Conditional probabilities for euro area sovereign default risk

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Disclaimer: Not necessarily the views of ECB, Eurosystem, or Riksbank.

Contributions

We propose a **novel modeling framework** to infer **conditional** and **joint probabilities** for sovereign default risk from observed CDS.

Novel framework? Based on a *dynamic GH skewed–t* multivariate density/copula with time-varying volatility and correlations.

Multivariate model is sufficiently flexible to be **calibrated daily** to credit market expectations. Not an "official opinion".

Analysis is based on **Euro area** CDS data, daily from 2008 to end-2012. **Event study**: SMP/EFSF announcement & initial impact on risk.

Literature

1. **Sovereign credit risk:** e.g. Pan and Singleton (2008), Longstaff, Pan, Pedersen, and Singleton (2011), Ang and Longstaff (2011).
2. **Risk contagion**, see e.g. Forbes and Rigobon (2002), Caporin, Pelizzon, Ravazzolo, Rigobon (2012).
3. **Observation-driven time-varying parameter models**, see Creal, Koopman, and Lucas (2011, 2012), Zhang, Creal, Koopman, Lucas (2011), Creal, Schwaab, Koopman, Lucas (2011), Harvey (2012).
4. **Non-Gaussian dependence/copula/credit modeling**, see e.g. Demarta and McNeil (2005), Patton and Oh (2011).

Empirical questions

(**Q1**) Financial stability information: Based on credit market expectations, what is ...

Pr(two or more credit events in Euro area)?

Pr(i|j)-Pr(i), for any i,j?

Spillovers, e.g. Pr(PT|GR) - Pr(PT|not GR)?

Corr_t(i,j) at time t?

(**Q2**) Model risk: For answering (a), how important are parametric assumptions? *Normal* vs *Student-t* vs *GH skewed-t*.

(**Q3**) Event study: did the May 09, 2010 Euro area rescue package change risk dependence? How?

Copula framework

Sovereign defaults iff benefits (v_{it}) exceed a cost (c_{it}), where

$$v_{it} = (\varsigma_t - \mu_\varsigma)\tilde{L}_{it}\gamma + \sqrt{\varsigma_t}\tilde{L}_{it}\epsilon_t, \quad i = 1, \ldots, n,$$

$\epsilon_t \sim$N$(0, I_n)$ is a vector of risk factors,
$\tilde{L}_{it}$ contains risk factor loadings,
$\gamma \in \mathbb{R}^n$ determines skewness,
$\varsigma_t \sim IG$ is an additional scalar risk factor for, say, *interconnectedness*.

A default occurs with probability p_{it}, where

$$p_{it} = \Pr[v_{it} > c_{it}] = 1 - F_i(c_{it}) \;\Leftrightarrow c_{it} = F_i^{-1}(1 - p_{it}),$$

where F_i is the CDF of v_{it}.

Focus on *conditional* probability $\Pr[v_{it} > c_{it} | v_{jt} > c_{jt}]$, $i \neq j$.

GH skewed-t dependence

$$y_t \;=\; \mu + L_t e_t, \quad t = 1, ..., T,\; e_t \sim \text{GHST},\; \mathrm{E}[e_t e_t'] = I_n,$$

$$p(y_t; \cdot) \;=\; \frac{v^{\frac{v}{2}} 2^{1-\frac{v+n}{2}}}{\Gamma\left(\frac{v}{2}\right) \pi^{\frac{n}{2}} \left|\tilde{\Sigma}_t\right|^{\frac{1}{2}}} \cdot \frac{K_{\frac{v+n}{2}}\left(\sqrt{d(y_t)\cdot(\gamma'\gamma)}\right) e^{\gamma'\tilde{L}_t^{-1}(y_t - \tilde{\mu}_t)}}{\left(d(y_t)\cdot(\gamma'\gamma)\right)^{-\frac{v+n}{4}} d(y_t)^{\frac{v+n}{2}}},$$

where

$$d(y_t) \;=\; v + (y_t - \tilde{\mu}_t)'\tilde{\Sigma}_t^{-1}(y_t - \tilde{\mu}_t),$$

$$\tilde{\mu}_t \;=\; -v/(v-2)\; \tilde{L}_t \gamma,$$

$$\tilde{\Sigma}_t \;=\; \tilde{L}_t \tilde{L}_t' \quad \text{is scale matrix}$$

If $\gamma = 0$, then GH skewed-t simplifies to Student's t density.
If in addition $v^{-1} \to 0$, then multivariate Gaussian density.
$\tilde{\Sigma}_t(f_t) = \tilde{L}_t(f_t)\tilde{L}_t(f_t)'$ is driven by 1st and 2nd derivative of the pdf.

Time varying parameters: score

Important: first two derivatives are available in closed form.

$$
\begin{aligned}
\nabla_t &= \partial \ln p(y_t; \tilde{\Sigma}(f_t), \gamma, \upsilon) / \partial f_t \\
&= \frac{\partial \mathrm{vech}(\Sigma_t)'}{\partial f_t} \frac{\partial \mathrm{vech}(L_t)'}{\partial \mathrm{vech}(\Sigma_t)} \frac{\partial \mathrm{vec}(\tilde{L}_t)'}{\partial \mathrm{vech}(L_t)} \frac{\partial \ln p_{GH}(y_t | f_t)}{\partial \mathrm{vec}(\tilde{L}_t)} \\
&= \ldots \\
&= \Psi_t' H_t' \mathrm{vec} \left\{ w_t y_t y_t' - \tilde{\Sigma}_t - \left(1 - \frac{\upsilon}{\upsilon - 2} w_t \right) \tilde{L}_t \gamma y_t' \right\}
\end{aligned}
$$

$$
\begin{aligned}
\text{where } \Psi_t &= \partial \mathrm{vech}(\Sigma_t) / \partial f_t' \\
H_t &= \text{messy} \\
w_t &= \frac{\upsilon + n}{2 \cdot d(y_t)} - \frac{k'_{\frac{\upsilon+n}{2}} \left(\sqrt{d(y_t) \cdot (\gamma'\gamma)} \right)}{\sqrt{d(y_t) / \gamma'\gamma}}; \; k_a'(b) = \frac{\partial \ln K_a(b)}{\partial b}.
\end{aligned}
$$

Time varying parameters: scaling

Scaling is by the inverse Fisher information matrix for a symmetric-t.

$$S_t = \left\{ \Psi' (I \otimes \tilde{L}_t^{-1})' \left[gG - \text{vec}(I)\text{vec}(I)' \right] (I \otimes \tilde{L}_t^{-1}) \Psi \right\}^{-1},$$

where

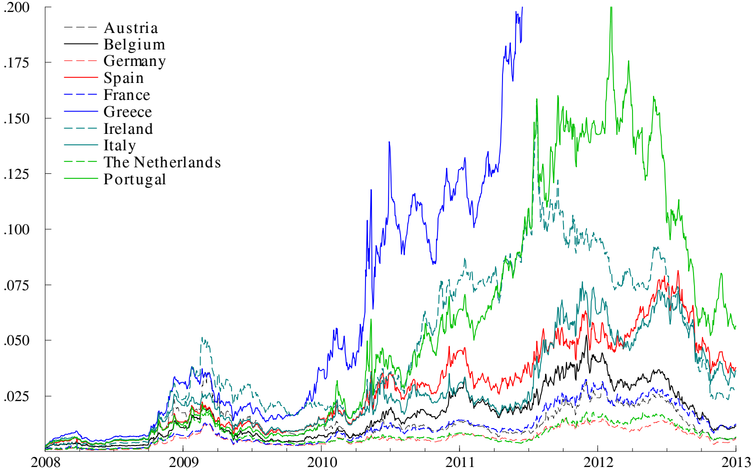
$$\begin{aligned} \Psi_t &= \partial \text{vech}(\Sigma_t) / \partial f_t', \\ g &= (\upsilon + n)(\upsilon + 2 + n) \\ G &= \mathrm{E}[x_t x_t' \otimes x_t x_t'] \ \text{ for } \ x_t \sim \mathrm{N}(0, I_n) \end{aligned}$$

Extracting marginal pd's from CDS

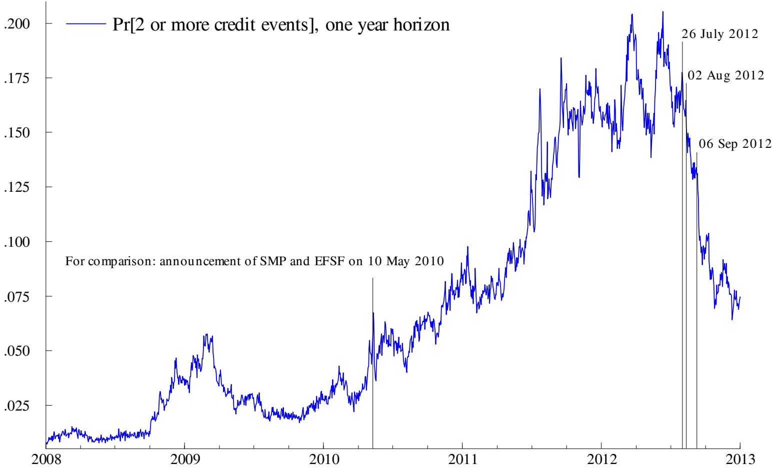
CDS fee equates a premium leg and a default leg given a default intensity, see Duffie (1999), O'Kane and Turnbull (2003).

- We use a nonlinear solver to find the default intensity that matches E[PV premium leg] = E[PV default leg].
- Intensity and annual pd are nonlinearly related.
- Use 25% recovery rate and interest rate flat at 1%, ignore counterparty credit risk.
- Overall, not that difficult.

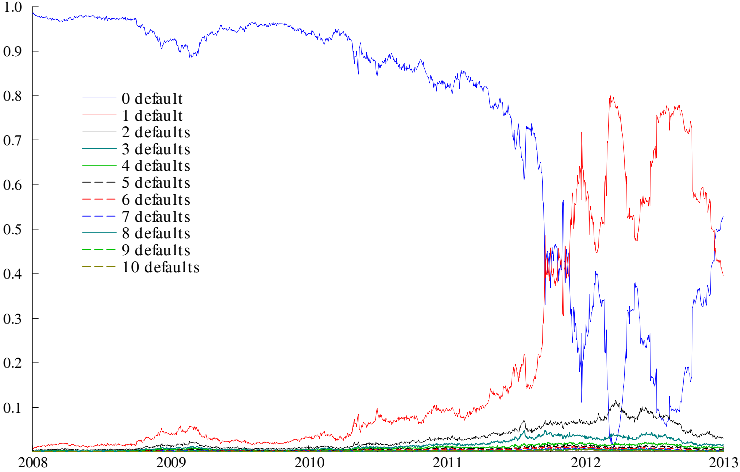
Marginal pd's from CDS



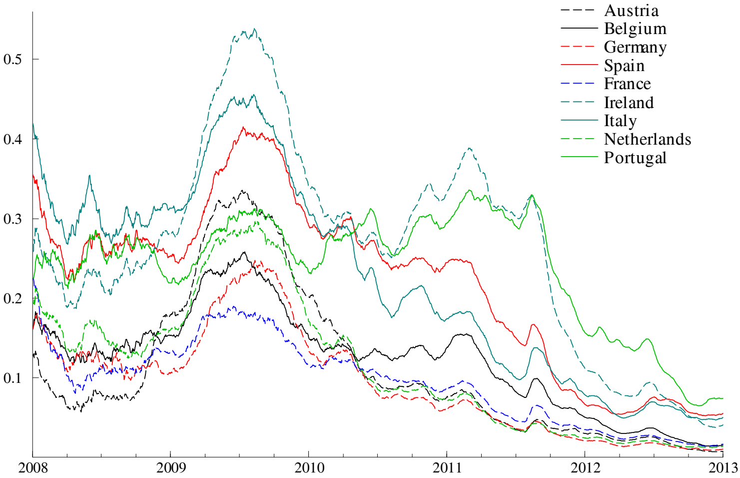
Pr[2 or more credit events]



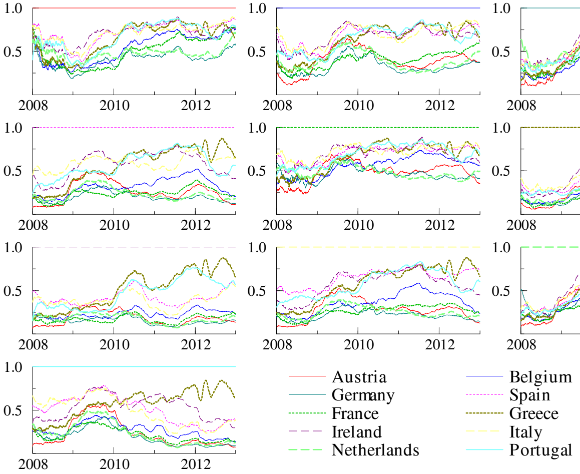
The probability of k=0,1,2,... failures



Conditional pds: Pr(country i|GR)



Conditional pds: Pr(all i| all j)



The May 09, 2010 package

Joint risk, $\Pr(i \cap j)$						
	Thu 06 May 2010			Tue 11 May 2010		
	PT	GR	DE	PT	GR	DE
AT	1.1%	1.1%	0.6%	0.6%	0.7%	0.4%
BE	1.2%	1.4%	0.7%	0.9%	1.0%	0.6%
DE	1.0%	1.1%		0.8%	0.8%	
ES	3.0%	3.3%	0.9%	1.5%	1.6%	0.6%
FR	1.0%	1.0%	0.6%	0.8%	0.9%	0.6%
GR	**4.8%**		1.1%	**2.3%**		0.8%
IR	**2.6%**	**3.1%**	0.8%	**1.4%**	**1.8%**	0.6%
IT	2.8%	2.9%	0.9%	1.4%	1.5%	0.6%
NL	0.9%	0.9%	0.5%	0.6%	0.7%	0.5%
PT		4.8%	1.0%		2.3%	0.8%
Avg	**2.0%**	**2.2%**	0.8%	**1.1%**	**1.2%**	0.6%

The May 09, 2010 package

Conditional risk, Pr(i | j)						
	Thu 06 May 2010			Tue 11 May 2010		
	PT	GR	DE	PT	GR	DE
AT	17%	8%	53%	22%	10%	46%
BE	20%	10%	60%	32%	15%	61%
DE	16%	8%		26%	12%	
ES	49%	25%	78%	50%	23%	63%
FR	16%	8%	58%	28%	12%	62%
GR	**78%**		99%	**80%**		86%
IR	**43%**	**23%**	75%	**49%**	**26%**	68%
IT	45%	22%	77%	49%	21%	64%
NL	14%	7%	49%	21%	10%	50%
PT		36%	91%		33%	81%
Avg	**33%**	**16%**	71%	**40%**	**18%**	64%

Bottom line: joint risks ↓↓, but dependence ↑. "Firewall"-analogy?

Conclusion

We proposed a **novel modeling framework** to infer **conditional** and **joint probabilities** for sovereign default risk from observed CDS.

Based on a *dynamic skewed–t* multivariate density with time-varying volatility and correlations. Application to euro area CDS.

Thank you