Lecture notes Schrijver: Answers to Exercises

The answers are usually not written out completely.

1.5. $2^{n(n-1)/2}$

1.6. Make a cycle through all vertices and add a perfect matching.

1.7 and 1.8. The number of cycles on $V = \{1, 2, \ldots, n\}$ is $\frac{1}{2}(n-1)!$.

1.11. The total degree is 80, hence the number of edges is 40.

1.12. $\frac{nk}{2}$.

1.13. No: the total degree would be odd.

1.15. $\frac{1}{2}\sum_{i=1}^{n} d_i$.

1.17. The solution of $3x + 5(14 - x) = 50$: $x = 10$.

1.18. 5.

1.20. The Peterson graph (find a drawing on wikipedia or google it).

1.23. $\frac{1}{2}n(n-1)$; i.e., $\left(\begin{array}{c} n \\ 2 \end{array} \right)$.

1.25. mn.

1.26. $m = n$.

1.27. $\frac{1}{2}2^n$; the number of ways to select U from V, taking care not to count U and $V \setminus U$ as two different graphs.

1.29. Yes, of degree $n - k - 1$.

1.30. $3, 2, 2, 1, 1, 1$.

1.31. The complement consists of two K_4.

1.39. 6.

1.42. $\sum_{i=2}^{n} \frac{(n-2)!}{(n-i)!}$.

1.43. Let $\{s, v_1, \ldots, v_k, t\}$ the walk from s to t. If no i and j exist such that $v_i = v_j$ on the walk, then the walk is in fact a path. If $v_i = v_j$ $(i < j)$, then $\{s, v_1, \ldots, v_{i-1}, v_i = v_j, v_{j+1}, \ldots, v_k, t\}$ is also a walk from s to t with one duplicated vertex less. Repeating this till no duplicated vertex is left gives a s-t path. □