Optimal Investment Policies
for Defined Benefit Pension Funds

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Abstract

This paper analyzes optimal investment policies for pension funds of a defined benefit (DB) type. The nature of a DB fund induces a natural modeling of preferences being of the mean-downside risk type. With compensation for inflation as an explicit goal of a pension fund, a natural reference point for the risk measure is the future (indexed) value of the liabilities. Results are presented for different levels of inflation uncertainty and its correlation with stock returns. The optimal decision rules show increased risk-taking for funding ratios moving away from the discounted value of the reference point. Furthermore, it is shown that the outcomes are comparable with those using a mean-downside deviation criterion. We provide intuition for the results and compare the outcomes with actual investment policies of six large Dutch pension funds.

Keywords: pension funds, optimal investment, defined benefit, mean-shortfall, loss aversion, asset-liability management.
JEL classification: G11, G23.

1   Introduction

In analyzing investment policies for pension funds, most of the financial literature has settled on the use of the mean and variance of investment returns to measure performance. For example, Randall and Satchell (1997) compare the efficiency of different portfolios for pension fund assets with respect to the mean and variance of the return. Such an analysis has the advantage of being able to use the standard toolkit of mean-variance analysis

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as developed by Markowitz (1952). The disadvantage, however, is that it does not take into account that pension funds experience a serious downside when returns are (too) low. See also Owadally and Haberman (2004), who minimize a quadratic measure of the variability of contribution and market value of plan assets. Randall and Satchell (1997) already notice that pension funds cannot afford "to lose a huge amount of money, even if they are frequently making small amounts of money. This asymmetry is not accounted for by a model which is defined over mean and variance only."

A similiar weak point is present in the literature on pension funding that uses stochastic control to solve for optimal contribution/investment policies, see Boulier(1995,1996), Cairns (2000), Haberman and Sung (1994), and Vigna and Haberman (2001). Using stochastic control is well motivated by the dynamic multi-period nature of the pension fund's problem. However, using a traditional utility function in terms of surplus or quadratic penalty function on the contribution rate is not in accordance with pension fund managers who care about the negative consequences of having a less-than-fully funded pension fund. Defining the funding ratio as the value of assets divided by the present value of liabilities, underfunded pension plans have a funding ratio below 100%. Depending on the regulatory situation, underfunding is implicitly or explicitly associated with costs and penalties. The implicit costs vary from a lost of trust to a lower fraction in stocks, while the explicit costs can range from higher contributions to explicit penalties. Taking this institutional reality into account is the aim of the present paper. Using a stylized model for a DB pension fund that incorporates an explicit penalty for underfunding, I show that the optimal investment policy differs remarkably from the outcomes obtained from a traditional approach.

That a pension funds is more sensitive to under- than to overfunding is already explicitly acknowledged operationally in the practice of Asset/Liability Management (ALM). Many defined benefit (DB) pension funds base their strategic investment policy on so-called ALM studies. ALM uses techniques from the field of operations research to optimize funding and investment policies under uncertainty. See Ziemba and J. M. Mulvey (Eds.) (1998) for an extensive overview of the use of and research into ALM. Relevant for the current paper is that the risk measures used in most ALM studies are downside risk

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measures. Downside risk measures explicitly define risk in terms of the downside to all possible outcomes. Good illustrations of the use of downside risk measures are Sortino and Van der Meer (1991), Harlow (1991), Cariño et al. (1994), and Boender (1997). However, given its strong roots in operations research, most literature on ALM focuses on the computational side of ALM problems only. See for example Zenios (1995) Kusy and Ziemba (1986), Hiller and Eckstein (1993), Maranas et al. (1997) and Zenios et al. (1998). The approach taken by Haberman et al. (2003a) and Haberman et al. (2003b) resembles the ALM approach, analyzing a mean-shortfall optimization and using indifference curves and efficient frontiers to evaluate the outcomes. In this paper, we take a downside risk measure from the field of ALM, but focus the analysis on the qualitative outcome in terms of the optimal investment policies. As such, the novelty of our approach is not in the risk measure or subject of analysis, but rather in the comparative statics that have not been analyzed in this context before.

Finally, an extra argument for using a downside risk measure is that it represents the notion of *loss aversion*. Introduced by Kahneman and Tversky (1979), loss aversion is a behavioral concept that captures the notion that people are more sensitive to losses than to gains. Although the debate on rationality of loss aversion is not finished, it does provide extra support for studying pension funding in a downside risk framework.

Section 2 introduces the representative model for a defined benefit pension fund. Section 3 derives implications for pension fund investment policy in a relevant setting, and considers the sensitivity of the results to the specification of uncertainty. In Section 4 we explore empirical evidence of loss averse preferences for pension funds by examining actual pension fund investment policies. Policy implications are discussed in Section 5 and Section 6 concludes.

2   Model

It is instructive to list the fundamental properties of DB pension funds, on which we base the formulation of the model that is introduced next. A DB fund can be characterized by three basic features:

(a) participants each pay the same fraction of their salary to obtain fixed pension rights as a percentage of their current or final salary,

(b) the fund is supervised by an independent regulator,

(c) there is a single investment and funding policy.

First, (a) implies that defined benefit pension funds embed a strong form of solidarity, in that the link between contributions paid and new pension rights is only an indirect one. Since contribution is a fixed fraction of wages and the same for all participants, not depending on age, younger workers partly pay for the older workers. Older workers are closer to retirement, so given the same salary, and extra pension dollar has a higher actuarial[2] value for older workers than for the younger ones. Hence, since the contributions are the same, the younger generations are partly financing the pension rights of the older generations. Another aspect that makes the link between contributions and pension rights complex, is that contributions will vary with the financial status of the fund. If the pension plan is underfunded, extra contributions are necessary without there being any added pension benefits.

The regulator mentioned in feature (b) checks the financial status of the fund, quality of the management, and serves as protector of the rights of the active and inactive participants. The financial status of a fund can be summarized by the *funding ratio*, which is the ratio of assets over the present value of the liabilities. Feature (c) follows from the solidarity embedded in the fund's setup. All workers pay an equal fraction of their salary that is determined once a year. The solidarity across workers and across generation implies that the investment and funding decisions also have to be taken for the collective.

In this paper, we analyze the investment problem for a stylized DB fund as defined above. Consider a pension fund at time 0 (now) with current wealth W_0. At time T in the future, pension liabilities of W^B need to be covered. The decision to be taken is the amount X_0 of wealth to invest in a risky asset yielding a gross return of u_t at each period. The rest $(W_0 - X_0)$ is invested in a risk-free asset with gross return r_f in each period. Finally, the objective of the fund favors more wealth over less and punishes shortfall below

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[2]The actuarial value of a future cash flow is just the net present value, corrected for survival.

the benchmark level W^B. The formulation of the optimization problem is given by

$$\max_{X_0} \quad \mathbb{E}[W_T] - \lambda \cdot \mathbb{E}[(W^B - W_T)^+], \tag{1}$$

$$\text{s.t.} \quad W_T = (W_0 - X_0)r_f^T + X_0 \cdot \prod_{t=1}^{T} u_t, \tag{2}$$

where $(y)^+$ is the maximum of 0 and y and λ the aversion to shortfall. The symbols r_f and u_t represent the returns on the investment categories available to the fund. In practice, pension funds will invest in more than only two categories, but we can take the uncertain return u_t to represent the return on a market portfolio of risky assets. W_T is a natural representation for the total value of assets of the fund at time T. The choice for the static model excludes the possibility of policy changes between time 0 and T. Thus, the decision X_0^* gives an initial asset mix that is the optimal starting mix for a buy-and-hold strategy over the whole planning period. The static model is straightforward to analyze, and the solution is representative of the solution to the multistage model, see Siegmann and Lucas (2005).

We will refer to the model in (1) and (2) as the mean-shortfall model. However, an explicit feature of the model is that it represents the notion of *loss aversion.* Loss aversion in economics was introduced by Kahneman and Tversky (1979). The work of Kahneman and Tversky has become the starting point for most literature on behavioral finance, for a good overview see Hirshleifer (2001) or Shleifer (2000). The relevance for the present paper is that the downside-risk model is thus rooted in behavioral evidence on decision making under uncertainty. This can be added to the motivation based on the institutional setting of defined benefit pension funds. See also Haberman et al. (2003a) and Haberman et al. (2003b), who use the mean-shortfall model in a simulation model of optimal pension funding.

Our objective function is quite different from most existing literature on optimal pension funding. Boulier et al. (1995, 1996), Cairns (2000) and Owadally and Haberman (2004) minimize a quadratic penalty function on the contribution rate. Haberman and Sung(1994, 2002), and Vigna and Haberman (2001) minimize a quadratic penalty function on the deviation of funding and solvency values from their respective targets, punishing

unfavorable outcomes as well as favorable ones.

A limitation of the objective function in (1) is that it is one-dimensional, i.e., it is defined in terms of the single variable W_T. The real objective for a defined benefit fund, however, consists of several competing objectives. Typical objectives are: minimal contributions, maximal indexation of pensions, and minimal risk with respect to funding. With respect to the latter, the key ratio that is reported as a measure of financial soundness of a fund is the actuarial funding ratio. It is simply the value of assets divided by the present value of the liabilities, W_0/W_0^B. Another measure is the surplus, which can be defined as $W_0 - W_0^B$. Now, if the funding ratio (or surplus) is high, the risk of underfunding is low, the contribution level can be lowered, and indexation can fully compensate for inflation. Hence, in the simple model of pension fund investment, we use an ALM objective that is only defined in terms of the surplus of the fund, which has a 1-1 relation with the funding ratio. If the surplus is high, the other objectives are met at the same time. If the surplus is low, or if there is a deficit, the associated consequences are carried over to the other objectives, e.g., for a fund with a low funding level, contributions are raised and indexation might be postponed. See Boender and Vos (2000) for an analysis of the mechanism of allocating financial risks over multiple objectives for a pension fund, which they call risk-budgeting.

Our approach compares to that of Leibowitz et al. (1992), who model shortfall-aversion of pension funds by incorporating a shortfall constraint on the 'surplus return'. The usefulness of their approach is that they explicitly reckon with the duration of pension liabilities versus the duration of available fixed-income instruments. However, the problem with their setup is that a solution does not always exist, namely when the constraint cannot be met. In our case, using a penalty on shortfall, we can solve the optimal policy for a pension fund that currently experiences a deficit.

Pension liabilities

Interpreting W^B is key to the present paper. It is the reference level relative to which either a gain or a loss is measured. For a defined benefit pension fund, a natural candidate for the reference level is the *future* value of the liabilities that need to be covered at time

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T. Such a value is the present value of the pension rights built-up by the participants, and is obtained by discounting the future pension payments by an appropriate discount rate.

At this point, we need to discuss the role of inflation and indexation in the objective of the fund. There are several factors that influence the liabilities of a defined benefit pension fund, W^B in our model, such as the career developments of individual (active) members, and wage inflation. However, for simplicity we only focus on the most important factor: inflation[3] Of course, it is possible for a pension fund to only offer a nominal guaranteed pension, but it is clear that participants are foremost interested in the pension rights in terms of the *future* purchasing power that the pension provides. This is the reason that most collective and company pension plans in the Netherlands, and still a significant number of pension plans in the US, offer indexed pension schemes.

The aim of indexation is to compensate for inflation, which eats into the purchasing power of a pension dollar. That brings about a direct link between inflation and the *future* nominal level of the liabilities, W^B. Given a level of inflation over the planning period of π, and a level $W^{B_{cur}}$ representing the liabilities at today's prices/wages, W^B is given by

$$W^B = (1 + \pi) \cdot W^{B_{cur}}, \tag{3}$$

where $1 + \pi$ is the gross inflation factor, assuming annual compounding. The setup in (3) makes it easy to study the effect of inflation on the optimal investment policy. Also, because indexation is an explicit aim of DB pension plans, and given the extra complexity of adding another decision variable, i.e., indexation-cuts, we only consider unconditional indexation in this paper.

Planning horizon

We choose the length of the planning period equal to $T = 15$ years, intended to match the duration of the liabilities of an average pension fund. In an attempt to explain the equity premium puzzle (about the historical risk-adjusted return on stocks being too high from an equilibrium perspective), Benartzi and Thaler (1995) postulate that investors and

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[3]In practice, it makes a large difference whether pension funds consider price or wage inflation. However, for the current paper this difference is not relevant, so we will just use the term 'inflation' to refer to either price or wage inflation.

pension funds are loss averse and myopic, i.e., $T = 1$ year. They motivate the suggestion of myopia by pointing out that investors account for their gains and losses at the end of each year. However, from the outset myopia is unrealistic for pension funds. Pension fund managers frequently express their view that they are, and should be, long-term investors, and should not be pinned down on 1-year performance figures. This view is consistent with the practice of motivating investment policies by ALM studies that calculate risk and return over a long-term horizon.

Note that even with the horizon of 15 years we still analyze a static problem, since recourse actions at intermediate stages are not allowed, see Section 2. For details on the multi-stage setup and its solution, see Siegmann and Lucas (2005). For traditional utility functions of constant relative/absolute risk aversion, Campbell and Viceira (2002) give the conditions for which a multi-period investment problem reduces to a repeated single-period problem. However, their results do not carry over to the mean-downside risk framework. Also, inSiegmann and Lucas (2005) it is shown that it suffices to the single-period problem for analyzing the qualitative outcomes of the mean-shortfall model.

Shortfall risk

The objective function in (1) involves the maximization of wealth (or surplus), with a penalty on the expected amount of deficit below 100% actuarial funding. Loss aversion with respect to deficits below W^B represents that, in a defined benefit system, the liabilities need to be covered by available assets. The formulation in (1) allows for a deficit. The parameter λ represents the aversion to shortfall. In the context of pension funds, the extent of shortfall aversion is influenced by the fund's flexibility in a situation of underfunding. A fund with much flexibility with regard to jumps in the contribution rate, additional capital injections by the sponsor, lowering of pension rights, and skipping of indexation, might be less shortfall-averse than a fund that does not have such flexibility.

3   Results

This section presents the optimal investment policies for the DB pension fund that was presented in Section 2. Subsection 3.1 deals with the basic model in which inflation π, and thus W^B, is assumed constant. We take r_f such that the actuarial discount rate of

4% is correct. Subsection 3.2 specifies a joint probability distribution of inflation and the stock return. Subsection 3.3 presents the results for an alternative specification of downside risk, one which is often used in ALM studies.

3.1 Base case

In the most simple case, we assume that W^B, the inflation-protected level of liabilities at time T, is known with certainty at time 0, so inflation π is a known constant. As the planning horizon is 15 years, a reasonable assumption for the expectation of inflation is 2% per year, the inflation ceiling for the medium term of the European Central Bank (ECB). That makes $W^B = (1.02)^T \cdot W^{B_{cur}}$. Assuming that the current practice of using an actuarial discount rate of 4%, we set the riskfree return to an annual 6.1%, i.e., $r_f = 1.061$. With a fixed inflation rate, the only uncertainty in the model now comes from the yearly return on the risky asset u_t. We take u_t to have a lognormal probability distribution with an average return of 10% and standard deviation of 17%, representing typical historical figures for stock returns.

The analytical characterization of the solution to model (1) is presented in the appendix. However, finding the actual optimum still involves computing an integral. Therefore, to find the numerical optimum we resort to simply optimizing model (1) numerically with a standard solver, like the one available in Excel. This needs a concrete distribution of the 15-year return $\Pi_{t=1}^{15} u_t$, for which we use the discretized lognormal distribution mentioned above. Doing this for different values of the initial wealth W_0 results in optimal (wealth, investment) pairs (W_0, X_0^*). Figure 1 shows the resulting optimal fractions X_0^*/W_0 as a function of the funding ratio W_0/W_0^B for different values of the loss aversion parameter λ. Table 1 lists the values of λ and the associated probabilities of shortfall or surplus.

The numbers in Tabel 1 show that for a high-degree of shortfall-aversion, the probability of underfunding is very low, but so is the probability of becoming funded again when underfunded. A larger tolerance for shortfall creates a higher probability of underfunding, but also increases the odds that an underfunded pension fund can get out of underfunding. These results follow intuition and are a feature of the risk measure.
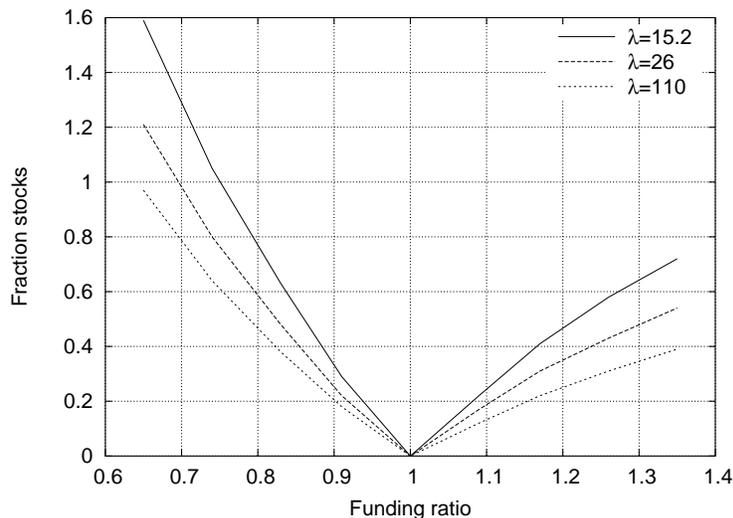
Figure 1: Optimal fraction invested in stocks for the base case

This figure shows the optimal investment in stock as a fraction of initial wealth against the funding ratio for different values of the loss aversion parameter λ. $T = 15$, and $u_t \sim lognorm(0.085, 0.16)$, which gives an average return of 10% and standard deviation of 17%.

Figure 1 shows that the optimal decision rules for the optimization problem have a typical V-shape: below 100% funding risk taking increases with shortfall, while above 100% funding risk taking increases with surplus. This behavior is closely linked to the

Table 1: The values of λ used to compute the optimal investment decisions in Figure 1

The second column lists the probability of underfunding, given a positive initial surplus. The third column gives the probability that the final surplus is positive, given a negative initial surplus.

λ	P(underfunding)	P(overfunding)
15.2	10%	16.8%
26	5%	11.8%
110	1%	7.8%

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mean-shortfall objective as formulated in (1). In a surplus situation, an increase in the funding ratio creates more room for risk taking for the same level of (expected) shortfall. Such a mechanism is also present in models with a traditional utility function, where the marginal utility of wealth is decreasing in wealth. However, a traditional approach cannot generate at the same time the left-hand side of the V-shape that we see in Figure 1. There, an initial shortfall position leads to more risk taking for a decreasing funding ratio. The intuition is that shortfall cannot be avoided with certainty, and thus a certain amount of risk taking is warranted, to decrease expected shortfall. To see that at least some risk taking under shortfall is optimal, consider the case with an infinite penalty on shortfall and no risk taking. Then, from an initial position of shortfall, taking a little risk decreases shortfall by an amount initially equal to the risk premium on the risky asset. Thus, the left-hand side does not correspond with a gambling-strategy. Models that result in such a strategy assume that an agent has nothing more to loose below a certain point, and thus gambles everything. In our model, however, an explicit penalty is given to shortfall, explicitly modeling that there is something to loose.

The figure also has a number of interesting consequences. First consider the shape of the optimal funding policy. For a funding ratio of 100%, the real pension liability is fully covered. In this case, the optimal allocation does not contain stocks and thus represents a minimum-risk portfolio. At a funding ratio of more than 100%, the surplus asset value leads to an increasingly higher fraction in stocks, using the risk premium on stock investment to make the fund even wealthier (in expectation). At funding ratios below 100%, stock investments increase in the extent of underfunding. When the pension liabilities cannot be guaranteed with certainty anymore, the fund has to take risk to prevent a further erosion of the pension claims. A higher percentage in stocks is the only way to reach the benchmark W^B with positive probability. In a real-world setting, an alternative to increasing X_0 can be an increased contribution from the side of the sponsor, or a more frugal pension claim by lowering indexation. Both alternatives can be visualized as having the effect of shifting the financial position of the fund to the right on the horizontal axis of Figure 1, either by increasing the numerator or by decreasing the denominator of the funding ratio.

Second, observe that the loss aversion parameter λ has a decreasing effect on the steepness of the V-shape in Figure 1. This corresponds to intuition: A higher loss aversion leads to a safer asset mix, as the certainty of the risk-free return becomes more attractive than the possible extra return on stocks. For the rest of this paper, we take $\lambda = 110$, representing a 1% probability of underfunding. It is important to note that an infinitely large λ does not make the left-hand side of the V-shape flat. In the appendix we show that the optimal strategy starting from a shortfall position for $\lambda \to \infty$ is a minimum-risk strategy with respect to expected shortfall, and always contains an investment in the risky asset (given a positive equity premium).

Third, pension funds usually use an ALM study to support an investment policy that is aimed at holding a "strategic mix". The figure shows, however, that the optimal initial asset mix for the 15 year period depends on the initial funding ratio. This suggests that holding the same mix whatever the funding ratio is not a good idea from an optimization point of view. The empirical results in Section 4 also show that the fraction of stocks is not constant for the funds examined.

So far, we have assumed a deterministic value for W^B. In the next subsection we consider the robustness of the shape of the decision rules to the introduction of an inflation rate that is uncertain.

3.2 Inflation uncertainty

The uncertainty surrounding future inflation is an import source of risk for a defined-benefit pension fund. Some developed countries have inflation protected bonds available, but for most countries inflation-linked bonds are just not available, or in any case not in the right quantities. In all, dealing with an uncertain inflation rate is relevant for most DB pension funds. Even if the market for inflation swaps becomes larger and more liquid, it remains to see whether pension funds would be inclined to bear the costs of protection given their large investment horizon.

With an uncertain inflation rate π, and the reference point W^B in the objective function (1) becomes a stochastic variable. Now, to have a simple and tractable modeling of the joint distribution of stock returns and inflation, we assume log-stock returns and
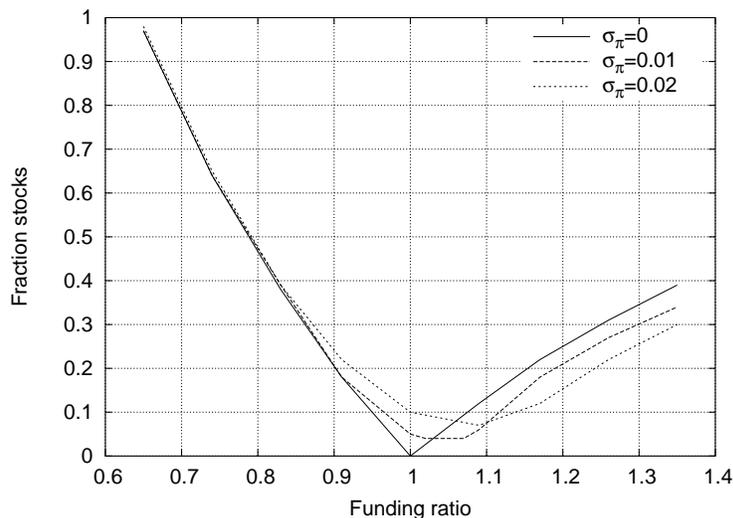
Figure 2: Optimal stock investment under inflation uncertainty.

In this figure the optimal stock fractions for model (1) are plotted for different values for σ_π, the standard deviation of the inflation. $\lambda = 110$, $T = 15$, $\rho = 0$, $u_t \sim lognorm(0.085, 0.16)$.

inflation are modeled as bivariate normal. This way, we can define the covariance matrix Σ of the log-series as

$$\Sigma = \begin{bmatrix} \sigma_\pi^2 & \rho \cdot \sigma_\pi \sigma_u \\ \rho \cdot \sigma_\pi \sigma_u & \sigma_u^2 \end{bmatrix}, \tag{4}$$

where ρ is the correlation coefficient between inflation π and the log-stock return u. For the stock return we take a mean of 10% and standard deviation of 17%. These correspond to a normal distribution with $\mu = 0.085$ and $\sigma = 0.16$ for the log-stock return. Mean inflation is set to 2% and $r_f = 1.061$. The following analysis will consider different values for the variance, and also, with sufficient controversy over the size of the correlation between inflation and stock returns the parameter ρ is varied in the analysis, starting from a value of zero. Results for varying values of the standard deviation σ_π of inflation are in Figure 2.

From Figure 2 we observe the same basic pattern as in Figure 1, i.e., the percentage in stocks has a V-shaped relation with the funding ratio. The difference with Figure 2 is that the kink of the V does not lie at a zero stock-investment. The minimum-risk
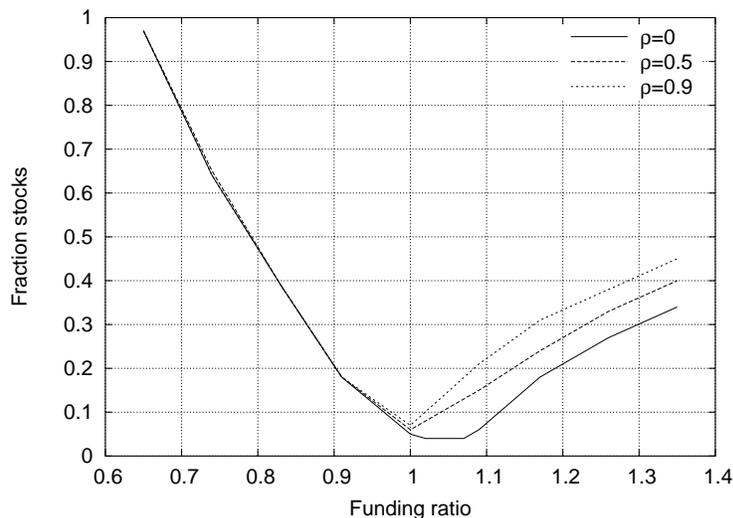
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Figure 3: Optimal stock investment for different correlation coefficients

In this figure the optimal stock fractions for model (1) are plotted for different values for ρ, the correlation coefficient between the log-stock return and inflation. $\sigma_\pi = 0.01$ $\lambda = 110$, $T = 15$, $u_t \sim lognorm(0.085, 0.16)$.

portfolio contains stocks. When inflation can only be hedged partially, the bottom line is that there is always uncertainty with respect to the level of the final pension payment.

Next, we consider the optimal solution to the loss averse model when the correlation between stocks and inflation is non-zero. In the long run stocks are considered a partial hedge for inflation risk. This is one of the motivations for investing in stocks in case of indexed pension claims, see Leibowitz et al. (1994). Randall and Satchell (1997) find a positive correlation for UK data of 0.16 between equity and paid-out pensions. However, for Dutch data from 1956 to 1994, Dert (1995) finds an annual negative correlation coefficient of -0.24 between log-stock returns and price inflation.

Figure 3 shows the effect of changes in the correlation ρ between stock returns and inflation. For a given funding ratio, a higher correlation results in a larger stock investment. The effect on the investment policy is only moderate, however. With a yearly correlation of 90%, which is very high, the maximum difference in fraction stocks compared to no correlation is 15 percentage points. It is surprising that this effect is so limited. As mentioned before: the presence of correlation between stock returns and inflation motivates

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stock investment as an inflation hedge. Given that the true correlation is suggested to lie between 0.1 and 0.5, this motivation seems to be of limited imporant in practice. The positive risk-premium of stocks is more important than the effect of a positive correlation with inflation.

3.3   Downside deviation

The mean-shortfall model was selected because of its simplicity, having an analytical characterization of the optimal solution. In practice, however, ALM for pension funds predominantly uses downside deviation as the risk measure. Therefore, we now explore the sensitivity of the outcomes to taking downside deviation. The analytical outcome for the case of a fixed benchmark is studied in Siegmann and Lucas (2005).

Using downside deviation as a risk measure boils down to taking quadratic shortfall. The popularity of its use is due to the fact that it punishes large losses more than pro-portionally heavy than small losses. In analogy to the formulation of the mean-shortfall problem in equations (1) and (2), the formulation of the objective becomes,

$$\max_{X_0} \quad \mathbb{E}[W_T] - \nu \cdot \mathbb{E}([(W^B - W_T)^+])^2, \tag{5}$$

$$\text{s.t.} \quad W_T = (W_0 - X_0)r_f^T + X_0 \cdot \prod_{t=1}^{T} u_t, \tag{6}$$

with the parameters defined as before, and $T = 15$. The difference with the objective function (1) is the power of 2 in the second term of (5), as well as a different loss aversion parameter, ν, representing the change in risk measure. Figure 4 shows 3 panels with the results of the sensitivity analyses of the previous section, but now for downside-deviation as the risk measure.

Starting with the top left panel and comparing with Figure 1, we recognize an effect of the V-shape moving in the upper-left direction. Also, while the left-hand side of the V more or less stays at the same position, the right-hand side is much more sensitive to values of ν. In the absence of inflation uncertainty, Siegmann and Lucas (2005) provide analytical results for the solution under a downside deviation risk measure.

The top-right, and lower-left panel show the effect of uncertainty in inflation on the outcomes. We find that σ_π and ρ have the same effect on the decision rule as in the mean-
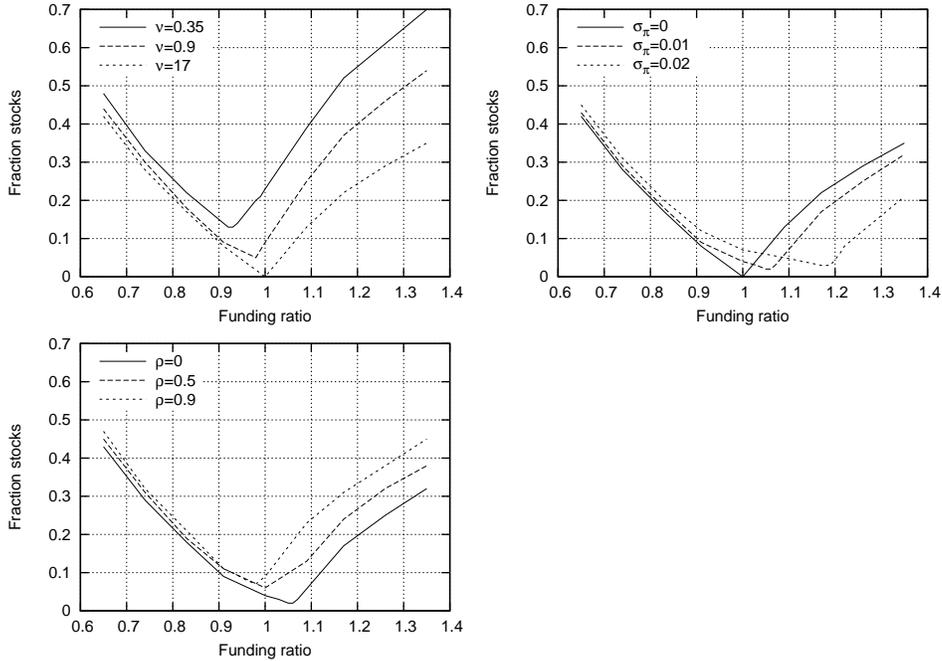
Figure 4: Optimal policies for downside-deviation objective

Top left panel shows the optimal stock investment fraction for different values of the loss aversion parameter ν, inflation constant at 0.02, $\rho = 0$, $u \sim logN(0.085, 0.16)$, $r_f = 1.061$. The values of ν have been chosen to roughly compare with the 1%, 5%, and 10% probabilities of underfunding, as in Figure 1. For $\nu = 8$, the top right panel shows the solution for different values of the standard deviation of inflation, as in Figure 2. The lower left panel has different values for the correlation between inflation and log-stock returns, ρ, while σ_π is kept at 0.01, compare Figure 3.

shortfall model. Increasing inflation uncertainty makes the minimum-risk point of the V-shape shift to the right. Increasing correlation between inflation and stock returns, the minimum stock allocation increases, as does the fraction of stocks for positive surpluses. As in Figure 3, the left-hand side of the V is not affected significantly by a positive correlation.

4   Empirical evidence

The previous sections have shown the specific investment policies that are optimal in the mean-shortfall framework representing a DB pension fund. To test if the framework is indeed representative of a DB fund, the strategies that are followed in reality should be

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Table 2: Six large Dutch pension funds

This table presents assets and funding ratios for six of the largest Dutch pension funds. Data is collected from annual reports and data from the Vereniging van Bedrijfstakpensioenfondsen (VVB).

Fund	Asset value(bln E)	Funding ratio ultimo 2001
ABP	147.3	1.12
Shell	12.2	1.39
PGGM	49.1	1.12
Philips	14.7	1.28
BPMT	17.5	1.18
SPF	9.9	1.45

comparable with our results. This section provides a first reality-check of the results, by analyzing historical investment policies followed by six of the largest Dutch pension funds.

Table 2 shows the asset values and funding ratios of the six pension funds. ABP is the pension fund for all Dutch government personnel, and the largest in terms of asset value in The Netherlands (and second in the world). The Shell pension fund is for employees of Royal Dutch Shell. PGGM is the fund for workers in the health sector. Philips is obviously for Philips workers. BPMT is the fund for people working in the steel and technical industries. Finally, SPF is the fund that covers the pension for workers at one of the companies associated with the Dutch railways. In total, these funds represent 58% of the total of pension assets in The Netherlands of 435 billion Euro in 2001.

Ultimo 2001, all six funds in Table 2 are overfunded, i.e., they have a funding ratio larger than 100%. However, an expected real interest rate lower than the actuarial rate used of 4% shifts the location of the kink of the V-shape to the right. Given a current interest rate that is far lower than 4% plus inflation, a V-shaped optimal policy would thus be located to the right of a 100% actuarial funding ratio based on 4% discounting. Put otherwise, a funding ratio of, for example, 110% need not be enough to protect the future value of the pension rights in a risk free manner. With this in mind, Figure 5 presents the scatter plots of the funding ratio versus fractional stock investment for the six pension funds listed in Table 2. Table 3 lists annual returns for representative stock

Table 3: Asset returns 1997-2001

This table gives the yearly stock returns as measured by the MSCI
world index, measured in local currency. Bonds is the Lehman
Brothers Aggregate Bond Index return. Interest is the 1-Year
Treasury Constant Maturity Rate

year	MSCI	Bonds	Interest
1997	20.8%	10.2%	5.6%
1998	19.2%	8.6%	5.1%
1999	26.3%	-2.0%	5.1%
2000	-10.8%	9.4%	6.1%
2001	-15.3%	10.4%	3.5%

and bond indices.

As can be seen in Table 3, the years 1997 to 2000 have witnessed an enormous surge
in stock prices, followed by a decline in 2000 and 2001. Bond returns have shown the
exact opposite patterns, while one-year interest rates have been fairly stable. For pension
funds, the variability of liability growth is very small compared to that of stock returns.
Therefore, the movement of the funding ratios in Figure 5 is for the most part determined
by the investment returns. For all funds we see the same relative movement of the funding
ratio: increasing from 1997 to 1999, decreasing from 1999 to 2001. The only exceptions
are PGGM and BPMT, who show a slight decrease of the funding ratio over the years
1997-1998. Note that stock investments do not have to be the only risky investments
that pension funds have. Several funds invest a significant part of their assets, i.e., more
than 5%, in risky investments such as private equity or real-estate. However, the fractions
invested in those categories were either too small or relatively stable over time, so they
are not considered in the analysis.

Ultimo 2001, the funds differ considerably in the fraction of stocks they have. ABP
and BPMT have around 40% stocks, with funding ratios of 1.1 and 1.2, respectively.
Philips and SPF have 55%, with funding ratios of 1.3 and 1.45. PGGM and Shell have
a high 70% stock investment, and funding ratios of 1.1 and 1.4. If we look at the three
pairs of funds with comparable stock investments, PGGM stands out as having one of the
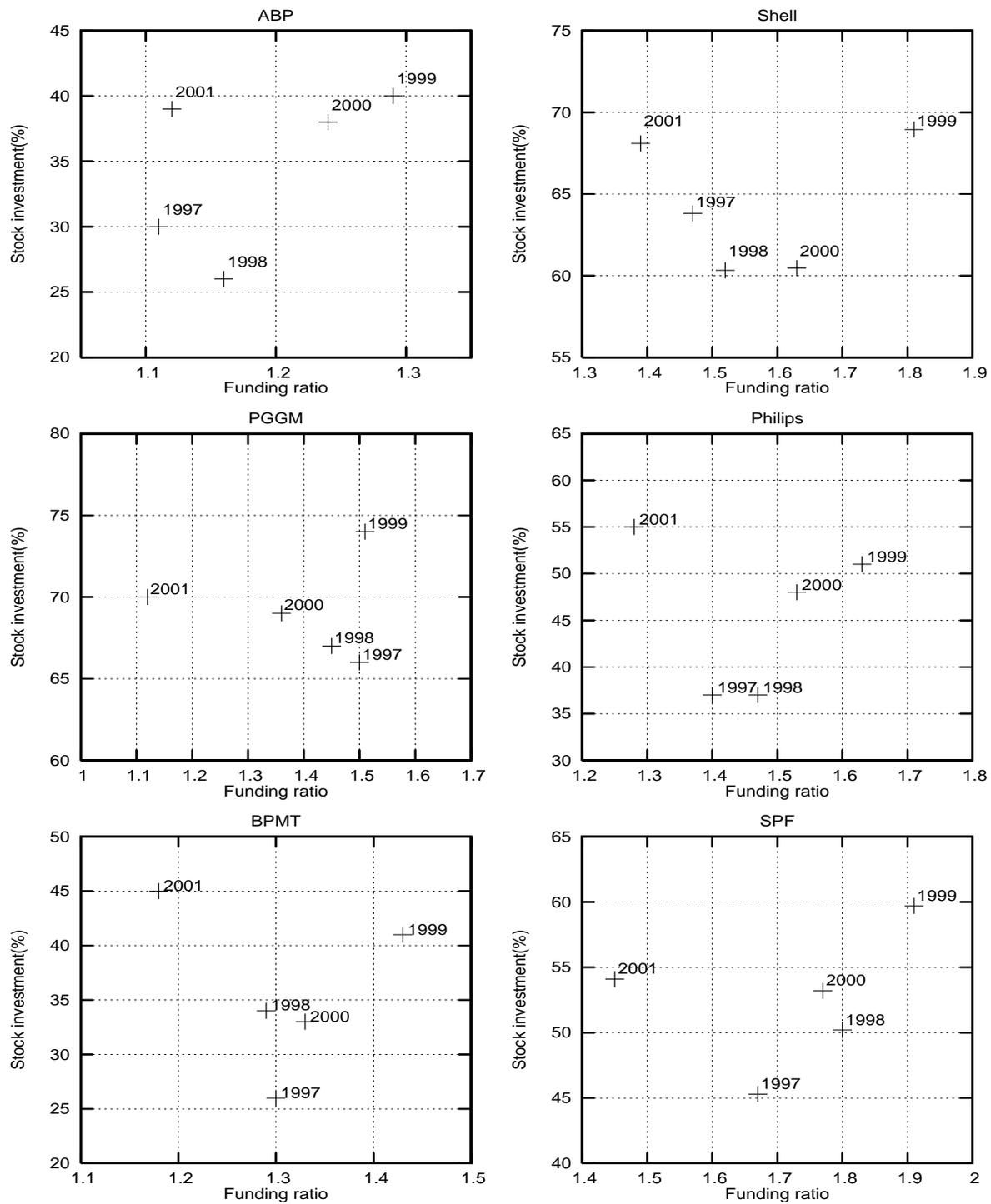lowest funding ratios and with the highest stock investment. It also has the most stable

Figure 5: Stock investment in relation to reported funding ratios

This figure shows funding ratios and fractions invested in stocks for six large Dutch pension funds. Data is from annual reports of the respective funds, Vereniging van Bedrijfstakpensioenfondsen(VVB). Where necessary, liabilities have been computed through (linear) extrapolation.

fraction of stocks over time, so it might be the result of taking a long-term calculated risk, rather than a double-or-nothing policy.

Turning to the investment patterns present in Figure 5, we see a diffuse behavior for the first period 1997-1998. Stock returns were very high in this year, but pension funds do not show a shared opinion on whether to increase or decrease the fraction of stocks. In the top panels, ABP and Shell had an increasing funding ratio, and decreasing stock fraction. The middle two panels show funding ratios and stock fractions with only little variation. The lower two panels show an increasing stock fraction with either a stable or increasing funding ratio. Clearly, investment policies differed to a large extent during the year 1997.

From 1998 onward, however, identical patterns emerge. At the end of 1999 the fraction of stocks has increased with the funding ratio, and has decreased again at the end of 2000. From 2000 to 2001, the funding ratios decrease further, but now the stock investments increase. The extent of the stock increase in the last year is only small for ABP, PGGM, and SPF, but given the large decreases in funding ratios (see the large negative returns on stocks), it represents a significant investment decision for all funds. The pattern can partly be explained by funds following a fixed-mix strategy. For four of the six panels, namely ABP, PGGM, Philips and SPF, one could imagine that the observed stock investments are the result of a fixed-mix strategy, where rebalancing occurs at the beginning of each year. For SPF, ABP, and Philips, though, we need to add the assumption that the strategic asset mix has changed after 1997 or 1998. For two of the six funds, namely Shell and BPMT, it is difficult to see how a fixed-mix policy would lead to the observed patterns. Over the period 1998-2001, both funds show a series of consecutive increasing and decreasing stock fractions of more than 7%-points.

Detecting V-shapes in the graphs asks to much from the current simple setup, although a formal test of loss-averse behavior could be accomplished with a more extensive dataset. Also, in assessing the figures, one needs to take into account the fact that funds differ not only in their funding ratio, but also in the composition of the members, financial position of the sponsor, and expectation of the economic fundamentals. This also motivates the

choice for different axes for each fund.

The relevance of the mean-shortfall results is illustrated in the situation of ABP and PGGM, which are well known for their sheer size. In newspaper interviews the managers of these funds indicate that they need a fraction of stock investments to fund the expected high liability growth. Thus, in the current model they can be seen as typical 'left-siders' in the terms of the found V-shape in this paper. Although the funds have a nominal surplus of around 10% ultimo 2001, their financial position in real terms (i.e., taking into account indexation) is one of shortfall, where risky investments are needed. This is well understood in the mean-shortfall analysis of this paper.

Clearly, the preliminary analysis of Figure 5 can at best be only suggestive about the validity of the mean-shortfall model for defined benefit pension funds. The main puzzle boils down to explaining the non-decrease in the fraction of stocks in the portfolio from 2000 to 2001, despite two consecutive large negative stock returns. So far, it appears that the mean-shortfall framework is helpful in explaining part of this puzzle.

5   Policy implications

The choice for the mean-shortfall model is rooted in the institutional setting of defined-benefit pension funds as well as actual preferences used in ALM studies. As such, the found results in this paper can be seen as indicative of the drivers behind investment policies of defined-benefit pension funds. With respect to the optimal V-shaped policy, at first sight it is not in accordance with current regulatory practice in for example the Netherlands. There, regulators require funds to always remain above 100% funding, which makes the interpretation of the left-hand side of the V-shape all the more interesting. An interpretation of the results should take into account (i) the possible interpretations of present value benchmark W_0^B for pension funds, and (ii) the reality of the fact that underfunding *can* happen.

To start with the first point, W_0^B was defined as the discounted value of the future nominal liability W^B with full indexation. Hence, the kink of the V-shape represents a

100% funding ratio when all future indexation is taken into account. However, in practice pension funds report a funding ratio where liabilities are discounted at current prices, not accounting for indexation explicitly. In the current model with 2% inflation, such a funding ratio should thus be as high as $1.02^{15} \approx 1.35$ to ensure indexed liabilities by only investing in risk-free instruments yielding 4%. Considering that most pension funds have funding ratios below 135%, the results in this paper suggest that the left-hand side of the V-shape is the relevant area under normal conditions. I.e., the more a fund approaches the level of 135% funding ratio, the less it will invest in stocks.

Secondly, the past few years with historically low stock returns have shown that pension funds do get in a situation of underfunding. This paper shows that under mean-shortfall preferences funds have a tendency to stay in stocks, even when funding ratios are low. The intuition provided is that a risky investment is necessary to ensure enough upside potential. Taking no risk at all might prove to give a larger downside risk than taking calculated risks in the investment policy. For regulators, the good news is that the results imply that pension funds would not want to sell their stocks when returns are disappointing. Given the size of pension funds, such behavior can have an undesirable effect on the stock market. Although the immediate impact of a single large trade on stock prices in any well-developed market is small, the combined effect of multiple pension funds selling stocks at the same time can be significant. The bad news is that the results show a tendency to increase risk in a worsening situation. And that is exactly the sort of behavior that regulators would want to prohibit.

6 Conclusions

In this paper I have introduced a simple mean-shortfall model for the investment decision of a defined benefit(DB) pension fund. Shortfall was measured relative to the real (indexed) value of the liabilities. A V-shaped policy was found to be the optimal investment strategy as a function of the initial funding ratio. This implies increased risk-taking for increasing initial shortfall. The sensitivity of the optimal investment strategy has been explored for changes in the economic assumptions on inflation, the correlation between stock returns and inflation, and the level of the real interest rate. Results for a different

risk measure, namely downside-deviation were also obtained, closely resembling those of the mean-shortfall model. The results have in common that the optimal investment policies all have a typical V- or U-shaped relation with the initial wealth of the fund. A lower risk free interest rate and higher inflation uncertainty both move the optimal policy to the right, i.e., the minimum-risk allocation is attained at higher wealth levels.

Section 4 considered patterns of portfolio adjustment versus funding ratios for six large Dutch pension funds. It is shown that the mean-shortfall model might explain the increasing stock fractions for funds that experienced a decreasing funding ratio.

Appendix: Analytical solution to the basic model

For clarity, here we present the solution to a 1-period model. The solution to the 15-year model is the same with the return distribution replaced by its 15-year equivalent.

Consider the one-period mean-shortfall model given by

$$\max_{X_0} \quad \mathbb{E}[W_1] - \lambda \cdot \mathbb{E}[(W^B - W_1)^+], \tag{A1}$$

$$\text{s.t.} \quad W_1 = W_0 r_f + X_0 \cdot (u_1 - r_f), \tag{A2}$$

Define $\bar{u}$ as the return for which $W_1 = W^B$. It is given by

$$\bar{u} = \frac{W^B - W_0 r_f}{X_0} + r_f. \tag{A3}$$

By definition, $\bar{u}$ gives the return value below which there is shortfall. I.e., $W_1 < W^B$ iff $u_1 < \bar{u}$. This way, we can write equation (A1) as

$$\max_{X_0} \quad \mathbb{E}[W_1] - \lambda \cdot \int_0^{\bar{u}} (W^B - W_1) dG, \tag{A4}$$

where $G(\cdot)$ is the distribution function of u_1.

To arrive at the first-order condition we take the first derivative of (A4), which is simplified by the fact that the integrand is zero when filling in $\bar{u}$ for u_1 (using Leibniz' rule for differentiating an integral). The resulting first-order condition to (A1) is given by

$$\int_0^{\bar{u}} (r_f - u_1) dG = \mathbb{E}[u_1 - r_f]/\lambda, \tag{A5}$$

Assuming that $G(\cdot)$ is unimodal, equation (A5) has two solutions in terms of $\bar{u}$, one for a positive and one for a negative surplus. We write them as $\bar{u}^+$ and $\bar{u}^-$, respectively.

Using the second-order conditions, it follows that the optimal solution is given by

$$X_0^* = \frac{r_f}{r_f - \bar{u}^*} S_0, \tag{A6}$$

where $\bar{u}^* = \bar{u}^+ < r_f$ for a positive surplus, and $\bar{u}^* = \bar{u}^- > r_f$ for a negative surplus. Hence, X_0^* has a V-shaped relation with the value of W_0. This completes the solution. ∎

To show that an infinitely high λ does *not* lead to zero risk taking when in underfunding, consider the following. For $\lambda \to \infty$, the first-order condition in (A5) simplifies to

$$\int_0^{\bar{u}} (r_f - u_1) dG = 0, \tag{A7}$$

which has solutions $\bar{u}^+ = 0$ and $\bar{u}^- > r_f$. So for positive surpluses, the optimal solution is degenerate and has zero investment in the risky asset. For a negative surplus, the solution is given by

$$X_0^m = \frac{r_f}{r_f - \bar{u}^m} S_0, \tag{A8}$$

where $\bar{u}^m$ is the $\bar{u} > r_f$ that solves (A7). This proofs the fact that the overall minimum-risk optimal investment in the risky asset, X_0^m, is strictly positive. ∎

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