Physical production functions

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Abstract

This paper addresses the relationship between physical and economic perspectives on production functions, in particular the inclusion of materials flows in the description of production. Production functions are studied on conceptual and operational levels by both economists and process technologists. Process technologists focus on physico-chemical characteristics of the production process, based on laws of thermodynamics. As such, these production functions are consistent. There is, however a difficulty in connecting them to models of economic structure, most notably partial and general equilibrium models, where substitution mechanisms are stressed. Economists define and use the concept of production in terms of values, in which it is not always clear whether mass balance and entropy constraints are adequately reflected. Furthermore, they regard inputs as homogeneous, and commonly assume one output. Attempts to reconcile physical and economic production functions have to assume a sort of dual form, covering the physical and value dimensions, and an implicit form, to address multi-output systems in a general, nonlinear way. The explicit linkage between these two dimensions and the embedding in equilibrium-type of models can guarantee consistency with accepted mechanisms in physics and economics.

1. Introduction

The special issue on *The contribution of Nicholas Georgescu-Roegen* (Daly, 1997) contains many interesting reflections from various sides of interest. This contribution concentrates on one aspect that is discussed by some of the contributors, namely Georgescu-Roegen's criticism on the neoclassical production function. We will start with presenting a number of statements from the various papers in the special issue.

First of all, Daly (1997, p. 263) explicitly states that the production function is expressed in – or at least based on – physical quantities: "The production function is a technical recipe with all terms in physical units, not value units." This sentence is not denied by Solow or Stiglitz, which one should of course not regard as their approval. Solow (1997, p. 268) at least asserts that Georgescu-Roegen's interpretation of production as a physical transformation "is, no doubt, one aspect of production." Other authors (*e.g.*, Opschoor, 1997, p. 282; Ayres, 1997, p. 286; Tisdell, 1997 p. 290) are more explicit in stating the material basis of production. This is then most often coupled to a discussion of the implications of this material basis: the danger of depletion, the inevitable production of waste, the need to develop technology when aiming at sustainable consumption or even sustainable growth. But no one, not even Daly himself, discusses the further implications of Daly's statement.

Secondly, Georgescu-Roegen's chapter on production functions (Georgescu-Roegen, 1971, p. 211*ff.*; see also van Gool, 1984 and van den Bergh, 1998) is of special interest, because the general formulation of his flow-fund model is one that lacks a normative context. With this we mean that inputs and outputs of the production process are listed without making reference to their value or potential scarcity. The elements of the three vectors of "agents" (input flows, output flows and funds) represent all material entities that enter or leave a process. As such, they naturally satisfy mass balancing conditions, because a description of an existing process can only be complete if these conditions are met. This is a fact that is almost never discussed. Instead, there have been a number of proposals to construct production functions under the restriction of mass balance (see, *e.g.*, Ayres & Kneese, 1969; Smith & Weber, 1989; Gross & Veendorp, 1990; van den Bergh & Nijkamp, 1994; Kandelaars & van den Bergh, 1996). All of these attempts suffer from being too specific. Most examples include a residual term to account for aggregated wastes, making no distinction between carbon dioxide and plutonium. Other examples pose themselves restrictions in the form of linear homogeneity by adopting an input-output framework.

In this paper, we summarize and expand on selected parts of (Heijungs, 1997) in so far as they relate to the topic of physical production functions. Before bringing in new elements, the reference situation needs to be described. The form of the production that Daly calls the Solow-Stiglitz form may be generalized as

$$Q = f(K, L, R) \tag{1}$$

where K denotes capital input, L denotes labour, R denotes natural resources, Q is produced output, and f is an as yet unspecified function of the input arguments. Solow-Stiglitz choose for the Cobb-Douglas form, but it is clear after an extensive forum that this choice is not appropriate in all circumstances. Therefore, a specification of the exact function will be left open at this point. The two main points that will be discussed are completeness and symmetry.

2. The first requirement: completeness

For production functions to be in mass balance, and therefore to be in accordance with the laws of nature, they should be free from normative elements. This means that inputs and outputs should be recorded, regardless their utility or value. Main classes that can be distinguished on this normative basis are:
- goods with private costs/proceeds (like steel and electricity);
- goods with social costs/proceeds (like air pollutants and forests);
- goods with no costs/proceeds (like oxygen in the air and water in the sea).

Inputs that are free goods, and in particular natural resources, should be recorded. But there are wider implications. Inputs which will not create environmental problems should also be recorded. Also in chemical and process technological literature, it is rare to find process descriptions that contain the oxygen used for combustion or the air or water used for cooling. Disregarding the oxygen will lead to a mass unbalance (Ayres, 1994). Land should also be in. But there is much more. Where are the normal input flows like steel, electricity and cleaning services?
Georgescu-Roegen (1971) discusses these aspects at length. He ends up with a production function (p. 236) which may be simplified as

$$Q = f(K, L, H, R, I, M, W) \tag{2}$$

Here, the first three categories (capital K, land L and labour H) are funds, while the other categories (natural resources R, material inputs from other processes I, maintenance requirements M, waste outputs W, and of course output Q) are flows.
Yet, also Georgescu-Roegen is not complete in his description. For instance, where are coproducts? Under Q together with the main product? And shouldn't we distinguish final waste (which is discharged to the environment) from intermediate waste (which is processed by an incinerator or recycling plant)? And, of crucial interest in determining external effects, where are the releases to the atmosphere, to watercourses, or to the soil, of which pollutants and radioactive substances are the most important ones, but which should also include less problematic ones, like water and concrete? It is clear that the criterion of completeness is not at all satisfied by the neo-classical production function, but that Georgescu-Roegen's extension is also incomplete.

3. The second requirement: symmetry

Production functions are highly asymmetric in appearance. This is already in the term production itself, which, in standard economics, refers to the act of making commodities, while consumption refers to the act of using them (Lipsey & Steiner, 1978, p. 6). If we remember the lessons of Ayres & Kneese (1969, p. 284) on the deceptive nature of the term consumption, we are forced to reconsider the idea of a production function.

The central idea in this reconceptualization is symmetry. Production can be regarded as the act of transforming commodities into different commodities. The difference may be one of quality (like producing cars from iron), one of form (like producing steel plate from steel), one of location (like producing a piano in Switzerland from one in Germany, normally denoted as transportation), or one of time (like producing beer now from beer last month, normally denoted as storage). It should be noted that all these types of differences in commodities have been recognized by neo-classical economics (Ginsburgh & Waelbroeck, 1981, p.3). But all the implications have not been recognized. All four examples listed above have goods as an input and goods as an output. The neo-classical production function is, however, unable to deal with input goods. Only Georgescu-Roegen's form contains a term for it (I).

The concept of symmetry has more implications. For instance, if we want to include the act of treating waste in the scheme of a production function, we should reserve a term for the input of (intermediate) waste. And why have input goods (I) a symbol that is different from output goods (Q), if we, after all, will have to match the inputs of one sector with the outputs of the same quality of another sector in an equilibrium-type of model? The same applies to the input of capital, which somehow needs to be produced somewhere. Samuelson (1967, p. 48) rightly writes: "A capital good differs from the primary factor inputs in that it is an input which is itself an output of the economy." The implication is that some production functions should look like $K = f(\cdot)$. Next, why is Q "privileged" in the sense of standing on the lefthandside of the equality sign? And why not define the act of providing labour as a process which converts input goods into labour and some waste goods (*cf.* Newman, 1962, p. 59)?

An important example of a symmetric production function is provided by von Neumann (1945-1946). A clear limitation of this model is that it is geared towards a monetary description of the flows, so that there is no completeness as defined in the previous section.

4. A complete and symmetric production function

The above considerations with respect to completeness and symmetry suggest a new type of production function. It is one in which flows of economic commodities (like steel and cars) and environmental commodities (like carbon dioxide and ores) enter on en equal footing, and in which there is no privileged commodity "on the other side of the equality sign". For that purpose we define the four essential elements of this function:

- goods (G), which are commodities that flow between two economic actors and that have a positive value (examples: steel, chairs, electricity, labour);
- waste (W), which are commodities that flow between two economic actors and that have a negative value (examples: sewage effluent, discarded chair, chemical waste);
- natural resources (R), which are commodities that flow from the environment to an economic actor (examples: iron ore, land, elephants, oxygen);
- emissions (E), which are commodities that flow from an economic actor to the environment (examples: carbon dioxide, phenol).

(Notice that the examples are not unique but represent typical situations. Living elephants may be traded between a hunter and a zoo, and then are a good; phenol may be traded between a chemical plant and a paint producer, and then is a good; carbon dioxide may be absorbed by agricultural production systems, and then is a natural resource.)

The form of the production function then in general involves several types of each of the four categories. These will therefore be indicated as vectors (**g**, **w**, **r** and **e**). The proposed form is

$$0 = \mathbf{f}(\mathbf{g}, \mathbf{w}, \mathbf{r}, \mathbf{e}) \quad (3)$$

A sign convention (*e.g.*, inputs are negative, outputs are positive; Georgescu-Roegen (1971, p. 215-216)) then suffices to indicate whether the process consumes or produces the commodity under concern. The fact that the production function **f** has not only vectors as arguments, but is also vector-valued itself (hence the null-vector **0** on the lefthandside) will be demonstrated later. The production function may be labelled as an implicit production function, because it does not single out one argument (say, g_1) on the "other side of the equality sign".
It should be observed that, although (3) is consistent with a materials balance condition, it not automatically satisfies such a condition. The lefthandside zero is in fact an arbitrary number, it could have been any other number without altering the argument. Hence, the materials balance condition must be imposed in addition to (3). It reads, for instance,

$$g_1 + g_2 + \cdots + w_1 + w_2 + \cdots + r_1 + r_2 + \cdots + e_1 + e_2 + \cdots = 0 \quad (4)$$

This form is, however, not true in general. This is so because nothing has been said with respect to the dimensions in which g_i, w_i, r_i and e_i are to be expressed. Among the infinite number of possibilities, we mention the most typical ones:

- mass in kg (or lbs, tonnes, ...), for instance for steel and carbon dioxide;
- volume in m^3 (or pints, barrels, ...), for instance for gasoline and water;
- time in s (or years, hours, ...), for instance for labour and music;
- dimensionless pieces, for instance for cars and elephants;
- energy in J (or kWh, BTU, ...), for instance for electricity and waste heat;
- time in s (or years, hours, ...), for instance for labour;
- surface in m^2 (or acres, ...), for instance for galvanizing and painting.

Almost all of these possibilities may be converted into mass, using the density of gasoline, the mass of an elephant, the mass-equivalent of electricity, etc. Denoting these "generalized densities" by $\rho(g_1)$, etc., (4) becomes

$$\rho(g_1)\cdot g_1 + \rho(g_2)\cdot g_2 + \cdots + \rho(w_1)\cdot w_1 + \rho(w_2)\cdot w_2 + \cdots$$
$$+ \rho(r_1)\cdot r_1 + \rho(r_2)\cdot r_2 + \cdots + \rho(e_1)\cdot e_1 + \rho(e_2)\cdot e_2 + \cdots = 0 \quad (5)$$

However, since a complete description of a process necessarily and automatically satisfies mass balance, there is no need to formulate explicit mass balance conditions. They can only provide a check on completeness.

5. Some examples

The production function that is defined according to (3) is quite general. In fact, it is so general that it may be hard to see the connection with the more well-known forms, like Cobb-Douglas, Solow-Stiglitz or Leontief. This section elucidates this connection.

a) Cobb-Douglas
The Cobb-Douglas production function can be constructed from (3) if we choose for $\mathbf{f}(\cdot)$ the one-dimensional form

$$f(g_K, g_L, g_Q) = (-g_K)^{\alpha} \times (-g_L)^{\beta} - g_Q \qquad (6)$$

This can be easily shown by equating this expression to 0, as suggested by (3):

$$g_Q = (-g_K)^{\alpha} \times (-g_L)^{\beta} \qquad (7)$$

where g_K and g_L are negative because they are an input.
Observe that we need to add the minus-sign in $(-g_K)$ and $(-g_L)$ to account for the fact that the usual Cobb-Douglas formulation is in disagreement with the sign convention assumed above.

b) Solow-Stiglitz
By extension, we can deduce from

$$f(g_K, g_L, g_Q, r) = (-g_K)^{\alpha} \times (-g_L)^{\beta} \times (-r)^{\gamma} - g_Q \qquad (8)$$

the Solow-Stiglitz variant

$$g_Q = (-g_K)^{\alpha} \times (-g_L)^{\beta} \times (-r)^{\gamma} \qquad (9)$$

c) Leontief
The function

$$\mathbf{f}(g_1, g_2, \dots, g_N, g_Q) = \begin{pmatrix} (-g_1) - \gamma_1 g_Q \\ (-g_2) - \gamma_2 g_Q \\ \dots \\ (-g_N) - \gamma_N g_Q \end{pmatrix} \qquad (10)$$

directly leads to the Leontief input-output-type of production function, with fixed technical coefficients γ_1, γ_2, ..., γ_N that are defined as

$$\gamma_i = \frac{(-\bar{g}_i)}{\bar{g}_Q} \qquad (i = 1, 2, \dots, N) \qquad (11)$$

for one empirical situation that is characterized by the values $\bar{g}_1, \bar{g}_2, \dots, \bar{g}_N, \bar{g}_Q$.

This form demonstrates the usefulness of specifying the production function as a vector-valued function $\mathbf{f}(\cdot)$ instead of a scalar-valued $f(\cdot)$. The scalar form would here lead to complications, related to the absence of substitution; see Miller & Blair (1985, p. 12), who undertake an attempt to cast the Leontief production function in the traditional form.

d) Ayres-Kneese

The materials balance production function that is the basis of the model of Ayres & Kneese (1969) and other environmentally extended input-output model, is one of the Leontief-type extended with environmental commodities (natural resources and emissions):

$$f(g_1, \dots, w_1, \dots, r_1, \dots, e_1, \dots, g_Q) = \begin{pmatrix} (-g_1) - \gamma_1 g_Q \\ \dots \\ (w_1) - \omega_1 g_Q \\ \dots \\ (-r_1) - \rho_1 g_Q \\ \dots \\ (e_1) - \varepsilon_1 g_Q \\ \dots \end{pmatrix} \tag{12}$$

The explicit materials balance condition is then

$$(-\gamma_1) + \dots + (\omega_1) + \dots + (-\rho_1) + \dots + (\varepsilon_1) + \dots + 1 = 0 \tag{13}$$

assuming that the technical coefficients are defined on a mass basis, *i.e.* ω_1 denotes the mass of waste of type 1 that is produced for one mass unit of output Q.

e) Perrings

Another input-output approach is given by Perrings (1987). Main modification on the scheme of Ayres & Kneese is the incorporation of time, as a factor of delay in production and consumption, and to account for technological change.

f) Victor

Victor (1972) makes a strong plea for implementing the make-use framework instead of the input-output framework. The essential modification is that the assumption of homogeneous production of industries is dropped, so that industries that make more than one type of good may be better described.
But the implications are wider, and are conceptually in agreement with the idea of symmetric production functions. In the input-output type of models, each sector is uniquely identified with "its" product Q. This enables the definition of technical coefficients on the basis of that unique Q; see (10). In the make-use type of models, there is not one output, but there may be many of them. In the traditional make-use framework, this provides no special complications, since Q may still be defined as the sum of proceeds of the different outputs of the sector. In a physical accounting scheme, the concept of proceeds is undefined. Furthermore, there is no other unambiguous measure for aggregating different types of outputs. Hence there is no special Q to be identified for defining technical coefficients. Yet, the concept of technical coefficients still makes sense. The active time of the process or sector is a very sensible candidate for defining the technical coefficients in a make-use model (*cf.* Georgescu-Roegen, 1971, p. 238 *ff.*). Thus, we may define

$$\gamma_i = \frac{g_i}{t} \qquad (\forall\ i) \tag{14}$$

for the inputs (g_i and γ_i both < 0) and for the outputs (g_i and γ_i both > 0). The same may be done for the wastes, the natural resources and the emissions.

6. A more sophisticated example

The list of 6 examples is illustrative to the extent that it shows how existing production functions fit into the new framework. But they do, of course, not demonstrate the power of that very framework. We will therefore introduce another example, which is – although still remote from real life – more realistic, and which hopefully gives guidance to applications to real examples.

Many processes (facilities, plants, installations, machines) possess some variable aspects on the input side as well as on the output side. Let us study the example of a machine that rolls steel plate from steel. To do so, it needs electricity. It will also produce an amount of waste steel scrap. The situation is summarized in Figure 1.

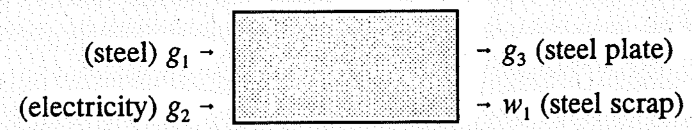

Figure 1
Representation of the inputs and outputs of a hypothetical process, labelled as "rolling of steel".

There is a number of *a priori* constraints, for instance $g_1 \leq 0$, $g_2 \leq 0$, $g_3 \geq 0$, $w_1 \geq 0$. Furthermore, the mass balance condition is

$$-g_1 = g_3 + w_1 \tag{15}$$

provided these quantities have been stated in the same units (and neglecting the mass-equivalent of electricity in g_2). We may also assume that there is some flexibility between the four flows, for instance, supplying more electricity will yield more steel plate and less steel scrap from the same amount of input steel. Or more steel plate and the same amount of scrap steel from the same amount of input steel. Or the same amount of steel plate and less scrap steel from less input steel. And there are many more combinations. We might, for instance, postulate that

$$\frac{g_3}{-g_1} = \frac{2}{\pi}\arctan\left(-g_2\right) \tag{16}$$

This results in a $g_3/-g_1$ ratio of 0 when $-g_2$ is zero (no electricity; all input steel becomes scrap steel), and a $g_3/-g_1$ ratio of 1 when $-g_2$ approaches infinity (lots of electricity; all input steel becomes rolled steel). The production function thus becomes

$$f(g_1, g_2, g_3, w_1) = \begin{pmatrix} g_1 + g_3 + w_1 \\ \dfrac{g_3}{g_1} + \dfrac{2}{\pi}\arctan\left(-g_2\right) \end{pmatrix} \tag{17}$$

This form clearly shows the central aspects of completeness and symmetry: all types of flows (input goods, output goods, wastes) fit into the production function. A more realistic process description would show similar features for multiple emissions, wastes, labour, etc.

7. Complete but still underdetermined

It is evident that the purely physical production function is underdetermined in itself. Given a certain external demand for steel plate, there is an infinite number of possibilities for running the process. Not all these production possibilities are equally desirable. Some will be illegal, for instance because they are in disagreement with emission regulations. But the most interesting aspect is to be found in economics. A huge input of electricity to save a small amount of input steel will from an economic point of view highly unattractive. It is likely that a plant manager will seek to maximize profit by choosing the working point of the process in such a way that proceeds minus costs of to all priced inputs and outputs assumes a maximum.

If we denote the prices per unit of g_1, g_2, g_3 and w_1 by $p(g_1)$, $p(g_2)$, $p(g_3)$ and $p(w_1)$ respectively, the expression for profit is

$$\pi = p(g_1)g_1 + p(g_2)g_2 + p(g_3)g_3 + p(w_1)w_1 \qquad (18)$$

Observe that the first, second and fourth term will be negative: the first and second because the g-values are negative (they are inputs for the process), and the fourth because the p-value is negative (it is waste to be processed). Only the second term is positive. The objective function associated with maximizing profit is then

$$\text{maximize} \qquad \pi \qquad (19)$$

The profit π is of course a function of g_1, g_2, g_3, w_1, $p(g_1)$, $p(g_2)$, $p(g_3)$ and $p(w_1)$.

8. Incorporation in an equilibrium model

It depends on the working of markets and the assumption in the model which quantities that determine profit are endogenous, and which are to be determined by the model. For instance, a general equilibrium model allows all quantities (three g-values, one w-value, four p-values) to vary, while a partial equilibrium model with endogenously determined external demand and prices only allows g_1, g_2 and w_1 to vary.

Let us first study this latter extreme situation. Given is a production function $\mathbf{f}(\cdot)$ that specifies two equations; see (17). Furthermore, we have a profit function (18) which is to be maximized. Finally, there is a set of five endogenous variables: g_3, $p(g_1)$, $p(g_2)$, $p(g_3)$ and $p(w_1)$. Summarizing: we have three equations in three unknowns: g_1, g_2 and w_1.

The other extreme is the general equilibrium model. If a treatise on the two-sector model of it already needs a hundred-and-odd pages (Dinwiddy & Teal, 1988), one can not expect that this paper contains a rigid formalism for including the complete and symmetric production functions in such a model. Nevertheless, some aspects will be highlighted at this place. All sectors (processes) are to be specified by means of vector-valued production functions. Sectors cover more than traditional industry, but include households/employees (that convert consumer goods into waste and labour) and waste processing (that convert waste into pollutants and recycled material) as well. Furthermore, all sectors need an objective function: for most sectors profit maximization, for consumers utility maximization. Finally there need to be a number of market clearing conditions: for each commercial commodity as well as for money

9. Conclusion

The general picture that emerges is as follows:

- There are hard constraints, which are captured in a physical production function. Such a production function simply represents the relationship between the different inputs and outputs of a process. Only possibilities that are physically and technically feasible are incorporated. As such, the physical production function necessarily satisfies materials balance, energy balance, entropy constraints, etc.
- There are all types of soft constraints. Most prominent are the legal ones, but there may be other types, originating from religious, ethical or political convictions ("no pork meat", "no child labour", "no communist steel"). These are called soft, because they may be violated, in contrast to the hard physical constraints.
- There are economic motives, which are captured in an objective function. These may be simply profit maximization, but a somewhat wider view, like welfare maximization, including externalities, is also possible.

The working point of a process is determined by simultaneous application of hard and soft constraints and the optimizing principles. Without a more exact specification concerning functional relationships or convexity, nothing can be said with respect to the existence of equilibria, their uniqueness, and their stability.

Acknowledgement

This paper is written in the context of a project funded granted by the Netherlands Organization for Scientific Research in the context of the Priority Programme on Sustainability and Environmental Quality. I am grateful to Jeroen van den Bergh, Department of Spatial Economics, Vrije Universiteit, for comments on an earlier version.

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