Teaching Note – Was the Champions League Draw Rigged?

Henk Tijms, Vrije University, Amsterdam

This teaching note gives a real-life example of Bayesian thinking. It discusses how credible accusations are that the outcome of the draw for the quarter-finals in the 2013 European Champions League football was manipulated.

1. Introduction

Football (“soccer”) is the most popular sport in the world, particularly in Europe, South-America, Africa and Asia. The most watched tournament is the UEFA Champions League, UEFA being the Union of European Football Associations. The UEFA Champions League is also the most-revenue generating tournament in football. Professional football has been troubled by multiple scandals in the past few years, including accusations of corruption and match-fixing. Recently, the UEFA fell again under a cloud of suspicion. The 2013 Champions League draw ceremony for the quarter-finals resulted in the following four matches:

- Málaga – Borussia Dortmund
- Real Madrid – Galatasaray
- Paris Saint Germain – Barcelona
- Bayern München – Juventus.

The outcome of the quarter-finals draw led to heated discussions in European sports programs on television and radio. Several football commentators accused the UEFA of manipulation in order to make possible the commercially most interesting semi-finals and final. Not surprisingly, in the light of earlier supposed UEFA draw scandals. The most explicit accusation of a rigged draw came from a former international football referee, see www.dailymail.co.uk/sport/football/article-2297502. The football commentators found it quite remarkable that the Big Four of the eight teams avoid each other in the quarter finals. The Big Four are the two Spanish teams Barcelona and Real Madrid and the two German teams Bayern München and Borussia Dortmund. What are the chances that this particular quarter-finals draw is rigged? A subjective answer to this question can be given by using Bayesian reasoning. This is done under the following basic assumption. If the UEFA Champions League draw of the quarter-finals 2013 draw would be rigged, then this would manifest itself in a match schedule in which the Big Four avoid each other.
2. Bayesian analysis

To answer the question whether the draw was rigged, let us first calculate the probability that the Big Four avoid each other when it is assumed that the eight teams are paired randomly. As a prelude to the calculation of this probability, consider the following probability model. A bowl contains four marked balls and four unmarked balls. Four times you pick at random two balls from the bowl without replacement. What is the probability that you pick each time a marked ball and an unmarked ball? Defining \( A_i \) as the event that the \( i \)th pick gives a marked ball and an unmarked ball, this probability is given by 

\[
P(A_1A_2A_3A_4) = \frac{4}{7} \times \frac{3}{5} \times \frac{2}{3} \times 1 \times \frac{1}{3}.
\]

This gives the value \( \frac{4}{7} \times \frac{3}{5} \times \frac{2}{3} \times 1 = \frac{8}{35} \).

This probability is not exceptionally small and therefore frequentists may argue that the result of the quarter-finals draw is no surprise when taking into account that there are many soccer tournament draw ceremonies. However, the discussion is not about many tournament draw ceremonies, but about a particular soccer tournament ceremony for which there is reasonable ground to believe beforehand that the draw could be manipulated. In this situation it is appropriate to use the Bayesian approach. Bayesian analysis requires that before the draw takes place, the sport journalist quantifies his personal belief that the draw will be manipulated. Once the prior has been specified, the rest is unambiguously determined. Suppose the sport journalist believes that the prior probability of a manipulated draw is at least 20%. Then, a generally valid rule from the theory of probability implies that for the sport journalist his revised personal belief in a rigged draw is expressed by a probability of at least 52.2% after he hearing the result of the draw. The easiest way to calculate this posterior probability is to use Bayes’ rule in odds form, being the most simple simple and insightful representation of the rule of Bayes:

\[
\frac{P(H | E)}{P(\overline{H} | E)} = \frac{P(H)}{P(\overline{H})} \times \frac{P(E | H)}{P(E | \overline{H})}.
\]

In our example the hypothesis \( H \) is the event that the draw is manipulated, \( \overline{H} \) is the complement event that the teams are paired at random, and the evidence \( E \) is the event that the draw yields a match schedule in which the Big Four avoid each other. It holds that \( P(E | H) = 1 \), by the assumption made above that the only way a rigged draw can manifest itself is through a match schedule in which the Big Four avoid each other. As shown above,
\( P(E \mid \overline{H}) = \frac{8}{35} \). Hence the likelihood ratio \( P(E \mid H)/P(E \mid \overline{H}) \) equals \( \frac{35}{8} \). Suppose the sport journalist has a prior probability of \( r\% \) that the draw will be manipulated. In other words, the prior odds \( P(H)/P(\overline{H}) \) is \( \frac{r/100}{1-r/100} \). This leads to the posterior odds

\[
\frac{P(H \mid E)}{P(\overline{H} \mid E)} = \frac{r}{100-r} \times \frac{35}{8}.
\]

Since \( P(\overline{H} \mid E) = 1 - P(H \mid E) \), it follows that the journalist’s posterior probability of a manipulated draw ceremony is given by

\[
P(H \mid E) = \frac{35r/8}{100 - r + 35r/8}.
\]

This posterior probability is 0.5224 if the prior probability of the sport journalist is given by \( r = 20\% \). The Bayesian analysis shows that the suspicion voiced in the sports programs after the announcement of the result of the 2013 Champions League quarter-finals draw is certainly not unwarranted.

### 3. Conclusion

This note stresses the importance of Bayesian thinking which is an indispensable part of statistical reasoning. Students should be more trained to think in the Bayesian way. Introductory probability courses should give greater recognition to the probabilistic ideas of Bayesian thinking and show that Bayes’ rule is the rational basis for answering many probabilistic questions from real life. The leading textbooks for introductory probability courses badly fail in the attention paid to the Bayesian approach. In my own introductory probability book *Understanding Probability* (Cambridge University Press, third edition, 2012) the Bayesian approach is advocated and is illustrated with several real-life applications.