Measuring Synchronisation and Convergence of Business Cycles

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we have GDP series from different economies in the Euro area and that of the U.S.;
these original integrated time series are band-pass filtered;
the resulting cyclical time series can be independent, correlated or common: we follow a model-based approach to assess the mutual dependence of cycles;
model allows that cycles may be lagging or leading each other;
• we introduce mechanisms in model for increasing or diminishing phase shifts and for time-varying association patterns in different cycles;

• using bivariate analyses, we find an increasing resemblance between the business cycle fluctuations within the Euro area. We also investigate Eurozone, U.K. and U.S. cyclical relationships;
Some introductory remarks

Comparisons of business cycle comovements are usually static, based on ad-hoc sub-samples using mostly non-parametric statistics.

We do model-based that avoids many of the shortcomings. Time-varying phase shifts and degrees of association are explicitly modelled. Regime switches are estimated simultaneously within a multivariate model. Transition to different regimes will take place smoothly.
Main motivation is the analysis of business cycles relations within the Euro area. Currency and common monetary policies in the Euro area questions whether resemblances of business cycles of the participant countries exist.

We document degree of association and synchronisation between the aggregate Euro area and individual European countries and U.S.

Results are mostly in line with those from similar studies: there is an increasing resemblance (higher degree of association and higher synchronisation) within Europe. U.S. is leading Europe.
Literature overview

- (Artis & Zhang 1997): using two subsamples and HP filter, ERM countries became more synchronised with the German one;
- (Angeloni & Dedola 1999): using subsamples and correlations of HP filtered economic indicators, 1993-1997 correlations are almost always higher;
- (Bayoumi & Eichengreen 1993): VAR analysis, no evidence was found on higher degrees in more recent period;
• (Belo 2001): using HP filter, confirming Wynne and Koo, there was an increase in the various measures of association and a leading cycle from the U.S. and the U.K. when compared to the Euro area.

Further, there has been a renewed interest in classical cycle analysis, spurred by (Burns & Mitchell 1946), mainly focusing on Markov-switching (vector) autoregressions (MS-VAR) approaches, using extensions of (Hamilton 1989): see (Diebold & Rudebusch 1996), (Krolzig & Toro 2002), (Harding & Pagan 1999)) and (Artis, Krolzig & Toro 2002).
Contribution of paper

We adopt the multivariate unobserved components model of (Harvey & Koopman 1997) for cycles. Phase shifts are introduced by (Rünstler 2002).

We incorporate mechanisms that model either increasing or diminishing phase shifts as well as mechanisms that model time-varying association patterns in the cyclical components.

Regime switches may appear as limiting cases. Time points of transition are estimated, not imposed by the researcher.

Focus is on business cycle, defined by (Lucas 1977). To obtain cyclical data we use best performing bandpass filter of (Christiano & Fitzgerald 2003).
Similar stochastic cycles

The stochastic cycle vector $\psi_t$ is modelled as

\[
\begin{bmatrix}
\psi_{t+1} \\
\psi^+_{t+1}
\end{bmatrix} = \phi \begin{bmatrix}
\cos(\lambda)I_N & \sin(\lambda)I_N \\
-\sin(\lambda)I_N & \cos(\lambda)I_N
\end{bmatrix} \begin{bmatrix}
\psi_t \\
\psi^+_t
\end{bmatrix} + \begin{bmatrix}
\kappa_t \\
\kappa^+_t
\end{bmatrix}
\]

The autocovariance function for $\psi_t$:

\[
\Gamma(\tau) = (1 - \phi^2)^{-1} \phi^\tau \cos(\tau \lambda) \Sigma_\kappa, \quad \tau = 0, 1, 2, \ldots,
\]

from which it follows that the variance matrix of the cycle is given by $\Gamma(0) = (1 - \phi^2)^{-1} \Sigma_\kappa$. 
Time series $y_t$ is modelled by

$$y_t = \mu + \psi_t + \varepsilon_t, \quad \varepsilon_t \sim \mathcal{N}(0, \Sigma_{\varepsilon}),$$

where $y_t$ is a $N \times 1$ vector of time series.

The cycle disturbance variance matrix is specified as

$$\Sigma_{\kappa} = CRC,$$

where matrix $C$ is diagonal and $R$ is correlation matrix, that is

$$C = \text{diag}\{\exp(\theta_{\kappa,1}), \ldots, \exp(\theta_{\kappa,N})\},$$
A possible specification is

\[
R = \begin{bmatrix}
1 & \rho_{\kappa,2,1} & \cdots & \rho_{\kappa,N,1} \\
\rho_{\kappa,2,1} & 1 & \cdots & \rho_{\kappa,N,2} \\
\vdots & \vdots & \ddots & \vdots \\
\rho_{\kappa,N,1} & \rho_{\kappa,N,2} & \cdots & 1
\end{bmatrix},
\]

and \( \theta_{\kappa,i} \) is log standard deviation.

A possible specification is

\[
R = K^{-1}LL'K^{-1}, \quad K = [\text{diagonal}(LL')]^{1/2},
\]

where \( L \) is a lower unity triangular.
Multivariate stochastic cycle model with shifts

(Rünstler 2002) generalises model to

\[ y_t = \mu + \text{diag}\{\cos(\lambda d_\xi)\}\psi_t + \text{diag}\{\sin(\lambda d_\xi)\}\psi_t^+ + \varepsilon_t, \]

where \( d_\xi \) is the real vector

\[ d_\xi = (\xi_1, \ldots, \xi_N)', \]

with its first element restricted to be equal to zero, that is \( \xi_1 = 0 \).

The variance of the cycle component is given by

\[ \Gamma(0) = \frac{1}{1 - \phi^2} \Sigma_{\kappa} \otimes \cos(\Lambda), \quad \Lambda = \lambda (1 d_\xi' - d_\xi 1'), \]
and the (multivarariate) autocovariance function is given by

\[
\Gamma(\tau) = \frac{\phi}{1 - \phi^2} \sum_\kappa \otimes \cos(\Lambda_\tau), \quad \Lambda_\tau = \lambda(\tau 11' + 1 d'_\xi - d\xi 1').
\]

When \( \Sigma_\kappa \) is diagonal, \( \Gamma(\tau) \) is also diagonal and does not depend on \( d\xi \) since the leading diagonal of matrix \( d\xi 1' - 1 d'_\xi \) is zero. In this case phase shifts are not identifiable.
For estimation purposes the shift element $\xi_i$ is transformed as

$$
\xi_i = \frac{\pi}{\lambda} \left[ \exp \theta_{\xi,i} (1 + \exp \theta_{\xi,i})^{-1} - 0.5 \right],
$$

that ensures $-\pi/2 < \lambda \xi_i < \pi/2$, $i = 2, \ldots, N$. Also, $\theta_{\xi,i} = 0$ implies $\xi_i = 0$. 
Synchronisation of multiple cycles

Time-variation of phase shift is logit:

\[
\xi_{i,t} = \frac{\pi}{\lambda}\left\{\exp \theta_{\xi_i} \left(1 + \exp \theta_{\xi_i}\right)^{-1} - 0.5\right\} \times \\
\exp(s_{\xi_i,t})\left\{1 + \exp(s_{\xi_i,t})\right\}^{-1},
\]

\[
s_{\xi_i,t} = s_{\xi_i} \times (t - \tau_{\xi_i}),
\]

for \(i = 2, \ldots, N\) and where \(s_{\xi_i}\) determines the shape of the logit function and \(\tau_{\xi_i}\) determines the mid-time position of the change. The parameters \(\theta_{\xi_i}, s_{\xi_i}\) and \(\tau_{\xi_i}\) are estimated by maximum likelihood.
Similar logit mechanisms are used for nonlinear smooth transition autoregressive models, see (van Dijk, Terasvirta & Franses 2002). The acf of cycle also depends on time since $\Lambda_\tau$ is time-varying:

$$\Lambda_{\tau,t} = \lambda(\tau 11' + 1d_{\xi,t}' - d_{\xi,t}1'),$$

$$d_{\xi,t} = (0, \xi_{2,t}, \ldots, \xi_{N,t})'.$$

This also applies to variance matrix $\Gamma(0)$. 
Convergence of multiple cycles

In bivariate case, correlation between two cycles is specified as

$$\rho_{\kappa,2,1} = \pm \left[1 - (1 - b) \times \exp(s_{\kappa,2,t}) \{1 + \exp(s_{\kappa,2,t})\}^{-1}\right],$$

$$s_{\kappa,2,1,t} = s_{\kappa,2,1} \times (t - \tau_{\kappa,2,1}),$$

for $i, j = 1, \ldots, N$, $i \neq j$, where coefficient $s_{\kappa,i,j}$ determines the shape of the function and $\tau_{\kappa,i,j}$ the midtime-point at which the transition takes place.

Coefficient $b$ adds further flexibility: ensures that correlation is between $b$ and one. For the purpose of estimation $b$ is specified as $b = [1 + \exp(\theta_b)]^{-1}$ where $\theta_b$ is an unknown coefficient.
Illustrations

We define business cycles as fluctuations within a range of periodicities (corresponding to a range of frequencies in the frequency domain). Finite sample ideal filter is not possible. We use approximation proposed by (Christiano & Fitzgerald 2003) that minimises, for each $t$, $Q$ wrt weights:

$$Q = \int_{-\pi}^{\pi} \left| W(\omega) - B_t(e^{-i\omega}) \right|^2 f_x(\omega) d\omega,$$

with $W(\omega)$ as freq response of ideal filter.
The freq response function $B_t(e^{-i\omega})$ is for filter

$$B_t(L) = \sum_{j=-(T-t)}^{t-1} b_{t,j}L^j,$$

with weights $b_{t,j}$. 


Empirical results for the Euro area

We consider band-pass filtered time series of (logs of) real, seasonally adjusted GDP for six European Union countries (Germany, France, Italy, Spain, The Netherlands and the U.K.) and also for the Euro area (12 countries) aggregate and the U.S. Sample consists of 32 years of quarterly observations and covers the periods 1970:1 to 2001:1.

We estimate model as described above with $N = 2$, applying it to bivariate combinations of GDP series for the Euro area and the individual European countries. In addition we consider some bivariate combinations that involve GDP series of the U.S.
Italy

(i)

(ii)

(iii)

(iv)

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France

(i)

(ii)

(iii)

(iv)
The Netherlands

(i)

(ii)

(iii)

(iv)
France and Germany display a high degree of association with the Euro area across the sample. Also, their cycles are now synchronised with those of the Euro area, although Germany displayed a leading cycle in the 1970’s.

Spain, Italy and the Netherlands had a relevant increase in the association with the Euro area, reaching levels of association close to those of Germany and France in the end of the sample.

Spain and Italy became more synchronised with the Euro area while The Netherlands displayed a small lead in the end of the sample.
United Kingdom

(i)

(ii)

(iii)

(iv)
The U.K. leads the Euro area by a bit more than three quarters in most of the sample. Only at the end of the sample the phase shift is estimated as zero. Phase adjusted correlation and contemporaneous correlation are also much lower than in previous cases but an increase in these measures of association takes place, reaching almost 1 in the end of the sample.

This increased synchronisation and association of the U.K. with the Euro area in the last 5 to 6 years were not reported so far.
United States

(i) Cycle US
(ii) Cycle Euro
(iii) Measuring Synchronisation and Convergence of Business Cycles – p.29
With respect to the U.S., a lead of 3 quarters compared to Euro area is estimated in most parts of the sample. Only until 1972 the phase effect is estimated as zero. Phase-adjusted correlation increases steadily. Contemporaneous correlation varies between 0.4 and 0.7 and are strikingly low.
References


