>
<td>1.0030</td>
<td>1.0014</td>
<td>1.0007</td>
<td>1.0013</td>
<td>1.0026</td>
<td>1.0024</td>
</tr>
<tr>
<td>12</td>
<td>1.0075</td>
<td>1.0073</td>
<td>1.0131</td>
<td>1.0100</td>
<td>1.0091</td>
<td>1.0033</td>
<td>1.0024</td>
<td>1.0012</td>
<td>1.0094</td>
<td>1.0093</td>
</tr>
<tr>
<td>13</td>
<td>1.0007</td>
<td>1.0025</td>
<td>1.0016</td>
<td>1.0019</td>
<td>1.0014</td>
<td>1.0012</td>
<td>1.0005</td>
<td>1.0024</td>
<td>1.0031</td>
<td>1.0038</td>
</tr>
<tr>
<td>14</td>
<td>1.0081</td>
<td>1.0079</td>
<td>1.0140</td>
<td>1.0139</td>
<td>1.0126</td>
<td>1.0032</td>
<td>1.0020</td>
<td>1.0105</td>
<td>1.0082</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>1.0044</td>
<td>1.0130</td>
<td>1.0077</td>
<td>1.0072</td>
<td>1.0054</td>
<td>1.0063</td>
<td>1.0038</td>
<td>1.0024</td>
<td>1.0187</td>
<td>1.0126</td>
</tr>
<tr>
<td>16</td>
<td>1.0028</td>
<td>1.0100</td>
<td>1.0051</td>
<td>1.0052</td>
<td>1.0040</td>
<td>1.0051</td>
<td>1.0021</td>
<td>1.0046</td>
<td>1.0126</td>
<td>1.0083</td>
</tr>
<tr>
<td>17</td>
<td>1.0022</td>
<td>1.0081</td>
<td>1.0040</td>
<td>1.0049</td>
<td>1.0035</td>
<td>1.0036</td>
<td>1.0020</td>
<td>1.0028</td>
<td>1.0092</td>
<td>1.0073</td>
</tr>
<tr>
<td>18</td>
<td>1.0034</td>
<td>1.0204</td>
<td>1.0088</td>
<td>1.0087</td>
<td>1.0060</td>
<td>1.0053</td>
<td>1.0027</td>
<td>1.0045</td>
<td>1.0158</td>
<td>1.0105</td>
</tr>
<tr>
<td>19</td>
<td>1.0063</td>
<td>1.0081</td>
<td>1.0049</td>
<td>1.0052</td>
<td>1.0034</td>
<td>1.0183</td>
<td>1.0061</td>
<td>1.0036</td>
<td>1.0112</td>
<td>1.0095</td>
</tr>
<tr>
<td>20</td>
<td>1.0120</td>
<td>1.0084</td>
<td>1.0090</td>
<td>1.0098</td>
<td>1.0073</td>
<td>1.0034</td>
<td>1.0065</td>
<td>1.0007</td>
<td>1.0114</td>
<td>1.0096</td>
</tr>
</tbody>
</table>