COMPUTING ALTERNATING OFFERS AND WATER PRICES IN BILATERAL RIVER BASIN MANAGEMENT

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Abstract

This contribution addresses the fundamental critique in Dinar *et al.* [1992, *Theory and Decision* **32**] on the use of game theory in river basin management: People are reluctant to monetary transfers unrelated to water prices and game theoretic solutions impose a computational burden. For the bilateral alternating-offers model, a single optimization program significantly reduces the computational burden. Furthermore, water prices and *property rights* result from exploiting the Second Welfare Theorem. Both issues are discussed and applied to a bilateral version of the theoretical river basin model in Ambec and Sprumont [2002, *Journal of Economic Theory* **107**]. Directions for future research are provided.

Bilateral River Basin Management; Alternating Offers; Computation; Water prices; Walrasian Equilibrium; Second Welfare Theorem; Property Rights; Non-Transferable Utility.

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1 Introduction

The international community has come to recognize that fresh water is scarce, witness the declarations at the Dublin Conference in 1992, the UN conference Johannesburg 2002 and the tri-annual World Water Forums starting in 1997. It is generally felt that the problem is not so much physical scarcity, but inefficient use and vested interests, in particular in case of the world's many international rivers. In some regions, flooding and pollution pose serious threats, whereas in water stressed regions, lack of agreement on how to share river waters and underground aquifers are a serious source of potentially violent conflict.[1]

International water law, i.e. the Helsinki Rules of 1966 and the UN Convention on the Law of the Non-Navigational Uses of International Watercourses of 1997, does not recognize claims by upstream countries of owing the water caught on its territory (absolute territorial sovereignty), confiscating headwaters by geopolitics or downstream nation's claims of "historical rights" (unlimited territorial integrity), see e.g., Ambec and Sprumont [2002]. Rather, international law states that the nations involved should mutually agree on sharing the river through negotiations, but it is left in the middle to what extend unilateral decisions can be made in the absence of agreement. Such negotiations are often deadlocked, because almost all governments in water stressed regions became aware of the water issues after having experienced serious shortages of water and a simple reshuffling of water is perceived as a "zero sum" game where giving up water is regarded as unacceptable. Unless politics either deepen or broaden the water agenda, the situation is most likely to stay put or might even deteriorate ending in conflict.

Coalition formation, the division of gains within coalitions and unilateral decisions prior to the negotiations, so-called threats, traditionally belong to the realm of game theory, which is also recognized by global institutions involved in river basin management such as the World Bank, e.g. Carraro *et al.* [2005a, b]. These references contain an extensive overview of the many documented studies in economics and game theory addressing the water issue. However, these surveys also recognize that there are only three applications of formal negotiation theory in which negotiation procedures are explicitly modelled: Rausser and Simon [1992], Thoyer *et al.* [2001] and Simon *et al.* [2001].

Although it is eminent that game theory offers a methodology to address water issues, the game theoretic profession did not seem to respond to the critique in Dinar *et al.* [1992]: stakeholders and policy makers are reluctant to game-theoretic transfers that are not related to water prices and, second, game theoretic solutions impose a huge computational burden upon the applied modeler. The computational burden in water issues arises because the physical economic problem has to be transformed into the so-called "utility-space", represented by the characteristic function form, before any of the game theoretic concepts can be applied and, then, requires

[1] See e.g. UNESCO's initiative "$PC \to CP$ From Potential Conflict to Cooperative Potential", http://webworld.unesco.org/water/wwap/pccp/cd/pccp_publications.html

a translation back into the original physical formulation. Also the computation of game theoretic concepts in utility space is computationally difficult. This criticism still stands today.[2]

The first critique goes beyond the lack of water prices. Since most existing international treaties, such as the Jordan-Israeli Peace Accords of 1994, are formulated in terms of minimal transboundary flows, water quality and financial transfers, this hints at that the framing of negotiation theory should be preferably close to physical variables and notions understood by negotiation parties. Roemer [1988] was among the first to demonstrate how our understanding of two axiomatic bargaining solutions, including the one proposed in Nash [1950], benefits from taking physical reality as the primitive.

Recent theoretical work by Houba [2005, 2007] for the alternating-offers model in Rubinstein [1982] provides a promising way to effectively deal with the critique in Dinar *et al.* [1992]. The bilateral case seems restrictive, but extends to multilateral negotiations requiring unanimity among all parties. The innovations are twofold.

First, the equilibrium proposals in the alternating-offers model, which form a fixed point problem in terms of the physical variables, can be solved directly as the optimum of a single optimization program for which excellent software is available.[3] Of course, every fixed point problem $f(x) = x$ can be reformulated as minimization of $(f(x) - x)^2$ that is minimized at every fixed point x of the function f. However, even for relatively small problems such procedure is known to be numerically cumbersome and might not produce any solution at all. The innovation in Houba [2005, 2007] is a different reformulation that does allow for a robust numerical implementation. Furthermore, this method is computationally superior to methods relying on truncating the model's infinite horizon, which is the approach in Rausser and Simon [1992], Thoyer *et al.* [2001] and Simon *et al.* [2001] to circumvent the fixed-point problem. The objective of the single program is the same asymmetric Nash product as first reported in Binmore *et al.* [1986] for instantaneously fast negotiations and this insight therefore extends to time-consuming or sluggish negotiations. The bargaining weights provide a theoretical measure for bargaining power in sluggish negotiations.

Second, this single program generates the player-dependent Pareto-efficient proposals. Therefore, the Second Welfare Theorem applies: Every Pareto efficient allocation can be regarded as a Walrasian equilibrium with Walrasian equilibrium prices and suitable financial transfers. This provides a sound underpinning of the game theoretic solution in terms of water prices. Moreover, for every equilibrium proposals, these Walrasian equilibrium prices coincide with the shadow prices in the optimal solution of the single program and these prices are automatically generated by the optimization software. The suitable financial transfers are equal to the difference in monetary value of the disagreement situation and the situation arising from agreement, both evaluated against the Walrasian equilibrium prices.

[2] Personal communication with Professor Dinar during Game Theory Practice 2006.

[3] For example, GAMS is popular in applied economics, see www.gams.com.

These transfers can be interpreted as transfers of *property rights*. This interpretation is well understood in Walrasian or applied general equilibrium (AGE) models, but novel to game theory. AGE modelling is popular in applied economics and formulated in physical variables that are close to the policy makers' concerns and understanding. The AGE framework is flexible to accommodate sectors or regions in and across economies as well as extensions involving uncertainty and dynamics, see e.g., Ginsburgh and Keyzer [2002]. Therefore, this framework is of relevance in modelling water related problems, as will be demonstrated for the bilateral version of the river basin management model proposed in Ambec and Sprumont [2002]. Finally, this reinterpretation of Pareto efficiency assumes non-transferable utility in dealing with the negotiation problem instead of the more restrictive transferable utility

This paper discusses the relevance of the results obtained for exchange economies in Houba [2005 and 2007] in dealing with the fundamental critique in Dinar *et al.* [1992]. First, the results for exchange economies are surveyed in Section 2. Then, production is added in the subsequent section. The Second Welfare Theorem, Walrasian equilibrium prices and transfers of property rights are discussed in Section 4. The bilateral version of the river basin model in Ambec and Sprumont [2002] in Section 5 serves as an illustration of the type of insights available for river basin management. Directions for future research are delegated to the final section.

2 Alternating Offers and the Single Program

The alternating-offers model in Rubinstein [1982] is formulated in terms of the division of a single dollar. In this model, two negotiators take turns in proposing how to divide the dollar until either an agreement ends the negotiations or they perpetually disagree. Under certain assumptions on time preferences (including discounting future payoffs), this model admits a unique subgame perfect equilibrium (SPE) in strategies that are stationary: Whenever a player proposes, this player always proposes the same identity-dependent SPE proposal that makes the responding player indifferent between immediately accepting this SPE proposal or the continuation SPE payoff in which agreement is postponed for an additional round. This specifies a fixed point problem that uniquely characterizes each player's SPE proposal, see for a survey e.g., Muthoo [1999] and Houba and Bolt [2002].

The alternating-offers model also allows for an interpretation of negotiations over an infinite stream of single dollars with discounting under everlasting stationary contracts, as pioneered by Fernandez and Glazer [1991] and Haller and Holden [1990] for wage bargaining. Recently, Houba and Wen [2006] point out that the Pareto frontier under an infinite stream of dollars and heterogeneous time preferences is supported by nonstationary contracts in which the impatient player obtains zero in the long run. Such contracts seem too unrealistic and the economic modeler has to impose justifiable restrictions upon the feasible divisions of such streams.

Stationary contracts represent just one of many choices and such contracts impose a constant division over time that are by default Pareto inefficient.

For river basin management, the interpretation in terms of an everlasting stream of surpluses is appropriate, because rivers typically are renewable resources that are exploited by its users over time. Reservoirs such as lakes, dams, cisterns or aquifers are of relevance in river basin management and these require the introduction of stock variables that link subsequent economies. However, we discuss such variables only in Remark 2 in Section 5. Even in the presence of stock variables, stationary contracts seem appropriate because many international agreements specify minimal annual river flows or cost sharing of annual operation and maintenance costs of operating installed infrastructure. Although the framework is flexible enough to also handle nonstationary contracts, such contracts are briefly discussed in Remark 1 in this section. In river basin management, initial endowments or property rights are typically ill-defined. In Section 5, we address the origin of these initial endowments in such situations and, until then, we assume these are given.

To establish notation, we consider an extension of the alternating-offers model in Rubinstein [1982] in which each of two agents discounts his per-period utility from a sequence of consumption bundles in an infinite stream of exchange economies (without stocks). The exchange economy consists of two agents, indexed $i = 1, 2$, called countries. The economy has $n \geq 2$ commodities, strictly monotonic and concave utility functions $u_i : \mathbb{R}^n_+ \to \mathbb{R}$, $i = 1, 2$, a vector of initial endowments $\omega^i \in \mathbb{R}^n_+$ for country i and total endowments $\omega = \omega^1 + \omega^2 > 0$.[4] A feasible allocation is denoted as $z = \left(z^1, z^2\right)$, $z^1, z^2 \in \mathbb{R}^n_+$, such that $z^1 + z^2 \leq \omega$. We assume that $\left(\omega^1, \omega^2\right)$ is Pareto inefficient meaning that the bargaining problem is essential.

The feasibility constraint is better known as the *aggregate commodity balance* and, whenever embedded in the single program, its shadow prices will play an important role in the application of the Second Welfare Theorem discussed in Section 4. In river basin management, it includes the so-called *water balances* that are determined by the hydrological experts, see e.g., Albersen *et al.* [2003].

In the alternating-offers model, time is discrete and indexed by $t \in \mathbb{N}$. The feasible allocation in period t is denoted as $z^t = \left(z^{1,t}, z^{2,t}\right)$ and endowments are constant over time. The subject of the negotiations is a feasible allocation $z = \left(z^1, z^2\right)$ that should be understood as an everlasting, binding and stationary contract, i.e., $\left\{\left(z^{1,\tau}, z^{2,\tau}\right)\right\}_{\tau=T}^{\infty}$ with $z^{i,\tau} = z^i$ for both i and period $t = T$ being the first period that the contract is implemented. In every period $t \in \mathbb{N}$ prior to agreement, each country consumes $z^{i,t} = \omega^i$. This means that country i's disagreement utility is given by $d_i = u_i\left(\omega^i\right)$, $i = 1, 2$. Country i's utility from $T \geq 0$ periods of disagreement followed by agreement on $z = \left(z^1, z^2\right)$ is given by

$$\left(1 - \delta_i^T\right) d_i + \delta_i^T u_i\left(z^i\right),$$

[4] We could allow for $\omega^1 + \omega^2 \leq \omega$ that would describe cases where cumulative property rights over several underdeveloped resources are less than is physically feasible. For example, the Israeli-Jordan Peace Treaty of 1994 further develops the excess seasonal flows of the Yarmuck River.

where $\delta_i \in (0,1)$ is country i's discount factor. Furthermore, each constraint is binding.

The negotiations proceed as follows: At t odd, country 1 proposes the feasible allocation and, then, country 2 accepts or rejects. Accept ends the negotiations. If rejected, then each country i consumes ω^i before the negotiations move to the next (even) round. At t even, the countries' roles are reversed. The equilibrium concept is SPE.

This extended alternating-offers model also admits a unique SPE in stationary strategies, denoted as SSPE, in which the proposing player makes the same identity-dependent SSPE proposal that makes the responding player indifferent between immediately accepting this SSPE proposal or the SSPE continuation in which agreement is postponed for one more round. The country-dependent SSPE allocations are denoted as $x = \left(x^1, x^2\right)$, respectively, $y = \left(y^1, y^2\right)$, for country 1 and 2. Then, accept allocation y is a best response for country 1 if and only if $u_1\left(y^1\right) \geq (1-\delta_1) d_1 + \delta_1 u_1\left(x^1\right)$. Similarly, accept x is a best response for country 2 if and only if $u_2\left(x^2\right) \geq (1-\delta_2) d_2 + \delta_2 u_2\left(y^2\right)$. Taking these equilibrium conditions and the feasibility constraints into account, we have that any pair of SSPE allocations (x,y) simultaneously solves the following pair of *convex* programs as a fixed point:

$$
\begin{aligned}
x &= \arg\max_z u_1\left(z^1\right), && (1)\\
&\quad \text{s.t.}\quad z^1 + z^2 \leq \omega, \quad u_2\left(z^2\right) \geq (1-\delta_2) d_2 + \delta_2 u_2\left(y^2\right), \\
y &= \arg\max_z u_2\left(z^2\right), && (2)\\
&\quad \text{s.t.}\quad z^1 + z^2 \leq \omega, \quad u_1\left(z^1\right) \geq (1-\delta_1) d_1 + \delta_1 u_1\left(x^1\right),
\end{aligned}
$$

where y^2, respectively, x^1 are exogenous in (1) and (2). Both x and y are Pareto efficient, see e.g., Houba and Bolt [2002]. Numerical implementation of this fixed point problem is computationally difficult.

Of greater significance therefore is the equivalence between any fixed point (x,y) of (1)-(2) and the solution to a single convex program, as first established in Houba [2007]. The equivalence is based upon the observation that any pair of SSPE allocations, the proposed allocations x and y have the same asymmetric Nash product associated with the bargaining weight $\alpha = \ln\delta_2 / (\ln\delta_1 + \ln\delta_2)$ for country 1. To see this, note that

$$\left(u_1\left(x^1\right) - d_1\right)^{\alpha}\left(u_2\left(x^2\right) - d_2\right)^{1-\alpha} = \delta_2^{1-\alpha}\left(u_1\left(x^1\right) - d_1\right)^{\alpha}\left(u_2\left(y^2\right) - d_2\right)^{1-\alpha}$$

and

$$\left(u_1\left(y^1\right) - d_1\right)^{\alpha}\left(u_2\left(y^2\right) - d_2\right)^{1-\alpha} = \delta_1^{\alpha}\left(u_1\left(x^1\right) - d_1\right)^{\alpha}\left(u_2\left(y^2\right) - d_2\right)^{1-\alpha},$$

where $\delta_1^{\ln\delta_2} = e^{(\ln\delta_1)\cdot\ln\delta_2} = \delta_2^{\ln\delta_1}$ implies that $\delta_1^{\alpha} = \delta_2^{1-\alpha}$. The asymmetric Nash product is the objective function in the single program and its constraints are obtained by combining the constraints in (1) and (2). However, a minor modification is needed, because the endogeniety of both y^2 and x^1 in each second inequality constraint in (1) and (2) would violate the convexity of the program. The convexity can be restored by introducing the additional variables s_i, $i = 1, 2$, replacing the utility functions $u_1\left(x^1\right)$ and $u_2\left(y^2\right)$ in these constraints and the Nash product at the costs of adding the additional constraints $s_2 \leq u_2\left(y^2\right)$ and $s_1 \leq u_1\left(x^1\right)$. Then, the single convex program is given by

$$\max_{s \geq d; x,y} \left(s_1 - d_1\right)^{\alpha} \left(s_2 - d_2\right)^{1-\alpha}, \tag{3}$$

$$\text{s.t.} \quad \begin{array}{rll} s_1 & \leq u_1\left(x^1\right) & \\ s_2 & \leq u_2\left(y^2\right) & \\ \left(1-\delta_1\right) d_1 + \delta_1 s_1 & \leq u_1\left(y^1\right), & \\ \left(1-\delta_2\right) d_2 + \delta_2 s_2 & \leq u_2\left(x^2\right), & \\ x^1 + x^2 & \leq \omega, & \left(p^x\right) \\ y^1 + y^2 & \leq \omega, & \left(p^y\right) \end{array}$$

where p^x and p^y denote vectors of shadow prices. The following result states the equivalence of the fixed point (1) and (2) with program (3), which is the main result in Houba [2007].

Proposition 1 $\left(s_1^*, s_2^*, x^*, y^*\right)$ *is a solution of (3) if and only if* $(x, y) = (x^*, y^*)$ *is a pair of SSPE allocations to (1)-(2). The SSPE proposals* x^* *and* y^* *are Pareto efficient and, in the optimum, all constraints in (3) are binding.*

Remark 1 *Reinterpretation of the* n *commodities in (3) can represent nonstationary contracts. Suppose the* k*-th component represents consumption of, say, water or money at period* k*. Then, the contract distinguishes consumption of a single commodity in* n *different periods. A normalized and constant stream of this single commodity would correspond to* $\omega_k = 1$*. Discounted utility over these* n *periods can be captured by introducing some per-period utility function* $\hat{u}_i : \mathbb{R}_+ \to \mathbb{R}$ *and redefining the utility function* $u_i\left(z^i\right)$ *as* $\sum_{k=1}^{n} \delta_i^k \hat{u}_i\left(z_k^i\right)$*. This would also allow for restrictions on the contract space through additional constraints. For example, stationarity imposes* $z_k^i = z_1^i$ *for all* k*, or limiting the growth of country* i*'s consumption over time to a maximum of* $\lambda \cdot 100$ *percent imposes* $z_{k+1}^i \leq (1+\lambda) z_k^i$ *for all* k*. Many of such restrictions preserve the convexity of the program.*

Since the program is convex, the Maximum Theorem implies that the shadow prices p^x and p^y are nonnegative. Program (3) lends itself for implementation in many of the optimization packages available today, such as e.g., GAMS. Since this program is convex, these packages offer robust computational algorithms designed to efficiently compute an accurate numerical approximation of the unique optimum. This almost exact numerical solution is superior to approximation of the fixed point of (1)-(2) through a T-period finite horizon truncation that involves solving a sequence of T single programs of n variables and n linear constraints being either (1) or (2). Rausser and Simon [1992], Thoyer *et al.* [2001] and Simon *et al.* [2003] assume a random proposer at the final bargaining round that eliminates the deadline effect and speeds up the convergence.

The single program states the formula for the Nash bargaining solution for *all* parameter values δ_1 and δ_2 in a modified exchange economy of "double" size. It also generalizes the well-known result in Binmore *et al.* [1986] for instantaneous negotiations to time-consuming sluggish negotiations. Instantaneous negotiations correspond to taking the limit of vanishing time between bargaining rounds. Formally, let $\Delta > 0$ denote the time between any two subsequent bargaining rounds and consider discount factors equal to δ_1^Δ and δ_2^Δ. Vanishing time means taking the limit Δ goes to 0, i.e., $\lim_{\Delta \to 0} \delta_1^\Delta = \lim_{\Delta \to 0} \delta_2^\Delta = 1$. Then, instantaneous negotiations correspond to the asymmetric Nash bargaining solution and feature $x = y$. Therefore, these can be implemented by less variables and constraints and solved as

$$\max_z \left(u_1\left(z^1\right) - d_1\right)^\alpha \left(u_2\left(z^2\right) - d_2\right)^{1-\alpha}, \quad \text{s.t.} \quad z^1 + z^2 \leq \omega. \tag{4}$$

So, the additional computational costs, in terms of additional variables and constraints, of solving (3) for sluggish negotiations instead of (4) for instantaneous negotiations amounts to $n+2$ variables and $n+4$ constraints, of which n are linear.

The bargaining problem in utility representation

Houba [2005] establishes similar results for convex bargaining problems in utility representation. This is theoretically relevant, because every bilateral negotiation problem that can be transformed into such convex bargaining problem is also solvable as a single convex program. Furthermore, it also provides valuable insights for non-convex bargaining problems.

Formally, a bargaining problem in utility representation is denoted as the pair (S, d) with $S \in \mathbb{R}^2$ the nonempty, compact set of feasible utility pairs and $d \in S$ the disagreement point. The curve $s_i = f_i(s_j)$, $i, j = 1, 2$, $i \neq j$, describes the Pareto frontier of S.

Any pair (s_1^*, s_2^*) of SSPE utilities simultaneously solves the following pair of programs as a fixed point:

$$s_1^* = \arg\max_{s \geq d} s_1, \quad \text{s.t.} \quad s_1 \leq f_1(s_2), \quad s_2 \geq (1-\delta_2) d_2 + \delta_2 s_2^*, \tag{5}$$

$$s_2^* = \arg\max_{s \geq d} s_2, \quad \text{s.t.} \quad s_2 \leq f_2(s_1), \quad s_1 \geq (1-\delta_1) d_1 + \delta_1 s_1^*. \tag{6}$$

Each program implies that the proposing country maximizes his own utility among the set of feasible and acceptable utility pairs. In each optimum, both constraints are binding. Houba [2005] establishes the following result for convex bargaining problems (S, d):

Proposition 2 *Let S be a convex set. Then, (s_1^*, s_2^*) is the unique pair of SSPE utilities of (5)-(6) if and only if*

$$\begin{aligned} (s_1^*, s_2^*) &= \arg\max_{s \geq d} (s_1 - d_1)^{\alpha} (s_2 - d_2)^{1-\alpha}, \qquad (7) \\ s.t. \quad s_1 &\leq f_1 ((1-\delta_2) d_2 + \delta_2 s_2), \\ s_2 &\leq f_2 ((1-\delta_1) d_1 + \delta_1 s_1). \end{aligned}$$

This proposition partly extends to the class of bargaining problems in utility representation that are strongly comprehensive or "non-convex" as in Herrero [1989]. Then, program (7) always yields a pair of SSPE allocations, because in the optimum both constraints are binding. However, the reverse may not hold as Herrero [1989] shows: Uniqueness of the pair of SSPE utilities (s_1^*, s_2^*) may break down and multiple non-stationary SPE strategies may exist as well. The Nash product associated to different pairs of SSPE utility pairs (s_1^*, s_2^*) are also different and may be less than the maximal attainable Nash product. For instantaneous negotiations, the (limit) pair of SSPE utilities (s_1^*, s_2^*) in program (7) coincides with the maximal Nash product as axiomatized in Kaneko [1980]. The limit set of all SSPE utilities of (5)-(6) is axiomatized in Herrero [1989]. What is needed for uniqueness in (non-stationary) SPE strategies is the stronger uniqueness in fixed points of (5)-(6).

3 Production Economies

The results for exchange economies might seem of limited interest for applied economics. The aim of this and later sections is to show the merits in applications. In this section, we address how to incorporate production.

Extending the exchange economy to allow for production activities is conceptually straightforward, see e.g. Varian [1984]. Production plans require inputs from the economy in order to produce outputs. These are represented in a single vector $q \in \mathbb{R}^n$ with positive and negative elements, where positive (negative) elements represent outputs (inputs). Production technologies are often represented by the production set $Q \subset \mathbb{R}^n$ of all technologically feasible input-output combinations. Often, production sets are represented by transformation functions. In our case, transformation functions more naturally fit the optimization framework. The function $F : \mathbb{R}^n \to \mathbb{R}$ is a transformation function representing Q if $q \in Q$ if and only if $F(q) \leq 0$. Furthermore, $F(q) = 0$ means q is efficient. The possibility of inaction and no free lunch translate into $F(0) = 0$. The technology is convex if the function F is quasi-convex. Otherwise, the technology is called nonconvex.

Since we later discuss bilateral river basin management in which the economy of each riparian country involves water related production, we assume that water related production is carried out by many small producers that are mainly active in one country. For explanatory reasons, we aggregate all producers in one country by assuming one production set for each country.[5] So, each country exclusively controls some production technology and country i's production plan is a vector $F_i\left(q^i\right) \leq 0$. The subject of the negotiations becomes a feasible allocation $z = \left(z^1, z^2, q^1, q^2\right)$ such that

$$z^1, z^2 \in \mathbb{R}^n_+, \quad F_1\left(q^1\right) \leq 0,\ F_2\left(q^2\right) \leq 0 \quad \text{and} \quad z^1 + z^2 \leq \omega + q^1 + q^2,$$

where negative components of either q^1 or q^2 lower the amount of that particular good available for consumption. The aggregate commodity balance implies that the demand for each good is at most equal to its supply. It is standard to have a single vector per producer instead of having the demand for inputs on the left-hand side.

In case of convex production technologies, we immediately have that the bargaining problem in utility representation is also convex, see e.g., Roemer [1988], and, hence, the equivalence stated in Proposition 2 immediately applies. Also in terms of the economic environment, the equivalence between the fixed point problem and program (3) remains in tact, where the single program remains convex. Including (convex) production per country requires the following modifications to the alternating-offers model, where we add an additional superscript x and y to the production plans to distinguish between country 1's and 2's proposal. The modifications to program (1) imply rewriting the commodity balance and adding both transformation functions such that the modified program includes the following constraints:

$$x^1 + x^2 \leq \omega + q^{x,1} + q^{x,2}, \quad F_1\left(q^{x,1}\right) \leq 0 \quad \text{and} \quad F_2\left(q^{x,2}\right) \leq 0.$$

Similar, the modified program (2) includes the following constraints:

$$y^1 + y^2 \leq \omega + q^{y,1} + q^{y,2}, \quad F_1\left(q^{y,1}\right) \leq 0 \quad \text{and} \quad F_2\left(q^{y,2}\right) \leq 0.$$

These modifications must also be made to obtain the modified program (3), which we omit.

Nonconvex production technologies can be implemented in the same manner. However, such technologies cause a breakdown of the convexity of the modified program (3), because such technologies are known to give rise to nonconvex bargaining problem in utility representation. As argued for the utility representation in Section 2, the theoretical results only partly extend. Its counterpart for program (3) with nonconvex production reads: The maximum of the modified single program (3) corresponds to one of possibly multiple SSPE strategies. The SSPE specified by this program is special in that it has the largest Nash product.

The modified program (3) with production can also be implemented in optimization software. Under convex production technologies, the software returns the unique optimum. However, for nonconvex programs it is fundamentally unclear

[5] If not, we would have the index set J_i of producers in country i and $F_j\left(q^j\right) \leq 0$ for every $j \in J_i$. In the text, we assume $J_i = \{i\}$.

whether a local or global optimum is found, even though most packages offer robust algorithms for such problems. Although this is a fundamental problem of any numeric optimization, the numerical solution returned, whether it is the global or a local optimum, has properties that are consistent with SSPE behavior.

4 Market Prices and Property Rights

This paper is motivated by the fundamental critique in Dinar *et al.* [1992], who report on the difficulties arising from applying cooperative game theory to several small-scale water issues. They state: "Clearly, the potential for additional income due to cooperation is higher when side payments are possible. However, the soundness of such transfers with no a prior reference to the price per unit of water may be questioned, especially considering the general resentment of farmers to adopt side payments as a policy." Since side payments or transfers are advocated by (cooperative) game theory as the universal remedy towards cooperation, the game theoretic society should treat this critique very seriously.

In this section, we discuss the merits of the Second Welfare Theorem in dealing with this fundamental issue. Since (cooperative) game theory developed autonomously from microeconomics, it does not refer to nor does it exploit the implications of the Second Welfare Theorem. For that reason, we discuss these implications in detail before turning our attention to bilateral river basin management in the next section.

The Second Welfare Theorem for economies with production states: Any Pareto efficient allocation is attainable as a price quasi-equilibrium or Walrasian equilibrium with transfers, see e.g., Varian [1984], Mas-Colell *et al.* [1995] and Ginsburgh and Keyzer [2002]. Mas-Colell *et al.* [1995] show that this theorem holds under convex and locally nonsatiated preferences and convex production technologies. The transfers can be achieved through many appropriate physical reallocations of the initial endowments or through financial lump-sum transfers evaluated against the Walrasian equilibrium prices. In terms of a Walrasian economy, these transfers take place before price-taking behavior by all agents on the economy's markets. Then, by the law of supply and demand, the Walrasian equilibrium prices support the Pareto efficient allocation under consideration. These Walrasian equilibrium prices can also be obtained as the shadow prices of a welfare program. The objective function in program (3) is the Nash social welfare function as axiomatized in Kaneko [1980]. The implication to river basin management is clear: Any Pareto efficient allocation can be reinterpreted in terms of lump-sum financial transfers and supporting water prices.

In order to focus the discussion, consider country 1's SSPE proposal $\left(x^{\star 1}, x^{\star 2}, q^{\star x,1}, q^{\star x,2}\right)$ with the vector of shadow prices $p^{\star x}$ as obtained by the modified program (3). The allocation $\left(x^{\star 1}, x^{\star 2}, q^{\star x,1}, q^{\star x,2}\right)$ is feasible and, being a SSPE proposal, is Pareto efficient. Therefore, the shadow prices $p^{\star x}$ can be regarded as the Walrasian equilibrium prices and these prices clear all the markets: aggregate demand $x^{\star 1}+x^{\star 2}$ equals aggregate (net) supply $\omega+q^{\star x,1}+q^{\star x,2}$.

Valued against $p^{\star x}$, country i's allocated consumption $x^{\star i}$ is worth $p^{\star x}\cdot x^{\star i}$ and can be regarded as country i's expenditure on all goods. This expenditure is financed from this country's market income obtained from selling against $p^{\star x}$ its endowments ω^i and its produce $F_i\left(q^i\right)\leq 0$ that are worth $p^{\star x}\cdot\omega^i+p^{\star x}\cdot q^{\star i}$. In general, a country's allocated (or allowed) expenditure and its market income will not be equal and this means that either a country is allowed to expend more than it earns, or less. This difference can be interpreted as country i's implicitly received lump-sum subsidy, or tax levied on this country. Formally, in country 1's SSPE proposal, country i receives the net lump-sum transfer $T_i^{\star x}=p^{\star x}\cdot x^{\star i}-p^{\star x}\cdot\omega^i-p^{\star x}\cdot q^{\star i}$, which is a subsidy if positive and a tax if negative. Before-tax market income is equal to $p^{\star x}\cdot\omega^i+p^{\star x}\cdot q^{\star i}$ and after-tax market income is $m^{\star i}=p^{\star x}\cdot\omega^i+p^{\star x}\cdot q^{\star i}-T_i^{\star x}=p^{\star x}\cdot x^{\star i}$. Note that there is a balanced budget for the fictitious tax authority. This follows directly from the aggregate commodity balance that appears as $p^x\cdot\left(x^1+x^2-q^{x,1}-q^{x,2}-\omega\right)$ in the Lagrangian of the optimization problem and this term is equal to 0 in the optimum.

In the Walrasian equilibrium all trade is voluntary and the markets respect property rights in the sense that, valued against the Walrasian equilibrium prices, each consumer's expenditure is equal to his (non-taxed) market income: $T_i^{\star x}=0$ in a Walrasian equilibrium. In any SSPE agreement, each country's expenditure and after-tax income satisfies the same property, but from moving from before-tax to after-tax market income a change in property rights occurs valued $T_i^{\star x}$ that is most likely different from zero. In the context of negotiations, the countries are rational and any agreement is reached on a voluntary basis. So, any such voluntary agreed upon contract implies agreement upon a redistribution of property rights.

As mentioned, the Pareto efficient allocation $\left(x^{\star 1},x^{\star 2},q^{\star x,1},q^{\star x,2}\right)$ can be thought of as arising from a Walrasian economy in which all parties act as price takers. Country i behaving as a price-taking consumer facing market prices $p^{\star x}$ and having after-tax income $m^{\star i}=p^{\star x}\cdot x^{\star i}$ solves

$$x^{\star i}=\arg\max_{x^i\geq 0}u_i\left(x^i\right),\quad \text{s.t.}\quad p^{\star x}\cdot x^i\leq m^{\star i},$$

where we take uniqueness of the maximizer for granted. So, country i acting as a price-taking consumer voluntarily purchases $x^{\star i}$ such that $\nabla u_i\left(x^{\star i}\right)=p^{\star x}$, where ∇u_i denotes the gradient of u_i in case of differentiability. Monotonicity of the utility function guarantees $p^{\star x}\cdot x^i=m^{\star i}$. This latter condition should also be fulfilled in the first-order conditions of the modified program (3). As mentioned, for convex programs the shadow prices p^x (and p^y) are nonnegative. This result generalizes to economies with non-convex production, because then the monotonicity of the utility functions guarantees the non-negativity of $p^{\star x}$ and $p^{\star y}$ through $p^{\star x}=\nabla u_i\left(x^{\star i}\right)\geq 0$ and $p^{\star y}=\nabla u_i\left(y^{\star i}\right)\geq 0$.

Similar to the Robinson Crusoe economy, country i is also producer i. Country

i behaving as a price-taking producer facing market prices $p^{\star x}$ solves

$$q^{\star x,i} = \arg\max_{q^{x,i}} p^{\star x} \cdot q^{x,i}, \quad \text{s.t.} \quad F_i\left(q^{x,i}\right) \leq 0,$$

where we once more take uniqueness for granted.[6] Similar as before, country i as a price-taking producer voluntarily chooses the production plan $q^{\star i}$ such that $\nabla F_i\left(q^{x,i}\right) = p^{\star x}$, which is also fulfilled by the first-order conditions of the modified program (3). Under convex production, firm i always makes a nonnegative profit that accrues to consumer i's before-tax market income. However, nonnegative profits are not automatically ensured under non-convexities. Then, we need to modify the profit maximization problem taking into account a lump-sum producer's subsidy $S^{\star i} = -p^{\star x} \cdot q^{\star x,i} \geq 0$ received by producer i to favour the producer's decision towards $q^{\star x,i}$ instead of inaction at $q^{x,i} = 0$. Of course, proper accounting requires that producers's subsidies and consumers' subsidies are counted just once. The producer subsidy $S_i^\star$ ensures that consumer i receives a net profit of at least 0 from operating the production plant, but this consumer pays for $S_i^\star$ through $T_i^{\star x}$, which requires a minor adjustment of the national accounts.

To summarize, since each SSPE proposal is Pareto efficient it can be supported by Walrasian equilibrium prices as an immediate consequence of the Second Welfare Theorem. The associated Walrasian equilibrium prices are the shadow prices of the modified program (3) and these resolve the lack of (water) prices. Although shadow prices are implicitly present in transforming the physical economy into the (often transferable) utility representation in game theoretic applications, their presence seems to be ignored. Also the richer interpretation of agreements in terms of reallocation of property rights remains behind a veil when taking the utility representation as the primitive of the analysis. In negotiations, parties benefit from voluntarily agreeing upon a redistribution of property rights, even in the absence of such rights as will be clear from the next section.

The Walrasian equilibrium prices suggest the possibility to decentralize all consumer and producer decisions through markets and suitable taxation. In river basin management, introducing water markets is often advocated as a solution to inadequate water management. Of course, whether it is advisable to do so should depend upon whether or not these agents have market power to manipulate market prices, which is a separate matter and outside the realm of the Walrasian equilibrium model.

5 Bilateral River Basin Management

Thus far, we established that program (3) can be implemented in economies with production and that SSPE proposals can be interpreted as Walrasian equilibria. In

[6] Under constant returns to scale, the Walrasian equilibrium prices are such that the firms make zero profit and then a set of maximizers exists.

this section, we illustrate the potentials of this insight to bilateral river basin management in a two-country version of the model proposed in Ambec and Sprumont [2002].

Consider a river that runs through two countries, where country 1 lies upstream of country 2 and all water users within the same country are aggregated as a single consumer. A more detailed model would allow for explicit production and water users that differ in their spatial location, representing different regions or cities, and differ in their use, such as agriculture, industrial and domestic. The territory of country i, $i = 1, 2$, captures $e_i > 0$ of water that is available for use. The subject of the negotiations concerns the allocation of water and an explicit financial transfer. Therefore, each country derives utility from consuming water and from holding money. Country i's utility from consuming z_i of water and the possibly negative transfer t_i is given by $u_i(z_i, t_i) = b_i(z_i) + t_i$, where b_i is monotonically increasing, homogenous ($b_i(0) = 0$), differentiable and strictly concave. The function b_i can also be regarded as an implicitly described production technology that can be separated as described in Section 4 at the cost of an additional variable and constraint. Following Ambec and Sprumont [2002], money is transferred utility meaning that the two-country economy does not have initial holdings of money. Total endowments are $\omega = (e_1, e_2, 0)$.

In terms of economic goods, the model distinguishes between good 1 representing water that is physically located in country 1, good 2 representing water located in country 2 and good 3 representing money. In Walrasian economies, each consumer expresses a demand for each of the available goods, but in case of rivers country 1 cannot consume good 2 and country 2 cannot consume good 1. To minimize on subscripts and superscripts, we continue denoting country i's consumption of good i simply as z_i. So, in terms of Section 4, we have $z^1 = (z_1, 0, t_1)$ and $z^2 = (0, z_2, t_2)$.

Since water disposed of by country 1 flows downhill and transforms good 1 into good 2 we should see the river as a giant production process governed by physical processes. In the model under consideration, the river accumulates e_2 on country 2's territory and, therefore, the river's production of downhill water between "locations" 1 and 2 takes place on country 1's territory. So, it is country 1 that produces good 2 with good 1 as input. Although some countries spend a significant proportion of the gross national income on pumping water uphill, such as Israel and the Kingdom of Jordan, we refrain from pumping as in Ambec and Sprumont [2002]. We only note that pumping should also be treated as a production process. In the present setup, country 2 cannot produce good 1.

With respect to production of good 2, country 1 produces q_2 from input q_1 and, under costless transformation, we have $q_1 + q_2 \leq 0$ and $q_1 \leq 0$, which implies the production vector $q^1 = (q_1, q_2, t)$ and the convex production technology $F_1\left(q^1\right) \leq$

$\max\{q_1+q_2,q_1,t\}\leq 0$.[7] The aggregate commodity balance is given by

$$\begin{array}{ll} z_1 \leq e_1+q_1, & (p_1) \\ z_2 \leq e_2+q_2, & (p_2) \\ t_1+t_2 \leq 0, & (p_3) \end{array} \tag{8}$$

where p_1, p_2 and p_3 refer to the shadow prices for the three goods. Consumption of $z_1 \leq e_1$ by country 1 and efficiency in production implies $q_2 \leq -q_1 \leq e_1 - z_1$. Substitution yields the mathematically equivalent feasibility constraints in Ambec and Sprumont [2002]:

$$\begin{array}{ll} z_1 \leq e_1, & (p_1) \\ z_2 \leq e_2+e_1-z_1, & (p_2) \\ t_1+t_2 \leq 0, & (p_3) \end{array} \tag{9}$$

Then, the subject of the negotiations is reduced to (z_1, z_2, t_1, t_2). The allocation should be extended to also include q_1 and q_2 in applications where the hydrological part of the model is provided by hydrologists.

The disagreement point

An essential ingredient of any bargaining problem is the disagreement point. For water issues, there is some modelling freedom in the choice of such point. An obvious choice is a disagreement point based upon property rights according to international law. An alternative choice for the disagreement point would be to assumes that this point is based upon the countries' unilateral decisions concerning water issues. We discuss both alternatives.

Suppose in modeling water issues we opt for a disagreement point based upon international law. As Ambec and Sprumont [2002] argue, international law is ambiguous and they discuss two conflicting doctrines. One of these doctrines is *absolute territorial sovereignty* and it assigns e_i as the property rights for country i and the river's production technology to country 1. This would imply $\omega^1 = (e_1, 0, 0)$, $\omega^2 = (0, e_2, 0)$ and the disagreement point $d = (b_1(e_1), b_2(e_2))$. This point is always feasible, because

$$b_1(e_1) + b_2(e_2) \leq \max_{z_1\in[0,e_1]} b_1(z_1) + b_2(e_1+e_2-z_1). \tag{10}$$

In order to have an essential bargaining problem, the inequality has to be strict. Then, the players negotiate for an agreement within the set IR of Figure 1.

The second doctrine is *unlimited territorial integrity* and it assigns incompatible property rights to the countries, namely e_1 to country 1 and e_1+e_2 to country 2. Also, the river's production technology accrues to country 2. This would translate into $\omega^1 = (e_1, 0, 0)$, $\omega^2 = (0, e_1+e_2, 0)$ and the unattainable aspiration point $(b_1(e_1), b_2(e_1+e_2))$, because

$$b_1(e_1) + b_2(e_1+e_2) > \max_{z_1\in[0,e_1]} b_1(z_1) + b_2(e_1+e_2-z_1). \tag{11}$$

[7] For completeness, $q^2 = (\hat{q}_1, \hat{q}_2, \hat{t}) \in Q^2$ if and only if $F_2(q^2) = \max\{\hat{q}_1, \hat{q}_2, \hat{t}\} \leq 0$.

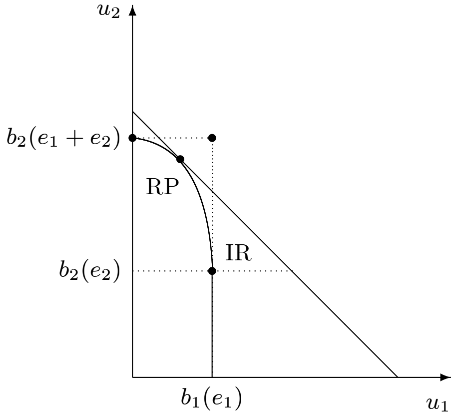

Figure 1: Absolute territorial sovereignty implies negotiations over the set IR, whereas unlimited territorial sovereignty implies negotiations over the set RP below $u_1 + u_2 \leq 1$.

In case both countries are fully committed to these "virtual" aspiration levels, then it is most likely that the negotiations remain deadlocked with disagreement as the only outcome according to e.g. Crawford [1982] and Muthoo [1992]. However, when parties are not committed, Mariotti and Villar [2005] propose the (cooperative) Nash rationing solution to negotiate concessions with respect to the infeasible claims in bankruptcy problems. Applied to bilateral river basin management, the countries negotiate concessions within the set RP of Figure 1. Obviously, country 2 prefers Nash rationing based upon unlimited territorial integrity, whereas country 1 prefers negotiations based upon absolute territorial sovereignty. This explains the often fierce disputes over these two doctrines commonly observed in reality.

As an alternative modelling approach that is also close to reality, we may assume that each country takes unilateral decisions concerning water issues in the absence of bilateral river basin management. In terms of noncooperative bargaining theory, the disagreement point is endogenous. Several theoretical models are available, see e.g. Busch and Wen [1995]. These bargaining models also assume the model under consideration as the disagreement game in which both countries take unilateral decisions. Under SSPE behavior and the impossibility of commitment to disagreement actions prior to the negotiations, the disagreement point coincides with a Nash equilibrium, which is $(z_i, t_i) = (e_i, 0)$, $i = 1, 2$, in our case. This Nash equilibrium coincides with absolute territorial sovereignty. Although international law suggests that countries should mutually agree on Pareto improvements through negotiations, this law seems to lack a doctrine how to treat unilateral decisions in absence of agreement.

The Second Welfare Theorem and water prices

The purpose of discussing this particular model is to arrive at water prices, water trade and transfers of property rights through financial money transfers. The interesting case assumes the nontrivial case in which the disagreement point corresponds to absolute territorial sovereignty: $d = (b_1(e_1), b_2(e_2))$. Application of the modified program (3) yields the single convex program

$$\max_{s \geq d; \left(x_1, x_2, q_1^x, q_2^x, t_1^x, t_2^x\right), \left(y_1, y_2, q_1^y, q_2^y, t_1^y, t_2^y\right)} (s_1 - d_1)^{\alpha} (s_2 - d_2)^{1-\alpha}, \tag{12}$$

s.t.

$$\begin{array}{rll} s_1 & \leq b_1(x_1) + t_1^x & (\mu_1) \\ s_2 & \leq b_2(y_2) + t_2^y & (\mu_2) \\ (1-\delta_1) d_1 + \delta_1 s_1 & \leq b_1(y_1) + t_1^y, & (\lambda_1) \\ (1-\delta_2) d_2 + \delta_2 s_2 & \leq b_2(x_2) + t_2^x, & (\lambda_2) \\ x_1 & \leq e_1 + q_1^x, & (p_1^x) \\ x_2 & \leq e_2 + q_2^x, & (p_2^x) \\ t_1^x + t_2^x & \leq 0, & (p_3^x) \\ y_1 & \leq e_1 + q_1^y, & (p_1^y) \\ y_2 & \leq e_2 + q_2^y, & (p_2^y) \\ t_1^y + t_2^y & \leq 0, & (p_3^y) \\ q_1^x + q_2^x & \leq 0, & (\gamma^x) \\ q_1^y + q_2^y & \leq 0, & (\gamma^y) \end{array}$$

where all Greek symbols between brackets denote shadow prices. As in Section 4, we only discuss country 1's SSPE proposal $(x_1, x_2, q_1^x, q_2^x, t_1^x, t_2^x)$. The part of the first-order conditions involving the partial derivatives of these six variables (maintaining the stated order) are given by

$$\begin{aligned} \mu_1 b_1'(x_1) - p_1^x &= 0, \\ \lambda_2 b_2'(x_2) - p_2^x &= 0, \\ p_1^x - \gamma^x &= 0, \\ p_2^x - \gamma^x &= 0, \\ \mu_1 - p_3^x &= 0, \\ \lambda_2 - p_3^x &= 0. \end{aligned}$$

Solving these equations yields

$$p_1^x = p_2^x = p_3^x b_1'(x_1) = p_3^x b_2'(x_2) > 0, \tag{13}$$

because utility is strictly increasing in money and, therefore, $p_3^x > 0$. Due to specificities of the model, we obtain the special case of a uniform water price for all locations. From (13) we observe that only relative prices matter and we may take money as the numeraire by dividing all shadow prices by p_3^x, where we denote the normalized (uniform) water price as $p_w^x = p_1^x / p_3^x$. Assuming that the bargaining problem is essential, i.e., Pareto improvements exist as depicted in Figure 1, the total gains from bilateral river basin management are maximized if the upstream country is willing to trade some of its water for money. Since also all shadow prices are positive, we must have that all constraints are binding, including efficient river production. So, $q_1^x = x_1 - e_1 < 0$ and $q_2^x = -q_1^x > 0$ implies $x_2 = e_1 + e_2 - x_1 > e_2$. Furthermore, $b_1'(x_1) = b_2'(x_2)$ in (13) implies that the joint surplus $b_1(x_1) + b_2(x_2)$ is maximized in this SSPE proposal and, hence, x_1 coincides with the unique maximizer of the right-hand side of (11), as could be expected. Finally, $t_1^x + t_2^x = 0$ and $q_1^x + q_2^x = 0$ imply that aggregate spending equals aggregate income $p_w^x(x_1 + x_2) = p_w^x(e_1 + e_2)$.

According to the Second Welfare Theorem, country i's before-tax income $p_w^x e_i$ and its expenditure or after-tax income is equal to $m_i^x \equiv p_w^x x_i + t_i^x$. Acting as a price-taking consumer, country i spends its income on water consumption and monetary liquidity by solving:

$$(x_i, t_i^x) \in \arg\max_{z_i, t_i} u_i(z_i, t_i), \quad \text{s.t.} \quad p_w^x z_i + t_i \leq m_i^x, \tag{14}$$

which yields $\frac{\partial u_i(x_i,t_i)}{\partial x_i} = b_i'(x_i) = p_w^x$ and $\frac{\partial u_i(x_i,t_i)}{\partial t_i} = 1$. Note that $b_i'(x_i)$ is also country i's marginal rate of substitution between water and money and it is equal to the relative price $p_w^x/1$. The monotonic preferences imply the budget constraint is binding. Country 1 receives the amount of money $t_1^x = -t_2^x > 0$ for its delivery of $e_1 - x_1$ to country 2. This implies a unit price of water of $(e_1 - x_1)/t_1$ that is unrelated to marginal costs and benefits. Note that it does not matter whether t_i represents money or some consumption good from which the countries obtain utility. Since the SSPE proposal is individually rational, we obtain that $t_i^x \geq b_i(e_i) - b_i(x_i) > 0$ for both $i = 1, 2$. Summation of these inequalities shows a nonempty range of transfers that are feasible, because $t_1^x + t_2^x = 0$ and $b_1(e_1) + b_2(e_2) - b_1(x_1) - b_2(x_2)$ is negative.

Next, consider producer 1 with its constant-returns-of-scale river production technology. Producer 1's profit under price-taking is equal to 0, because he buys input $q_1 = x_1 - e_1 < 0$ against price p_w^x and sells exactly this amount at exactly the same price to country 2. We refer to Albersen *et al.* [2003] for non-trivial financial accounts derived from shadow pricing and non-convex physical processes.

Although money is usually regarded as a special economic good, it is just one of the goods in the economy, also according to the Second Welfare Theorem. This theorem provides an interpretation of the allocation $(x_1, x_2, q_1^x, q_2^x, t_1^x, t_2^x)$ in terms of market trade (or marginal cost/benefit pricing) against the price vector $(p_w^x, p_w^x, 1)$ and a redistribution of property rights equal to $T_i^{\star x} = p_w^x x_i + 1 \cdot t_i^x - p_w^x e_i$, being the difference between expenditure and before-tax income. Whether $T_i^{\star x}$ is positive or negative is an empirical matter. This redistributive effect consists of the combined value of net trade in water $p_w^x(x_i - e_i)$ and the net trade in money $1 \cdot (t_i^x - 0)$ that is opposite in sign. The negotiated outcome combines two effects: redistribution of wealth and wealth improving trade, which is incorporated in the before-tax market income. Even though the initial rights in river basin management might be ill-defined, both countries agree on a redistribution of the implicitly defined initial property rights by establishing "final" property rights associated with

$(x_1, x_2, q_1^x, q_2^x, t_1^x, t_2^x)$. Although the shadow prices can be thought of to represent Walrasian equilibrium prices as if established by water markets that are governed by the law of supply and demand, this interpretation assumes the countries refrain from exercising market power.

In general, the Second Welfare Theorem deals with non-transferable utility instead of transferable utility or money, as e.g., in Ambec and Sprumont [2002]. For the bilateral case, the transferable utility value of cooperation, denoted as $v(1,2)$, is equal to the right-hand side of (10). The Pareto frontier is described by $f_i((1-\delta_j) d_j + \delta_j s_j) = v(1,2) - (1-\delta_j) d_j - \delta_j s_j$. Then, direct application of program (2) yields

$$\begin{gathered}\max_{s_1,s_2} (s_1 - d_1)^{\alpha} (s_2 - d_2)^{1-\alpha}, \\ \text{s.t.} \\ s_1 \leq v(1,2) - (1-\delta_2) d_2 - \delta_2 s_2, \\ s_2 \leq v(1,2) - (1-\delta_1) d_1 - \delta_1 s_1.\end{gathered}$$

In this program, any reference to prices and marginal benefits has vanished from the model description. In this simple case, this crucial information can be retrieved from (10), but for less transparent applications a holistic approach in physical variables as in e.g. Roemer [1988] yields more information to policy makers.

Finally, Ambec and Sprumont [2002] show that there is a unique Pareto efficient utility vector consistent with both doctrines: In Figure 1 it is the intersection of the line $u_1 + u_2 = 1$ and the line through the disagreement point and the aspriration point. These utilities are related to the downstream incremental distribution that assigns utility d_1 to country 1 and $v(1,2) - d_1 > d_2$ to country 2. This solution coincides with the SSPE outcome (in utilities) in the alternating-offers model with disagreement point $(b_1(e_1), b_2(e_2))$ and bargaining weight $\alpha = 0$, or in terms of the primitives either $\delta_1 = 0$ or $\delta_2 = 1$. This indicates that it is highly unlikely that this axiomatic solution will prevail in an alternating-offers perspective. Note that even at $\alpha = 0$ the Second Welfare Theorem applies.

We conclude this section with a remark on stock variables.

Remark 2 *Optimal river basin management includes the optimal release and recharge of lakes and dams as reservoirs of water. Reservoirs would introduce stock variables to the model. A reservoir can also be seen as a production process that produces "future" water from "current" water and it can be represented as before by a transformation function* $F(q) \leq 0$. *As an illustration, reinterpret* z_1, *respectively,* z_2 *as water at present and in the future, say the wet and dry season. Then, the reservoir produces future water* q_2 *from present water* q_1 *as input and, under absence of evaporation, we have* $q_1 + q_2 \leq 0$ *and* $q_1 \leq 0$ *as before. Then* q_1 *is the end stock of period* 1 *and* q_2 *the initial stock at period* 2.

6 Concluding Remark

This contribution deals with the fundamental critique in Dinar *et al.* [1992] that alienates game theory from the language and concerns of policy makers and stakeholders: Easy to implement solutions that are based upon common notions of water pricing; and insight in the gains and losses for every stakeholder from policy reforms toward efficient river basin management. For bilateral negotiations modeled as alternating-offers, as pioneered in Rubinstein [1982], a powerful computational innovation in a physical representation of real-world issues is available that simultaneously allows for an interpretation of water prices as Walrasian equilibrium prices, which is a direct consequence of the Second Welfare Theorem. Unfortunately, the utility representation in game theory washes away any notion of water prices from the physical model, as illustrated in Section 5. The physical model can be regarded as the popular AGE framework that allows for further differentiation of water users, water related production and consumption goods by time (within the hydrological cycle), by space and uncertainty about extreme weather conditions (droughts and floodings), although the latter would assume the existence of contingent contracts and a discrete number of events.

Identifying water prices as Walrasian equilibrium prices should not be mistaken as naively suggesting to decentralize decisions through water markets. For that to be the best policy recommendation, it should be made clear first that all participants on these markets do not posses significant market power. For river basin management, also the role of governments is crucial even in case these do not trade themselves on the water market, because upstream countries might initiate development of plans for, say, expanding the area under irrigation affecting future downstream flows.

Although this paper identifies a promising route for further developing tools for water policy research, the bilateral case is just a first step that generalizes to multilateral river basin management negotiations requiring unanimity. This is the case in negotiations on interstate river compacts between US states, where each state has an option to appeal to the US Supreme Court, see Heintzelman [2006]. Future research should be directed to deal with coalition formation among countries or among different stakeholders within and across countries. Also the issue of regulating water markets when some parties have market power is a relevant issue. As in all areas of economic policy, lobbying is a matter of related interest. The bilateral model in Houba [2005] captures such negotiations in which the probability of success in lobbying means a higher probability to propose during the negotiations. This model reduces to the standard alternating-offers model after a suitable transformation and, therefore, the approach advocated in this contribution also applies.

In river basin management, meteorological and hydrological data are often considered as classified by the national authorities involved. Also, water users often tap illegally into the river or aquifer, or keep secret about the waste they dilute into the river. Incomplete and imperfect information is therefore an important research theme. The standard alternating-offers model has proven itself in these extensions, see e.g., Muthoo [1999]. This hints at good prospects for extending the equilibrium analysis to such extensions in the context of bilateral river basin management.

The issue of imperfect contracts should also be addressed in future work, but differently from stationary versus nonstationary contracts as in Houba and Wen [2006] and discussed in Remark 1. With imperfect contracts within each period, governments should also be given the right incentives to comply or prosecute offenders in order to preserve the treaty. Suppose incentive constraints can be expressed similar as the transformation functions F_i, then these can be added to program (12) as the "incentive functions" $I_i\left(x^1, x^2\right) \leq 0$ and $I_i\left(y^1, y^2\right) \leq 0$, $i = 1, 2$. Although convexity and feasibility of this modified program (12) become an issue, this program is still applicable to compute SSPE proposals. Then, the commodity balances still provide marginal values for water, which have an interpretation as water prices. Therefore, program (12) addresses the critique in Dinar *et al.* [1992] even for imperfect contracts.

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