

2 Reinout Heijungs

3 Probability, Statistics
4 and Life Cycle Assessment

5 Guidance for Dealing with Uncertainty
6 and Sensitivity

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1 **Preface**

2 This book is devoted to the topic of what is often called ‘uncertainty in LCA’
3 including the ramifications into ‘sensitivity in LCA’. The importance of these topics
4 was stressed already in the early 90s, when the *Society of Environmental Toxicology*
5 *and Chemistry* (SETAC) and the *International Organization for Standardization*
6 (ISO) started to develop guidelines and standards for life cycle assessment (LCA).
7 However, while the resulting texts mention that an uncertainty and sensitivity analysis
8 needs to be carried out, they give few guidelines for how to actually do so.

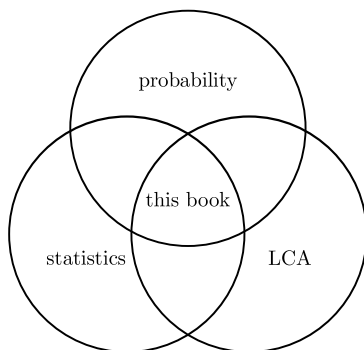
9 The results of this lack of guidance can be subdivided into two classes: positive
10 and negative. On the negative side, we see that still many LCA case studies report
11 results without or with only a very limited analysis of uncertainty and sensitivity. The
12 positive side is that in the past 25 years a large number of articles, Ph.D. theses and
13 reports have been published on the inclusion of uncertainty and sensitivity analysis
14 in life cycle assessment. Sadly, these texts provide a fragmented picture. They all
15 use their own terminology and notation, and they focus on specific elements. What
16 is lacking is an embracing, coherent and critical treatment of the topics. This book
17 aims to fill this gap.

18 At the same time, some of the publications that have appeared in the past 25 years
19 are more valuable than others. This can be for practical reasons (e.g., issues related
20 to computer time), but there are also more fundamental reasons. Uncertainty and
21 sensitivity are topics that originate from considerations of probability and statistics.
22 Any approach for dealing with uncertainty and sensitivity should therefore build
23 upon the foundations of probability theory and statistics. That is much more than
24 just the adoption of terms, symbols and equations, for standard deviations and similar
25 concepts. It requires a complete reconsideration of the principles of LCA, based on
26 probabilistic and inferential thinking. It is the author’s experience that many LCA
27 practitioners, long ago, took a course in these subjects, and now have forgotten most
28 of it. That is unfortunate, because probability and statistics are fields that do more than
29 just delivering a formula for a standard deviation. They provide a way of thinking,
30 in terms of random variables, samples and estimation.

31 The lack of a solid background in probability and statistics is evident from several
32 of the published documents. For instance, they frequently use terms like ‘parameter’,

33 'bias', 'error', 'confidence interval' and 'true value' in an incorrect way. The result
 34 is that the validity of some of these approaches can be doubted. But it requires a long
 35 path before we get to that point.

36 This book can be truly seen as discussing the three fields of the title: probability,
 37 statistics and LCA, but only to the extent that they are relevant in relation to each
 38 other. Graphically:



40 Or symbolically, as a pseudo-equation:

42
$$\text{this book} = \text{probability} \cap \text{statistics} \cap \text{LCA}$$

43 Readers interested in the general theories of probability, statistics or LCA should
 44 consult other books. Of course, in writing this book, the author has consulted such
 45 books, and reference will be provided.

46 Part I of the book is a primer. Indeed, one aim is to present in a concise, coherent
 47 and reader-friendly way the basic ingredients of probability theory and statistics,
 48 but not for the purpose of offering a basic textbook on probability or statistics.
 49 The purpose is to single out those topics that are relevant to the incorporation of
 50 uncertainty and sensitivity in LCA. Typical undergraduate textbooks on statistics are
 51 either highly mathematical or very thick, and they contain several aspects that are
 52 not directly relevant to our topic. For instance, moment generating functions and
 53 exponential smoothing are introduced in many such books, but we will not need
 54 them, and therefore skip them.

55 Part II is a critique. As a matter of fact, the author believes that a critical analysis
 56 is badly needed. The last decade or so, developments in uncertainty analysis of LCA
 57 have been going around in circles, author *B* building on what author *A* writes, and
 58 the other way around. As a result, we now see many strange things in the published
 59 articles: notation, terminology and concepts are blurred. We give just one example
 60 here: several authors in the field of LCA think that the squared geometric standard

61 deviation is the standard (or best, or only) way to address uncertainty. A fresh treat-
 62 ment, based on established foundations, is wanted, and that is exactly what this book
 63 offers.

64 Part III accumulates all findings into a guidance document: guidance for including
 65 the lessons from probability and statistics in LCA. It is, in contrast to the middle part,
 66 a constructive text.

67 The author has been teaching probability, statistics and LCA, and his emphasis in
 68 the field of LCA has always been the mathematical side. Few other people combine
 69 these three angles, and he feels it as a unique opportunity to edify a solid foundation
 70 for the treatment of uncertainty and sensitivity in LCA. His first steps in the study of
 71 uncertainty and sensitivity analysis (then phrased as reliability and marginal analysis)
 72 were published in 1994 (Heijungs (1994)). Of course, he has not been alone in
 73 conceiving this work. In particular, his (former) colleagues; Ph.D. students; and
 74 students Angelica Mendoza Beltrán, Arjan de Koning, Carlos Felipe Blanco, Evelyne
 75 Groen, Jeroen Guinée, Patrik Henriksson, Sietske Lensen, Stefano Cucurachi, Tristan
 76 Senga Kiessé and Valentina Prado have contributed substantially to the development
 77 of the ideas described in this book.

78 The ideas were—in part—previously published in papers, mainly Blanco et al.
 79 (2020a), Brandão et al. (2022), Cucurachi et al. (2016, 2022), Groen et al. (2014,
 80 2016, 2017), Heijungs (1994, 1996, 2010, 2017, 2017a, 2017b, 2020a, 2021a, 2022b),
 81 Heijungs and Kleijn (2001), Heijungs and Suh (2002), Heijungs and Frischknecht
 82 (2005), Heijungs et al. (2005, 2016, 2017, 2019), Heijungs and Tan (2010), Heijungs
 83 and Lenzen (2014), Heijungs and Dekker (2022), Henriksson et al. (2015a, 2015b),
 84 Mendoza Beltrán et al. (2016, 2018a, 2018d), Senga Kiessé et al. (2022) and Wolf
 85 et al. (2017). But there are also several parts that were not published before. More-
 86 over, these previous works have been reassessed and reworked into a consistent and
 87 coherent set-up.

88 All computations were performed in *Excel*, without any add-ins except from those
 89 that are available by default (*Analysis ToolPak* and *Solver Add-in*). This is important
 90 information, because it shows that all methods that are illustrated can be used without
 91 any specialized software. This also applies to the figures in this book. The large
 92 majority of graphs was made with *Excel*; only a few diagrams were made with
 93 *TikZ*.¹ The text itself was typeset in L^AT_EX, using *MikTeX*.²

94 The book itself has its beginnings in a seminar, *Probability, Statistics and the*
 95 *Environment*, that the author gave on May 9, 2017 at *Technische Universität Wien*
 96 (thanks go to Helmut Rechberger and Oliver Cencic for the invitation). The argument
 97 was later developed in a manuscript *Probability, Statistics and Industrial Ecology*
 98 that was sent to the *Journal of Industrial Ecology*. But it was found unsuitable for a
 99 journal due to its length and mainly pedagogical set-up (thanks go to Richard Wood
 100 for recommending this). So in the end the article was reworked into the present book,
 101 and indeed, the reader will agree that it was a bit too much for an article, even while
 102 the breadth went down from *Environment* to *Industrial Ecology* and then even further

¹ See <https://www.ctan.org/tex-archive/graphics/pgf/>.

² See <https://miktex.org/>.

103 down to LCA. The lock-down of the university from March 2020 onward gave the
104 author the possibility to recluse and concentrate on writing it.

105 The book's title is a hint to von Mises' *Probability, Statistics and Truth* (Von
106 Mises, 1939). The book, originally written in 1928 in German as *Wahrscheinlichkeit,*
107 *Statistik und Wahrheit*, is a classic semi-popular exposition of the basic ideas of
108 probability and statistics. Its title has inspired many authors for follow-ups: *Proba-*
109 *bility, Statistics and Mathematics*; *Probability, Statistics and Analysis*; *Probability,*
110 *Statistics and Econometrics*; *Probability, Statistics and Estimation*, so inserting *Life*
111 *Cycle Assessment* seemed to be appropriate. Essential to von Mises' argument is that
112 probability and statistics are to be understood in the frequentist's sense: repeated
113 observations from a phenomenon that is inherently variable. This book will continue
114 along that line: LCA relies on data that display an inherent variability. As a result,
115 probability theory is the only correct way to do LCA. There is no 'true' LCA result,
116 and the methods developed in probability and statistics are essential reading for
117 anyone involved in the theory and practice of LCA.

118 It seems appropriate to close this Preface with the remark in von Mises' Preface
119 of 1939, which has unfortunately been removed in the second English edition that
120 is available today (Von Mises, 1981). *Clear thinking, the scrupulous testing of all*
121 *propositions by comparison with objective phenomena and the repudiation of all*
122 *empty phrases may be rare qualities at the present time, but there are still some*
123 *philosophers who endeavor to maintain these principles, and with such, I think, we*
124 *shall easily find a common language.* Clear thinking, repudiating empty phrases and
125 finding a common language are indeed what this book seeks to achieve.

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Reinout Heijungs

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1 **Contents**

2	1 Introduction	1
3	1.1 Life Cycle Assessment	1
4	1.2 Uncertainty in LCA	2
5	1.3 Sensitivity in LCA	3
6	1.4 Uncertainty and Sensitivity in LCA: Past, Present	
7	and Future	4
8	1.4.1 Early Recommended Practice	5
9	1.4.2 Developments	8
10	1.4.3 Application by Case Studies	10
11	1.4.4 Review Papers	11
12	1.4.5 Current Recommended Practice	13
13	1.4.6 Current Practice	18
14	1.4.7 The Future	20
15	1.4.8 The Role of Software	22
16	1.4.9 Fixing the Errors	22
17	1.4.10 This Book	23
18	1.5 Terminology and Notation	27
19	1.6 Outline	28
20	References	28
21	Part I A Primer	
22	2 Probability 1: Basics	45
23	2.1 Probability and Random Variables	46
24	2.1.1 Probability Defined	46
25	2.1.2 Random Variables	47
26	2.1.3 Infinite and Finite Populations	51

27 2.2 Probability Distributions and Probability Distribution

28 Functions 53

29 2.2.1 Probability Distributions 53

30 2.2.2 Discrete Probability Distribution Functions 54

31 2.2.3 Continuous Probability Distribution Functions 57

32 2.3 Cumulative Distribution Functions 60

33 2.3.1 Discrete Cumulative Distribution Functions 60

34 2.3.2 Continuous Cumulative Distribution Functions 61

35 2.3.3 Inverse Cumulative Distribution Functions 63

36 2.3.4 Complementary Cumulative Distribution

37 Functions 64

38 2.4 Probability Distributions, Dimensions and Units 64

39 2.4.1 A Brief Digression on Dimensions and Units 64

40 2.4.2 Dimensions and Units for Probability Theory 66

41 2.5 Moments and Other Functions of a Distribution 67

42 2.5.1 Expected Value 67

43 2.5.2 Variance 69

44 2.5.3 Raw, Central, Higher and Standardized

45 Moments 70

46 2.5.4 Other Functions of a Distribution 72

47 2.5.5 Chebyshev’s Inequality 76

48 2.6 Model Distributions 77

49 2.6.1 The Binomial and Bernoulli Distributions 78

50 2.6.2 The Multinomial Distribution 80

51 2.6.3 The Normal Distribution 81

52 2.6.4 The Standard Normal Distribution,

53 the Standardization Procedure

54 and the Interpercentile Interval 84

55 2.6.5 The Lognormal Distribution 88

56 2.6.6 The Two Triangular Distributions 90

57 2.6.7 The Beta Distribution 92

58 2.6.8 The Gamma Distribution 93

59 2.6.9 The PERT Distribution 94

60 2.6.10 The t -distribution 96

61 2.6.11 The χ^2 -distribution 97

62 2.6.12 The F -distribution 98

63 2.6.13 The Two Uniform Distributions 99

64 2.6.14 Summary of Main Continuous Probability

65 Distributions 100

66 2.6.15 A Few Relationships Between Probability

67 Distributions 100

68 2.6.16 Other Distributions 102

69 2.7 More Than One Random Variable 102

70 2.7.1 Probabilities Relating to Multiple Events 102

71 2.7.2 Combining Random Variables: The Sum 109

Author Proof

72	2.7.3	Dependence and Covariance	112
73	2.7.4	Comparing Two Random Variables	114
74	2.7.5	The Correlation Between Two Random	
75		Variables	120
76	2.7.6	Simple Regression	121
77	2.7.7	Multiple Regression	124
78	2.7.8	Bivariate Probability Distributions	
79		and Copulas	125
80	2.7.9	Multivariate Probability Distributions	127
81	2.7.10	Sums of Several Random Variables	128
82	2.7.11	Functions of Probability Distributions	129
83	Recap		133
84	References		133
85	3	Probability 2: Alternatives	137
86	3.1	Alternative Interpretations of Probability	138
87	3.1.1	Bayesianism	139
88	3.1.2	The Classical View	143
89	3.1.3	Other Extreme Points of View	144
90	3.1.4	Imprecise Probability	145
91	3.1.5	Second-Order Uncertainty	145
92	3.1.6	Where is the Stochastic Process?	146
93	3.2	Alternatives to Probability	147
94	3.2.1	Possibility Theory and Fuzzy Numbers	147
95	3.2.2	Dempster–Shafer Theory	150
96	3.2.3	Rough Sets	150
97	3.2.4	Grey Relational Analysis	151
98	3.2.5	Set Pair Analysis	151
99	3.2.6	Interval Arithmetic	151
100	3.2.7	Information Theory	152
101	3.3	Combination of Paradigms	154
102	Recap		155
103	References		155
104	4	Statistics 1: Descriptive	161
105	4.1	Data	162
106	4.1.1	The Data Matrix	162
107	4.1.2	Some Issues of Notation	164
108	4.1.3	Data and Summaries	164
109	4.1.4	Data and Samples	165
110	4.1.5	Statistics and Their Symbols	166
111	4.1.6	Order Statistics and Ranks	166
112	4.2	Measures of Centrality	169
113	4.2.1	The Mean	170
114	4.2.2	Other Measures of Central Tendency	172

Author Proof

115 4.3 Measures of Dispersion 175

116 4.3.1 The Absolute Deviation 176

117 4.3.2 The Variance and the Standard Deviation 176

118 4.3.3 Other Measures of Dispersion 178

119 4.4 Other Univariate Descriptive Statistics and Visualizations 183

120 4.4.1 The Five Quartiles and the Box Plot 183

121 4.4.2 The Empirical Distribution Function 185

122 4.4.3 Kernel Density Estimation 186

123 4.4.4 Histogram, Frequency Polygon and Ogive 186

124 4.4.5 The Q-Q Plot and P-P Plot 190

125 4.4.6 Standardized Scores and Other

126 Transformations 192

127 4.4.7 Measures of Shape 194

128 4.5 Univariate Statistics for Categorical Data 196

129 4.5.1 Frequencies, Proportions and Odds 196

130 4.5.2 Bar Charts and Pie Charts 197

131 4.5.3 The Case of $k = 2$ Levels 199

132 4.5.4 Bar Charts and Pie Charts for Numerical Data 199

133 4.6 Bivariate and Multivariate Descriptive Statistics 199

134 4.6.1 Dependent and Independent Variables 200

135 4.6.2 Two Numerical Variables 202

136 4.6.3 One Numerical and One Categorical Variable 213

137 4.6.4 Two Categorical Variables 217

138 4.6.5 Multivariate Analysis 221

139 Recap 226

140 References 227

141 **5 Statistics 2: Inferential** 233

142 5.1 The Estimation Problem 234

143 5.1.1 Estimators, Estimates and Estimands 234

144 5.1.2 Estimation Principles 237

145 5.1.3 The Distribution of Estimates 240

146 5.2 The Central Limit Theorem 240

147 5.2.1 An Introduction to the Central Limit Theorem 241

148 5.2.2 Statement of the Central Limit Theorem 241

149 5.3 The Sampling Distribution 244

150 5.3.1 The Sampling Distribution of the Mean 244

151 5.3.2 The Sampling Distribution of the Mean

152 in Case σ is Unknown 246

153 5.3.3 The Sampling Distribution of the Standard

154 Deviation 247

155 5.3.4 The Sampling Distribution of the Proportion 248

156 5.3.5 Other Sampling Distributions 250

157 5.4 Point Estimates and the Standard Error of the Estimate 252

158 5.4.1 Point Estimates 252

Author Proof

159	5.4.2	The Standard Error of the Estimate	254
160	5.4.3	The Standard Error of the Mean	254
161	5.4.4	The Standard Error of Other Statistics	255
162	5.4.5	The Relative Standard Error	255
163	5.5	Confidence Intervals	256
164	5.5.1	The Idea of a Confidence Interval	256
165	5.5.2	Interpretation of a Confidence Interval	258
166	5.5.3	The Confidence Interval of the Mean	259
167	5.5.4	The Confidence Interval of the Mean in Case	
168		σ is Unknown	259
169	5.5.5	The Confidence Interval of the Variance	261
170	5.5.6	The Confidence Interval of the Standard	
171		Deviation	262
172	5.5.7	The Confidence Interval of the Proportion	262
173	5.5.8	Confidence Intervals of the Correlation	
174		Coefficient	262
175	5.5.9	Confidence Intervals of the Regression	
176		Coefficient	263
177	5.5.10	Confidence Intervals of Other Statistics	264
178	5.6	Estimating Probability Distribution Functions	265
179	5.6.1	Selecting an Appropriate Probability	
180		Distribution Functions	265
181	5.6.2	Tails or Central Part?	267
182	5.6.3	Estimating Parameters of a Probability	
183		Distribution Functions	268
184	5.7	Hypothesis Tests	271
185	5.7.1	Hypothesis Tests in Science	272
186	5.7.2	Null and Alternative Hypothesis, One-Sided	
187		and Two-Sided Hypothesis	273
188	5.7.3	The Test Statistic	275
189	5.7.4	Tails and Sides	275
190	5.7.5	Significance Level, Critical Region, Critical	
191		Value, and Null Distribution	276
192	5.7.6	The Realized Value of the Standardized Test	
193		Statistic	279
194	5.7.7	A Worked Example	279
195	5.7.8	The Statistical Decision	280
196	5.7.9	The p -Value	282
197	5.7.10	The Significance Level α and Type I Errors	283
198	5.7.11	Type II Errors and Power	284
199	5.7.12	Asterisks (*, **, ***) and Other Conventions	
200		to Denote Significance	286
201	5.7.13	Further Requirements and Assumptions	287
202	5.7.14	Non-parametric Tests	288

Author Proof

203 5.7.15 Bootstrap Tests 290

204 5.7.16 A Catalogue of Null Hypothesis Significance

205 Tests 291

206 5.8 Univariate Hypothesis Tests 292

207 5.8.1 Hypothesis Tests for the Mean (μ) 292

208 5.8.2 Hypothesis Tests for the Median (v) 295

209 5.8.3 Hypothesis Tests for the Standard Deviation

210 (σ) and the Variance (σ^2) 295

211 5.8.4 Hypothesis Tests for the Proportion (π) 297

212 5.8.5 Goodness-of-Fit Tests 302

213 5.8.6 Other Hypothesis Tests 303

214 5.9 Bivariate Hypothesis Tests 303

215 5.9.1 Independent Samples Tests 308

216 5.9.2 Dependent Samples Tests 309

217 5.9.3 Comparing Two Distributions 310

218 5.9.4 The Association Between Two Numerical

219 Variables 315

220 5.9.5 The Association Between Two Categorical

221 Variables 316

222 5.10 Multivariate Hypotheses Tests 317

223 5.10.1 The Problem of Multiple Testing 318

224 5.10.2 Comparing Several Numerical Variables 321

225 5.10.3 Hypothesis Tests for Multiple Regression

226 Analysis 323

227 5.10.4 Multi-way Analyses 324

228 5.10.5 Comparing Several Categorical Variables 325

229 5.10.6 Other Multivariate Methods 326

230 Recap 326

231 References 333

232 **6 LCA** 334

233 6.1 The Mathematical Model for LCA 335

234 6.1.1 The Mathematical Structure of a Model 339

235 6.1.2 LCA Without Maths? 340

236 6.1.3 Parameters, Arguments, Inputs and Outputs 344

237 6.1.4 Black Box or Not? 346

238 6.1.5 Goal and Scope Definition 347

239 6.1.6 Inventory Analysis 349

240 6.1.7 Impact Assessment 355

241 6.1.8 Interpretation 363

242 6.2 More Refined LCA 363

243 6.2.1 Regional/spatial Differentiation 366

244 6.2.2 Temporal/dynamic Differentiation 367

245 6.3 Alternative Methods to Calculate LCA Results 367

246 6.3.1 Matrix-Based LCI Versus Other Approaches 367

Author Proof

247	6.3.2	Matrix-Inversion-Based LCI Versus Other	
248		Matrix-Based Approaches	370
249	6.3.3	Process-Based LCI Versus IO-Based LCI	374
250	6.3.4	Matrix-Based LCIA	375
251	6.4	Data in LCA	376
252	6.4.1	Types of Data	376
253	6.4.2	Data for LCI	377
254	6.4.3	Characterization Data	386
255	6.4.4	Linear and Non-linear LCI and LCIA	388
256	6.4.5	Data Defined by Scenarios	390
257	6.5	Derivatives of the Mathematical Model	394
258	6.5.1	Derivatives at the Inventory Level	394
259	6.5.2	Derivatives at the Characterization Level	395
260	6.5.3	Derivatives at the Normalization Level	396
261	6.5.4	Derivatives at the Weighting Level	396
262	6.5.5	Higher Order Derivatives	397
263	6.5.6	Numerical Derivatives	398
264	6.6	Dealing with Uncertainty and Sensitivity in LCA	400
265	6.6.1	What Are the Issues?	400
266	6.6.2	Frameworks	402
267	6.6.3	Estimation of Probability Distributions	403
268	6.6.4	Software and Databases for Including	
269		Uncertainty and Sensitivity in LCA	404
270	6.6.5	Representing Statistical Information in LCA	
271		Databases	406
272	6.6.6	From Format to Calculations	413
273	6.6.7	Probability Distributions in LCA	413
274	6.6.8	From Statistics to Decisions	422
275	6.7	Uncertainty and Sensitivity in IOA and Related Models	423
276	6.7.1	The Case of IOA	424
277	6.7.2	The Case of EIOA and IO-Based LCA	424
278	6.7.3	Is IO-Based LCA More Precise Than	
279		Process-Based LCA?	425
280	6.8	Visualization in LCA	426
281	6.8.1	Graphs for Visualizing Uncertainty	427
282	6.8.2	Graphs for Visualizing Sensitivity	428
283	6.8.3	Miscellaneous Graphs	429
284	Recap		429
285	References		430
286	7	Error and Quality	459
287	7.1	Measurements and Their Errors	461
288	7.1.1	Terminology of Measurements	462
289	7.1.2	Terminology of Errors	462
290	7.1.3	Error and Variability	464

Author Proof

291 7.1.4 Measurement Errors Across the Sciences 464

292 7.1.5 Numerical and Categorical Measurements 465

293 7.1.6 What is a Measurement? 466

294 7.1.7 Models and Their Errors 467

295 7.1.8 Alternative Categorizations of Errors 468

296 7.1.9 Uncertainty > Error 470

297 7.2 Quantification of Error 471

298 7.2.1 Quantification of Random Error 472

299 7.2.2 Quantification of Systematic Error 477

300 7.2.3 Quantification of Total Error 478

301 7.3 Error Propagation 479

302 7.3.1 Notation 479

303 7.3.2 The Distribution of the Output Error 480

304 7.3.3 Methods for Error Propagation 482

305 7.4 Gaussian Error Propagation 483

306 7.4.1 First-Order Gaussian Error Propagation 483

307 7.4.2 Proof of the Gaussian Error Propagation

308 Formula 484

309 7.4.3 Second-Order Correction 486

310 7.4.4 Larger Errors 487

311 7.4.5 Correlated Errors 487

312 7.4.6 Estimating the Partial Derivatives 487

313 7.5 Sampling-Based Error Propagation 489

314 7.5.1 Monte Carlo for Error Propagation 490

315 7.5.2 More Refined Sampling Methods 492

316 7.5.3 The Number of Replications for Sampling

317 Approaches 493

318 7.6 Some Other Techniques for Error Propagation 498

319 7.6.1 Naive Numerical Error Propagation 499

320 7.6.2 Error Propagation Using Fuzzy Numbers 500

321 7.6.3 Error Propagation Using Interval Arithmetic 501

322 7.6.4 Error Propagation Using Scenarios 502

323 7.6.5 Miscellaneous and Combined Methods 503

324 7.7 Comparisons of Different Propagation Methods 505

325 7.7.1 Comparison of Conclusions 506

326 7.7.2 Comparison of Requirements 507

327 7.7.3 Combining the Strengths 507

328 7.8 Suspicious and Missing Measurements 508

329 7.8.1 Detecting Suspicious Measurements 508

330 7.8.2 Managing Suspicious Measurements 510

331 7.8.3 Missing Observations and Data Gaps 510

332 7.8.4 Data Estimation 515

333 7.8.5 Data Reconciliation 518

Author Proof

334 7.8.6 Estimation of Systematic Error 519

335 7.8.7 Outliers, Once More 520

336 7.8.8 The Role of Suspicious and Missing

337 Measurements in LCA 520

338 7.9 Truncation, Aggregation and Approximation Errors 522

339 7.9.1 Cut-off 523

340 7.9.2 Truncation Error 525

341 7.9.3 Streamlining, LCA for Design and Prospective

342 LCA 526

343 7.9.4 Aggregation Error 529

344 7.10 Other Types of Error 530

345 7.10.1 Sampling Error 530

346 7.10.2 Errors in Computation 530

347 7.10.3 Power Series Expansions 532

348 7.10.4 Database Errors 534

349 7.10.5 Software Implementation Errors 535

350 7.10.6 Model Errors 536

351 7.10.7 Trade-off of Errors 537

352 7.11 Validity, Reliability, Repeatability and Reproducibility 537

353 7.11.1 Validity and Reliability in the Social Sciences 537

354 7.11.2 Repeatability, Reproducibility and Reliability

355 in the Quality Management and Production

356 Engineering 538

357 7.11.3 Data Quality in LCA 539

358 7.12 Significant Digits 540

359 7.13 Data Quality Indicators 542

360 7.13.1 The NUSAP Scheme and the Pedigree 543

361 7.13.2 DQIs in LCA 544

362 7.13.3 Conversion of DQIs into Overall Data Quality

363 Scores 549

364 7.13.4 Conversion of DQIs into Probability

365 Distributions 552

366 7.13.5 Conversion of DQIs into Possibilistic

367 Information 554

368 7.13.6 System-Wide DQIs 554

369 7.13.7 The ‘Basic Uncertainty’ 557

370 7.13.8 Assessment Factors 558

371 Recap 560

372 References 560

373 **8 Uncertainty, Risk and Decisions 587**

374 8.1 Decisions 588

375 8.1.1 The Decision Matrix 588

376 8.1.2 Expected Value 589

Author Proof

377	8.1.3	Expected Utility	590
378	8.1.4	Value of Information	592
379	8.2	Uncertainty	592
380	8.2.1	Uncertainty in Daily Life and Society	593
381	8.2.2	Uncertainty in Science and Engineering	593
382	8.2.3	Uncertainty in Policy and the Post-normal	
383		Science Movement	594
384	8.2.4	Uncertainty = Error + Variability	595
385	8.2.5	Is Uncertainty Quantifiable?	599
386	8.2.6	Types of Uncertainty	600
387	8.3	Modeling Uncertainty	603
388	8.3.1	Uncertainty = Error + Variability: A Closer	
389		Look	604
390	8.3.2	Combining Variability and Measurement Error	605
391	8.3.3	Inferring the Underlying Measurand	607
392	8.3.4	Non-normal Distributions	608
393	8.4	Risk	609
394	8.4.1	What Is Risk?	609
395	8.4.2	Risk and Decisions	612
396	8.4.3	The Role of Risk in LCA	612
397	8.5	Multi-criteria Methods	614
398	8.5.1	Basics of Multi-criteria Decision Analysis	615
399	8.5.2	Multi-criteria Decision Analysis	
400		and Uncertainty	616
401	8.5.3	Use of Multi-criteria Decision Analysis in LCA	617
402	8.5.4	Use of Multi-criteria Decision Analysis	
403		in Uncertain LCA	617
404	8.6	Decisions in LCA	618
405	8.6.1	From Numbers to Decisions	618
406	8.6.2	Decision Situations and Application Areas	620
407	8.6.3	LCA as an Optimization Problem	621
408	8.7	Single-Product Decisions	622
409	8.7.1	Provision of Product Information	622
410	8.7.2	Benchmarking a Product	623
411	8.7.3	Benchmarking the Mean Product	623
412	8.7.4	Benchmarking 'all' Products	625
413	8.7.5	Benchmarking a 'Significant' Share	
414		of Products	626
415	8.7.6	The Problem of Defining Benchmarks	627
416	8.7.7	Product Improvement with a Contribution	
417		Analysis	627
418	8.7.8	Product Improvement with a Sensitivity	
419		Analysis	630
420	8.7.9	Product Improvement with Mathematical	
421		Optimization	630

Author Proof

422 8.8 Multi-product Decisions 631

423 8.8.1 Pairwise Comparisons 632

424 8.8.2 The Best Product Alternative 632

425 8.8.3 The Product Alternative with the Lowest

426 Mean Impact 634

427 8.8.4 The Product Alternative with the Significantly

428 Lowest Mean Impact—Two Alternatives 635

429 8.8.5 Dependent and Independent Comparisons 637

430 8.8.6 Issues with ‘Significance’ 640

431 8.8.7 Effect Size 641

432 8.8.8 ‘Modified’ Significance Tests 643

433 8.8.9 The Product Alternative with the Significantly

434 Lowest Mean Impact—Three or More

435 Alternatives 644

436 8.8.10 Medians Instead of Means 645

437 8.8.11 Ranking a Set of Product Alternatives 647

438 8.8.12 Rank-Based Procedures 647

439 8.8.13 Pareto Fronts and Dominated

440 and Non-dominated Products 649

441 8.8.14 Overlap 650

442 8.8.15 Discernibility and the Comparison Indicator 653

443 8.8.16 Probabilities 658

444 8.8.17 A Tableau of Pairwise Comparisons 659

445 8.8.18 Multiple Products and Multiple Impacts 660

446 8.8.19 Other Types of LCA Research 661

447 Recap 663

448 References 663

449 **9 Sensitivity** 683

450 9.1 Defining Sensitivity Analysis 684

451 9.1.1 Sensitivity Versus Uncertainty 684

452 9.1.2 Local Versus Global Sensitivity Analysis 686

453 9.1.3 One-at-a-Time Versus All-at-a-Time 691

454 9.1.4 OAT and LSA Are Not the Same, and Neither

455 Are AAT and GSA Synonyms 693

456 9.1.5 Sensitivity, Uncertainty and Influence 694

457 9.1.6 Uncertainty Apportioning 696

458 9.1.7 Discrete Changes and Scenarios 697

459 9.1.8 Subdividing the Field 699

460 9.1.9 Nominal Value Versus Probability Distribution 700

461 9.1.10 Is Sensitivity Analysis Probabilistic? 701

462 9.1.11 A Note on Notation 702

463 9.1.12 Visualizing Sensitivity 702

464 9.1.13 Screening 703

Author Proof

465 9.2 Local Sensitivity Analysis, One-at-a-Time 704

466 9.2.1 The Basic Idea of LSA-OAT 705

467 9.2.2 Derivatives, Multipliers, the Gradient

468 and the Jacobian Matrix 706

469 9.2.3 Non-linear Effects and the Second Derivative 709

470 9.2.4 Derivative-Based Measures of Sensitivity 710

471 9.2.5 Data Requirements for Local Sensitivity

472 Analysis 713

473 9.2.6 The Choice of the Nominal Value 714

474 9.3 Local Sensitivity Analysis, All-at-a-Time 715

475 9.3.1 Two-at-a-Time 716

476 9.3.2 All-at-a-Time 719

477 9.4 Discrete Sensitivity Analysis 720

478 9.4.1 Sensitivity for Choices 721

479 9.4.2 Sensitivity for Changes in Discrete Variables 724

480 9.4.3 Sensitivity for Changes in Discretized

481 Continuous Variables 725

482 9.4.4 Sensitivity for Arbitrary Changes 725

483 9.4.5 All-at-a-Time Choices: Scenarios 725

484 9.4.6 Visualizing DSA 726

485 9.5 Global Sensitivity Analysis, One-at-a-Time 727

486 9.5.1 Sensitivity Functions and Their Visualization 727

487 9.5.2 Numerical Indicators 732

488 9.5.3 Stability Setting 734

489 9.5.4 Reliability Theory 737

490 9.6 Global Sensitivity Analysis, All-at-a-Time 737

491 9.6.1 Scatter Plots, Correlation and Regression 738

492 9.6.2 Some Other Approaches 740

493 9.6.3 Discretized Continuous Variables: Factorial

494 Design 741

495 9.6.4 Discretized, Two-at-a-Time: Visualization 743

496 9.6.5 Morris' Elementary Effects 744

497 9.6.6 Screening Methods 747

498 9.7 Uncertainty Apportioning 748

499 9.7.1 The Basic Idea of UA 748

500 9.7.2 Regression-Based UA 751

501 9.7.3 Variance-Based UA 755

502 9.7.4 Moment-Independent UA 757

503 9.7.5 Estimating Conditional Variances

504 in Variance-Based UA 757

505 9.7.6 Probing the Input Space 762

506 9.7.7 Other Variations 765

507 9.7.8 When is a Contribution Large? 769

Author Proof

508 9.8 Computational Aspects 770

509 9.8.1 Change of One Element 771

510 9.8.2 Change of One Row or Column 772

511 9.8.3 Change of Several Rows or Columns 773

512 9.8.4 Change of a Block of a Partitioned Matrices 773

513 Recap 774

514 References 774

515 **Part II A Critique**

516 **10 Statistical Concepts, Terminology and Notation** 793

517 10.1 Parameters, ‘true’ Values and ‘deterministic’ Values 794

518 10.1.1 Parameter Uncertainty Does Not Exist 794

519 10.1.2 What If the Parameters of the Input Data Were

520 Uncertain? 796

521 10.1.3 The Double Meaning of ‘parameter’ 797

522 10.1.4 The ‘true’ Value 798

523 10.1.5 Is the Central Value the ‘true’ Value? 799

524 10.1.6 The Roles of Error and Variability 800

525 10.1.7 The ‘deterministic’ Value 801

526 10.1.8 What Is the ‘deterministic’ Value? 803

527 10.1.9 Deterministic Input Value Versus

528 Deterministic Output Result 805

529 10.1.10 The Case of Non-sampling Methods 806

530 10.1.11 Do We Need a ‘deterministic’ Value? Yes,

531 But Let’s Call it the ‘nominal’ Value 807

532 10.2 The Use of Confidence Intervals 808

533 10.2.1 Statistical Intervals: Some Theory 808

534 10.2.2 A Short History of Confidence Intervals in LCA 809

535 10.2.3 Terminology Related to ‘confidence’ 813

536 10.3 Uncertainty Factors 816

537 10.3.1 Qualitative Interpretations 817

538 10.3.2 Uncertainty Factors Related to the Geometric

539 Standard Deviation 817

540 10.3.3 Uncertainty Factors Related to a Specific

541 Probability Range 820

542 10.3.4 Uncertainty Factors that Change the Central

543 Value 822

544 10.3.5 Unspecified or Unclear Uncertainty Factors

545 and Other Variations 823

546 10.4 Means, Variances and Standard Deviations 825

547 10.4.1 The Many Faces of Means, Variances

548 and Standard Deviations 826

549 10.4.2 Issues of Notation 827

Author Proof

550	10.4.3	Expected Values and Other Central Values	829
551	10.4.4	Standard Deviations and Other Measures	
552		of Dispersion	830
553	10.5	Hypotheses and Significance	834
554	10.5.1	Hypotheses Without a Hypothesis Test	834
555	10.5.2	Hypotheses with a Hypothesis Test	834
556	10.5.3	Significance Without a Hypothesis Test	838
557	10.5.4	Significance with a Hypothesis Test	840
558	10.5.5	Hypothesis Tests Without Significance	847
559	10.6	Correlation, Dependence and Regression	847
560	10.6.1	Correlation Versus Regression	847
561	10.6.2	Significant Correlations	848
562	10.6.3	Dependence and Association	850
563	10.6.4	Other Meanings of 'dependence'	854
564	10.7	Numbers, Graphs and Notation	854
565	10.7.1	Numbers and Units	855
566	10.7.2	Graphs	862
567	10.7.3	The Use of $x \pm \sigma$	877
568	10.7.4	Mathematical Notation	878
569	10.8	Some Other Issues	880
570	10.8.1	Sensitivity Analysis	880
571	10.8.2	Misuse of Terms	882
572	10.8.3	The Role of the Central Limit Theorem	883
573	10.8.4	The Use of the t -Distribution for Input	
574		Variables	883
575	10.8.5	The Ostrich Phenomenon	884
576	10.8.6	More Curiosities	884
577	10.9	Conclusion	886
578		References	887
579	11	The Lognormal Distribution in LCA	919
580	11.1	Parametrizations	920
581	11.1.1	Parametrization Ia: Abstract Definition	921
582	11.1.2	Parametrization Ib: Definition	
583		from an Underlying Normal Distribution	922
584	11.1.3	Parametrization IIa: Definition with Geometric	
585		Parameters	923
586	11.1.4	Parametrization IIb: Definition in Logarithmic	
587		Units	924
588	11.1.5	Relation Between the Different Forms	925
589	11.1.6	Other Definitions	927
590	11.1.7	Implementation in LCA Databases	931
591	11.2	Some Properties of the Lognormal Distribution	933
592	11.2.1	The Cumulative Distribution Function	933
593	11.2.2	Graphs of the pdf and cdf	933

594	11.2.3	Moments and Other Properties	934
595	11.2.4	Is the Median Equal to the Geometric Mean?	935
596	11.2.5	Interpercentile Intervals	936
597	11.2.6	Combining Lognormal Distributions	938
598	11.2.7	In Search of a ‘most representative’ Value	939
599	11.3	Estimation of the Parameters of the Lognormal	
600		Distribution	942
601	11.3.1	Maximum Likelihood Estimation	942
602	11.3.2	Method of Moments Estimation	943
603	11.3.3	Example of Parameter Estimation	943
604	11.3.4	Other Estimation Methods	944
605	11.3.5	Lognormal Estimation in LCA	945
606	11.3.6	Confidence Intervals for the Parameters	945
607	11.4	Alleged Confidence Intervals, the Dispersion Factor	945
608	11.4.1	Lognormal Distributions and ‘confidence	
609		intervals’ in LCA	945
610	11.4.2	The Dispersion Factor	947
611	11.4.3	The Role of GSD^2	950
612	11.4.4	Why Not Another Power of GSD ?	954
613	11.5	The Issue of Dimensions and Units	955
614	11.5.1	Dimensions and the Normal Distribution	955
615	11.5.2	Units and the Normal Distribution	956
616	11.5.3	Dimensions and the Lognormal Distribution	956
617	11.5.4	Units and the Lognormal Distribution	958
618	11.5.5	Implications for Parameters	959
619	11.6	Variants of the Lognormal Distribution	960
620	11.6.1	The Lognormal Distribution for Negative	
621		Numbers	961
622	11.6.2	The 3-Parameter Lognormal Distribution	962
623	11.6.3	The 4-Parameter Lognormal Distribution	963
624	11.6.4	Other Variations	964
625	11.7	Comparison with the Beta Distribution	964
626	11.7.1	Comparison of Lognormal and Beta	964
627	11.7.2	Dimensions and Units for the Beta Distribution	965
628	11.8	Is the Lognormal Distribution a Natural Choice for LCA?	966
629	11.8.1	The Case of Input Data	966
630	11.8.2	The Case of Output Results	973
631	11.9	Conclusion	975
632		References	975
633	12	The Quantitative Pedigree Approach	985
634	12.1	The NUSAP Scheme and the (Qualitative) Pedigree	985
635	12.2	The Pedigree in LCA	986
636	12.2.1	NUSAP or Pedigree?	986

Author Proof

637 12.2.2 Pedigree, Pedigree Matrix, or Pedigree Approach? 987

638

639 12.2.3 Weidema and Wesnæs (1996) 988

640 12.2.4 Meier (1997) 990

641 12.2.5 Frischknecht et al. (2004/2005/2007) 991

642 12.2.6 GHG Protocol (2011) 993

643 12.2.7 Weidema et al. (2013) 994

644 12.2.8 Muller et al. (2016a) and Ciroth et al. (2016) 996

645 12.2.9 Muller et al. (2016b) 998

646 12.2.10 Adoption by Others 1003

647 12.3 Alternative Quantitative DQIs 1004

648 12.3.1 Kennedy et al. (1996/1997) 1004

649 12.3.2 Van den Berg et al. (1999) 1005

650 12.3.3 Rousseaux et al. (2001) 1006

651 12.3.4 Ardente et al. (2004) 1007

652 12.3.5 Chen and Lee (2021) 1007

653 12.3.6 Zheng et al. (2019) 1008

654 12.3.7 Other Reviews and Modifications 1008

655 12.4 Critical Observations 1009

656 12.4.1 The Combination Rules 1009

657 12.4.2 A Proof of the Addition Rule for Squared Geometric Standard Deviations—With a Few Side Notes 1013

658

659 12.4.3 Adding GSD Unsquarred? 1015

660 12.4.4 The Role of Sample Size 1016

661 12.4.5 What Is the Basic Uncertainty? 1017

662 12.4.6 Comparison with Observed Data 1018

663 12.4.7 Some Other Objections 1018

664

665 12.5 The Quantitative Pedigree Approach in Practice 1019

666 12.5.1 Use of the Quantitative Pedigree 1019

667 12.5.2 Perception of the Quantitative Pedigree 1020

668 12.6 Conclusion 1021

669 References 1022

670 **13 Statistical Analysis of Non-stochastic LCA** 1029

671 13.1 Univariate Analysis 1029

672 13.2 Correlation and Regression Analysis 1030

673 13.3 Multivariate Statistics and Machine Learning 1032

674 13.3.1 Principal Component Analysis 1032

675 13.3.2 Other Multivariate Methods 1033

676 13.3.3 Machine Learning 1033

677 13.4 Meta-analysis 1034

678 13.4.1 What is Meta-analysis? 1034

679 13.4.2 Meta-analysis in LCA 1034

Author Proof

680 13.5 Conclusion 1035

681 References 1036

682 **Part III A Guidance**

683 **14 Including Uncertainty and Sensitivity in LCA 1045**

684 14.1 The LCA Model Equations as a Basis for Uncertainty

685 Modeling 1046

686 14.1.1 A Concrete Example of $\phi(\cdot)$ 1047

687 14.1.2 The LCA Model as a Black Box 1049

688 14.1.3 From Deterministic LCA to Probabilistic LCA 1050

689 14.2 Towards a Guide for Including Uncertainty

690 and Sensitivity 1050

691 14.2.1 Elements of a Guide 1051

692 14.2.2 Why a True Guide on Uncertainty

693 and Sensitivity Is Not Feasible 1053

694 14.2.3 A Guidance Document 1054

695 14.3 Overall Guidance 1055

696 References 1057

697 **15 Guidance for Standard LCA 1059**

698 15.1 Uncertainty Analysis 1060

699 15.1.1 Input Data and Choices in the Framework

700 of LCA 1060

701 15.1.2 Uncertain Data 1068

702 15.1.3 Uncertainty Propagation 1070

703 15.1.4 Non-comparative LCA 1072

704 15.1.5 Comparative LCA 1076

705 15.2 Sensitivity Analysis 1077

706 15.2.1 Local Sensitivity Analysis, One-at-a-Time 1077

707 15.2.2 Global Sensitivity Analysis, One-at-a-Time 1078

708 15.2.3 Discrete Sensitivity Analysis 1079

709 15.2.4 Global Sensitivity Analysis, All-at-a-Time 1079

710 15.2.5 Uncertainty Apportioning 1080

711 References 1080

712 **16 Guidance for Special Types of LCA 1083**

713 16.1 Input–Output-Based LCA and Hybrid LCA 1084

714 16.2 Attributional and Consequential LCA 1085

715 16.3 Parametrized and Non-linear LCA 1086

716 16.4 Agent-Based LCA 1088

717 16.5 Fleet-Based LCA 1088

718 16.6 Life Cycle Optimization 1089

719 16.7 Other Pillars of Sustainability 1089

720 16.7.1 Life Cycle Costing 1090

Author Proof

721	16.7.2	Social Life Cycle Assessment	1091
722	16.7.3	Environmental and Economic Aspects	1093
723	16.7.4	All Three Pillars	1094
724		References	1095
725		Appendix A: Symbols	1103
726		Appendix B: Matrix Topics	1109
727		Appendix C: Speical Functions	1121
728		Index	1131

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Author Proof