# A New Semiparametric Volatility Model

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Workshop on Dynamic Models driven by the Score of Predictive Likelihoods,
Amsterdam, January 2013

# Introduction

- We propose a new volatility model in which:
  - the skewed and fat-tailed shape of the innovation distribution directly affects volatility dynamics;
  - the innovation distribution is estimated by kernel density method.
- related literature: Creal, Koopman, and Lucas (2012), Engle and Gonzalez-Rivera (1991), Drost and Klaassen (1997), Harvey(2008).

# Model specification

- We apply the GAS framework to our need. We consider a univariate return series $y_t$ and

$$y_t = \mu + \xi_t = \mu + \sigma_t \epsilon_t, \epsilon_t \sim p_\epsilon(\cdot) \tag{1}$$

- In order to make sure that the volatility $\sigma_t$ is always positive, we let $f_t = \log \sigma_t^2$ and choose a GAS(1,1) specification,

$$f_{t+1} = \tilde{\omega} + \tilde{\alpha} s_t + \tilde{\beta} f_t \tag{2}$$

where $\tilde{\omega}, \tilde{\alpha} \in \mathbb{R}$, $|\tilde{\beta}| < 1$ and we choose unit scaling ($S_t = 1$) for the density score,

$$s_t = S_t \nabla_t = 1 \times \frac{\partial \ln p_\epsilon(y_t | f_t, \mathcal{F}_t; \theta)}{\partial f_t}. \tag{3}$$

# Estimation

- Parameter estimation of the model is straightforward, since the model is defined in conditional terms similar to the standard GARCH model.
- Calculation of $\nabla_t$:

$$\frac{\partial \ln p_y(y_t|f_t, \mathcal{F}_t; \theta)}{\partial f_t} = -\frac{1}{2} - \frac{1}{2}\frac{p'_\epsilon(e^{-\frac{f_t}{2}}(y_t - \mu))(y_t - \mu)e^{-\frac{f_t}{2}}}{p_\epsilon(e^{-\frac{f_t}{2}}(y_t - \mu))}. \quad (4)$$

- We can iteratively update $s_1, f_2, s_2, f_3, \cdots, f_{n-1}, s_{n-1}, f_n$. Then we can evaluate the likelihood as

$$L = \frac{1}{n}\sum_{t=1}^{n} l_t = \frac{1}{n}\sum_{t=1}^{n} \ln \frac{1}{\sigma_t} p_\epsilon\left(\frac{y_t - \mu}{\sigma_t}|f_t, \mathcal{F}_t; \theta\right). \quad (5)$$

# Semiparametric estimation

- We first estimate the model assuming normality, then we calculate standardized residuals and use kernel density estimator to determine the error density and replace $p_\epsilon$ by its estimate $\hat{p}_\epsilon$,

$$\hat{p}_\epsilon(x) = \frac{1}{nh}\sum_{t=1}^{n} k\left(\frac{\hat{\epsilon}_t - x}{h}\right), \tag{6}$$

- We use the standard normal kernel

$$k(v) = \frac{1}{\sqrt{2\pi}} e^{\frac{1}{2}v^2}, -\infty < v < \infty, \tag{7}$$

with a bandwidth of $h = 0.5$.

  - bandwidth: reasonable changes of the bandwidth, say $0.3 \leq h \leq 0.8$;

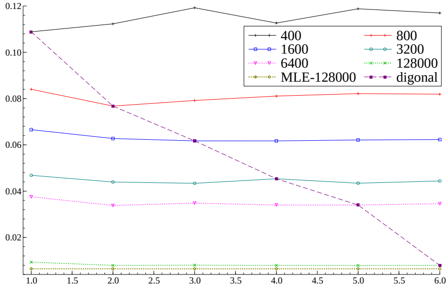
# Results under correct specification

- we use the new model as DGP to simulate return series and investigate volatility forecast accuracy of this model.
- simulation scheme
  - $B = 100$ samples; length of $n = 2000$; parameter $(\mu, \omega, \alpha, \beta) = (0, 0.2, 0.3, 0.9)$;
  - error density: standard normal, Student's $t$ with $\nu = 3$, and 5 degrees of freedom, and a mixture of normals.
- we estimated the parameters by four different estimation methods: GAS-true, GAS-normal, GAS-t($\nu$) and semi-GAS.

**Table 1: Simulation results under correct specification**

| median of RMSE of $\sigma_t$ | N | t(5) | $MN(\chi^2(6))$ |
|---:|---|---|---|
| GAS-true | 0.104 | 0.128 | 0.116 |
| GAS-normal | 0.104 | 0.474 | 0.430 |
| GAS-$t(\nu)$ | 0.105 | 0.147 | 0.419 |
| semi-GAS | 0.207 | 0.241 | 0.257 |

# what matters: number of iteration or sample size? $t(5)$, $\alpha$



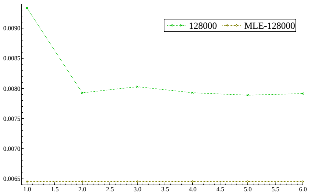




Figure : what matters: number of iteration or sample size?

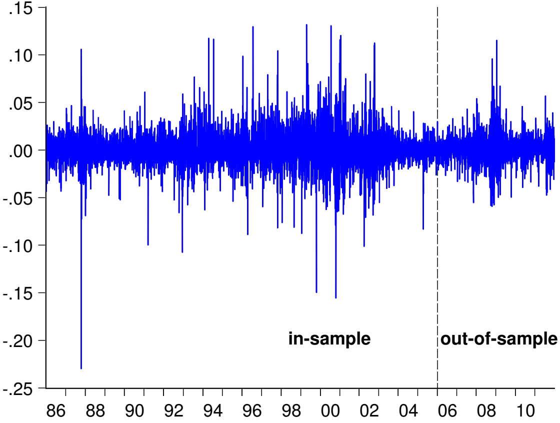
# Simulation with stochastic volatility

- In reality, we do not know the DGP and models are only approximate to the DGP.
- Therefore, we choose a stochastic volatility process such that the volatility models are only statistical models to approximate time-varying volatility.
- The stochastic volatility DGP SV is specified as $y_t \sim p(0, \sigma_t^2)$ with $\sigma_t^2 = exp(\alpha_t)$ and $\alpha_t = 0.01 + 0.98\alpha_{t-1} + \eta_t$, where $\eta_t \sim N(0, 0.1^2)$, for $t = 1, \dots, n$.

## Table 2: Simulation results under mis-specification: stochastic volatility

| median of RMSE of $\sigma_t$ | | GARCH | GAS |
|---|---|---|---|
| N | GAS-normal | 0.237 | 0.236 |
| | GAS-$t(\nu)$ | 0.237 | 0.235 |
| | semi-GAS | 0.237 | 0.243 |
| t(3) | GAS-normal | 0.351 | 0.375 |
| | GAS-$t(\nu)$ | 0.345 | 0.280*** |
| | semi-GAS | 0.334 | 0.295** |
| $MN(\chi^2(6))$ | GAS-normal | 0.247 | 0.246 |
| | GAS-$t(\nu)$ | 0.255 | 0.251 |
| | semi-GAS | 0.243 | 0.231** |

# Empirical application with IBM daily return series



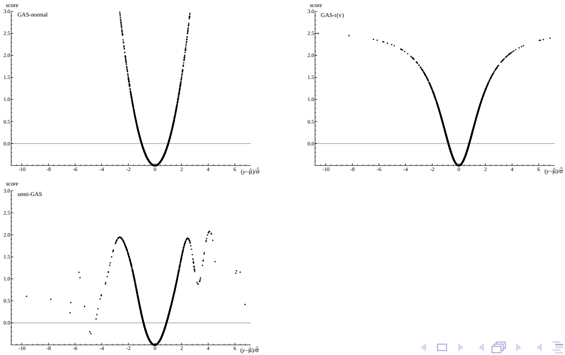
# Estimation results

Table 3: Empirical Estimation Results

| | GAS normal | GAS* $t(\nu)$ | GAS $t(\nu)$ | GAS semi | | GARCH normal | GARCH $t(\nu)$ | GARCH semi |
|---|---|---|---|---|---|---|---|---|
| $\mu$ | 0.061 | 0.012 | 0.017 | 0.041 | $\mu$ | 0.067 | 0.012 | 0.033 |
| | (0.023) | (0.020) | (0.019) | (0.019) | | (0.021) | (0.019) | (0.020) |
| $\tilde{\omega}$ | 0.016 | 0.007 | 0.007 | 0.012 | $\omega$ | 0.041 | 0.015 | 0.024 |
| | (0.003) | (0.002) | (0.003) | (0.002) | | (0.009) | (0.005) | (0.006) |
| $\tilde{\alpha}$ | 0.059 | 0.016 | 0.131 | 0.142 | $\alpha$ | 0.079 | 0.033 | 0.046 |
| | (0.007) | (0.002) | (0.019) | (0.010) | | (0.010) | (0.006) | (0.006) |
| $\tilde{\beta}$ | 0.985 | 0.993 | 0.994 | 0.991 | $\alpha+\beta$ | 0.995 | 0.995 | 0.994 |
| | (0.002) | (0.002) | (0.002) | (0.002) | | (0.003) | (0.002) | (0.002) |
| $\nu$ | | 4.743 | 5.174 | | $\nu$ | | 5.006 | |
| | | (0.300) | (0.345) | | | | (0.330) | |
| log-lik | -10000.80 | -9631.35 | -9576.68 | -9570.8 | | -9917.2 | -9599.78 | -9597.3 |

# To know more about the volatility dynamics

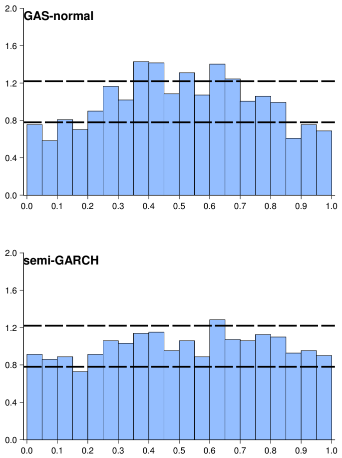
- We want to know: how does the volatility react to the news, $\epsilon_t$? Does the volatility react to positive news and negative news equally?
- We plot score functions against standardized residuals for each estimation method.



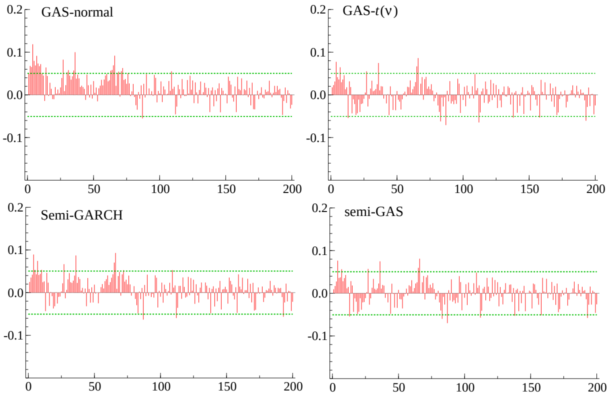
# To generate density forecasts and evaluate them

- After we estimate the model by one method, we freeze it, and use it to generate out-of-sample volatility forecasts and density forecasts. We denote the forecast of $p_\epsilon$ at time $t$ as $\hat{p}_t$.
- For GAS-normal, GAS-$t(\nu)$, semi-GAS and Semi-GARCH.
- Method by Diebold, Gunther, and Tay (1998). True density is $\{p_t(y_t|\mathcal{F}_t)\}_{t=1}^m$; Density forecast is $\{\hat{p}_t(y_t|\mathcal{F}_t)\}_{t=1}^m$. We can evaluate density forecasts by assessing the distribution of a series called the probability integral transform, $z_t$, with $z_t = \int_{-\infty}^{y_t} \hat{p}_t(u)du$.

# Density forecast evaluation results: histogram



# Density forecast evaluation results: Correlogram of $(z_t - \bar{z})^2$



# Conclusion: main results

- We introduce a new semiparametric time-varying volatility model. In this model,
  - we use kernel density methods to estimate the error density;
  - the form of the error distribution also governs the specification of volatility dynamics;
- Monte carlo evidence and application to real data:
  - simulations results show that the new model provides accurate volatility forecasts.
  - the new model does a good job of generating density forecasts